

6.5 EXERCISES

BMED6420

Brani Vidakovic; Fall 2018

Consult the class slides, hints, and cited literature for the solution of exercise problems.

1. Bayes' Net - Pancreatic Cancer.

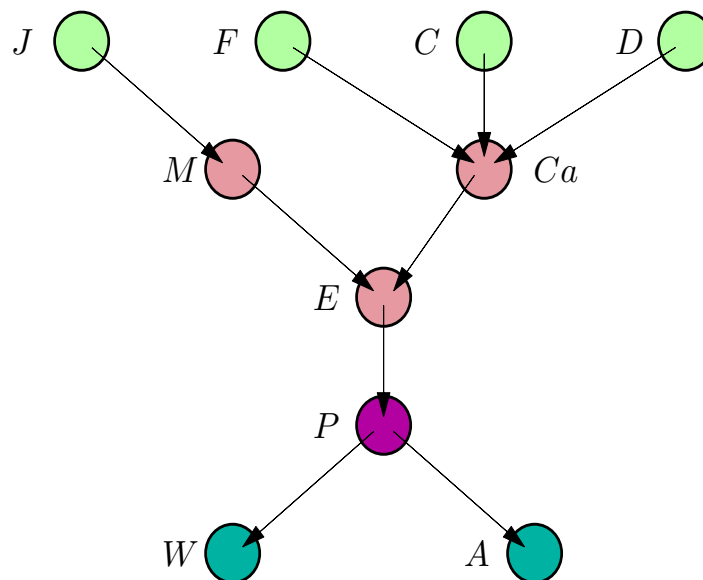
Project by BME Student Team: Claire Alpaugh, Jeff Bair, Catherine Gu, Sheng Jiang, Yeonghoon Joung, Saswat Panda, Sarah Reed, Jose Antonio Vasquez

Pancreatic Cancer 4th leading cause of cancer deaths in the US. Majority of diagnoses happen in late stage

1 Year survival rate: 25%

5 Year survival rate: 6%

Risk Factor	Pancreatic Cancer (P)	
	Medical test	Symptoms
Diabetes (D)	EUS-FNA (E)	Abdominal Pain (A)
Family History (F)	Antigen CA 19-9 (Ca)	Weight Loss (W)
Chronic Pancreatitis (C)	MRI (M)	
Jaundice (J)		



```

model
{
jaundice ~ dcat(p.jaundice [ ] );
family ~ dcat(p.family [ ] );
pancreatitis ~ dcat(p.pancreatitis [ ] );
diabetes ~ dcat(p.diabetes [ ] );
mri ~ dcat(p.mri [ jaundice, ] );
ca19 ~ dcat(p.ca19 [ family, pancreatitis, diabetes, ] );
eus ~ dcat(p.eus [ mri, ca19, ] );
cancer ~ dcat(p.cancer [ eus, ] );
weight ~ dcat(p.weight [ cancer, ] );
abdominal ~ dcat( p.abdominal [cancer,]);
}

list(p.jaundice= c( 0.9996, 0.0004),
p.family= c(0.50,0.50),
p.pancreatitis= c(0.9981,0.0019),
p.diabetes=c(0.917,0.083),
p.mri= structure(.Data = c(0.99, 0.01,0.20, 0.80), .Dim = c(2,2) ),
p.ca19=structure(.Data = c(0.9999, 0.0001, 0.60, 0.40, 0.50,0.50,
0.25, 0.80, 0.35,0.65, 0.30, 0.70, 0.30, 0.70, 0.15, 0.85), .Dim =c(2,2,2,2) ),
p.eus= structure(.Data = c(0.999, 0.001, 0.20, 0.80, 0.40, 0.60, 0.10, 0.90),
.Dim = c(2,2,2) ),
p.cancer= structure(.Data = c(0.90, 0.10, 0.40, 0.60), .Dim =c(2,2) ),
p.weight= structure(.Data = c(0.50, 0.50, 0.05, 0.95), .Dim =c(2,2) ),
p.abdominal = structure(.Data = c(0.50, 0.50, 0.10, 0.90), .Dim =c(2,2) ),
family=(nhe, 1, or 2), pancreatitis =(nhe, 1, or 2), eus =(nhe, 1, or 2),
ca19 =(nhe, 1, or 2),mri =(nhe, 1, or 2), jaundice =(nhe, 1, or 2),
diabetes =(nhe, 1, or 2), abdominal =(nhe, 1, or 2))
#nhe= no hard evidence/just commented out

```

1. Sally is a healthy individual with no prominent risk factors or biomarkers for pancreatic cancer. What is the probability that she will get pancreatic cancer?

Scenario 1 (hard evidence): family = 1, pancreatitis = 1, eus = 1, mri = 1, jaundice = 1, diabetes = 1, ca19 = 1, weight = 1, abdominal = 1

Probability: 0.3%

2. Robert has diabetes and has a family history of pancreatic cancer. Because of these risk factors, he underwent a test for pancreatic scan and was found to have a positive MRI. What is the probability that he will get pancreatic cancer?

Scenario 2 (hard evidence): family = 2, pancreatitis = 1, mri = 2, jaundice = 1, diabetes = 2, weight = 1, abdominal = 1

Probability: 2.1%

3. Jennifer has a family history of pancreatic cancer as well as chronic pancreatitis. It is known that she has high levels of the biomarker CA 19-9 in the body. She has recently been experiencing abdominal pains. What is the probability that she will get pancreatic cancer?

Scenario 3 (hard evidence): family = 2, pancreatitis = 2, jaundice = 1, diabetes = 1, ca19 = 2, weight = 1, abdominal = 2

Probability: 15.7%

4. Sam has lost weight recently due to the fact that he has been diagnosed with jaundice. Because jaundice is a risk factor to pancreatic cancer he had a screening done. One of the results from his screening stated that he had elevated levels of CA 19-9. What is the probability that he has pancreatic cancer?

Scenario 4 (hard evidence): family = 1, pancreatitis = 1, jaundice = 2, diabetes = 1, ca19 = 2, weight = 2, abdominal = 1

Probability: 31.2%

5. Ava has been developing many health problems recently. She has been diagnosed with diabetes, chronic pancreatitis, and jaundice. She also has a family history of pancreatic cancer. She has recently been having abdominal pains as well as unexpected weight loss. When tested for pancreatic cancer the results show elevated levels for CA 19-9 and positive results with both EUS-FNA(biopsy) and MRI. What is the probability she has pancreatic cancer?

Scenario 5 (hard evidence): family = 2, pancreatitis = 2, eus = 2, mri = 2, jaundice = 2, diabetes = 2, ca19 = 2, weight = 2, abdominal = 2

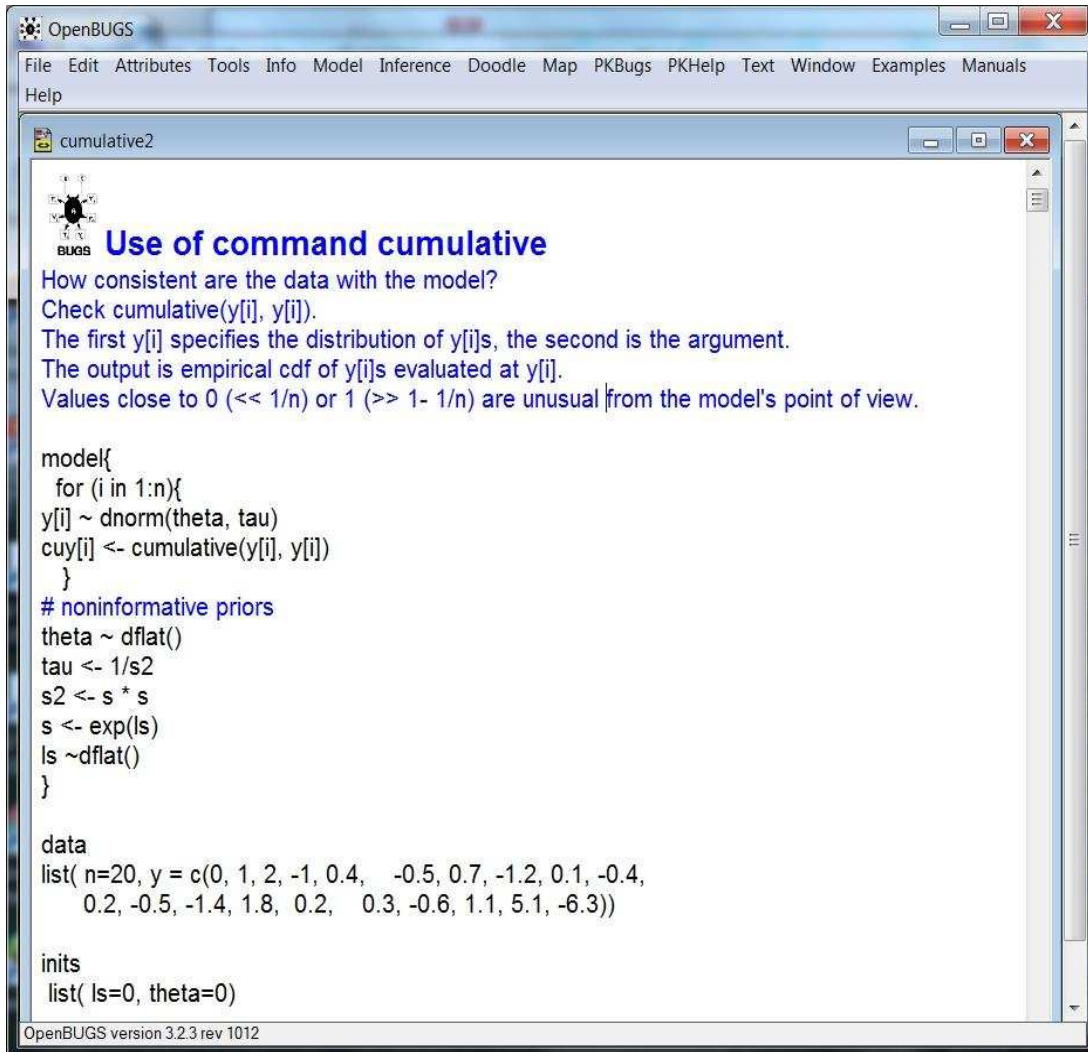
Probability: 83.9%

References

- Lee, Michael X., and Muhammad W. Siah. "Screening for Early Pancreatic Ductal Adenocarcinoma: An Urgent Call!" *J. Pancreas* (online) 10.2 (2009): 104-08. 17 Mar. 2009. Web. 8 Apr. 2012.
- Permuth-Wey, Jennifer, and Kathleen M. Egan. "Family History Is a Significant Risk Factor for Pancreatic Cancer: Results from a Systematic Review and Meta-analysis." *Familial Cancer* 117th ser. 8.109 (2008). 2 Sept. 2008. Web. 8 Apr. 2012.
- Turowska, Aldona, Urszula Lebkowska, Bozena Kubas, Jacek R. Janica, Jerzy R. Ladny, and Kazimierz Kordecki. "The Role of Magnetic Resonance Imaging (MRI) with Magnetic Resonance Cholangiopancreatography (MRCP) in the Diagnosis and Assessment of Resectability of Pancreatic Tumors." *Med Sci Monit* 13: 90-97. 2007. Web. 8 Apr. 2012.
- Everhart, J. and D. Wright (1995). "Diabetes Mellitus as a Risk Factor for Pancreatic Cancer." *JAMA: The Journal of the American Medical Association* 273(20): 1605-1609.
- Fuchs, CS, GA Colditz, MJ Stampfer, EL Giovannucci, DJ Hunter, EB Rimm, WC Willett, and Fe Speizer. "A Prospective Study of Cigarette Smoking and the Risk of Pancreatic Cancer." *Archives of Internal Medicine* 156.19 (1996): 2255-266. Print.

Fernandez, E., C. La Vecchia, et al. (1995). "Pancreatitis and the risk of pancreatic cancer." *Pancreas* 11(2): 185-189.

Use of Command *cumulative*.



OpenBUGS

File Edit Attributes Tools Info Model Inference Doodle Map PKBugs PKHelp Text Window Examples Manuals Help

cumulative2

Use of command cumulative

How consistent are the data with the model?
Check `cumulative(y[i], y[i])`.
The first `y[i]` specifies the distribution of `y[i]`s, the second is the argument.
The output is empirical cdf of `y[i]`s evaluated at `y[i]`.
Values close to 0 ($\ll 1/n$) or 1 ($\gg 1 - 1/n$) are unusual from the model's point of view.

```
model{
  for (i in 1:n){
    y[i] ~ dnorm(theta, tau)
    cuy[i] <- cumulative(y[i], y[i])
  }
  # noninformative priors
  theta ~ dflat()
  tau <- 1/s2
  s2 <- s * s
  s <- exp(ls)
  ls ~dflat()
}
```

data


```
list( n=20, y = c(0, 1, 2, -1, 0.4, -0.5, 0.7, -1.2, 0.1, -0.4,
  0.2, -0.5, -1.4, 1.8, 0.2, 0.3, -0.6, 1.1, 5.1, -6.3))
```

inits

```
list( ls=0, theta=0)
```

OpenBUGS version 3.2.3 rev 1012

How Many Trick.



"HOW MANY" TRICK

Suppose the cost of a repair is gamma distributed with mean \$100 and standard deviation \$50.
The alotted amount for repairs is \$1000 dollars. How many repairs can you have untill the alotment is depleted.
 $100 = EY = a/b$; $50 = \text{sigma}(Y) = \sqrt{a}/b \rightarrow a=4, b=1/25=0.04$

Start with N large -- say 20, so 1000/100 is much smaller.

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```
model {  
  for (i in 1:20) {Y[i] ~ dgamma(4, 0.04)}  
  cum[1]      <- Y[1]  
  for (i in 2:20) {  
    cum[i]      <- cum[i - 1] + Y[i]  
  }  
  for (i in 1:20) {  
    cum.step[i]  <- i*step(1000 - cum[i])  
  }  
  number <- ranked(cum.step[], 20) # maximum number in cum.step  
  check <- equals(cum.step[20], 0) # always 1 if N=20 big enough  
}
```

Birnbaum-Saunders Distribution.

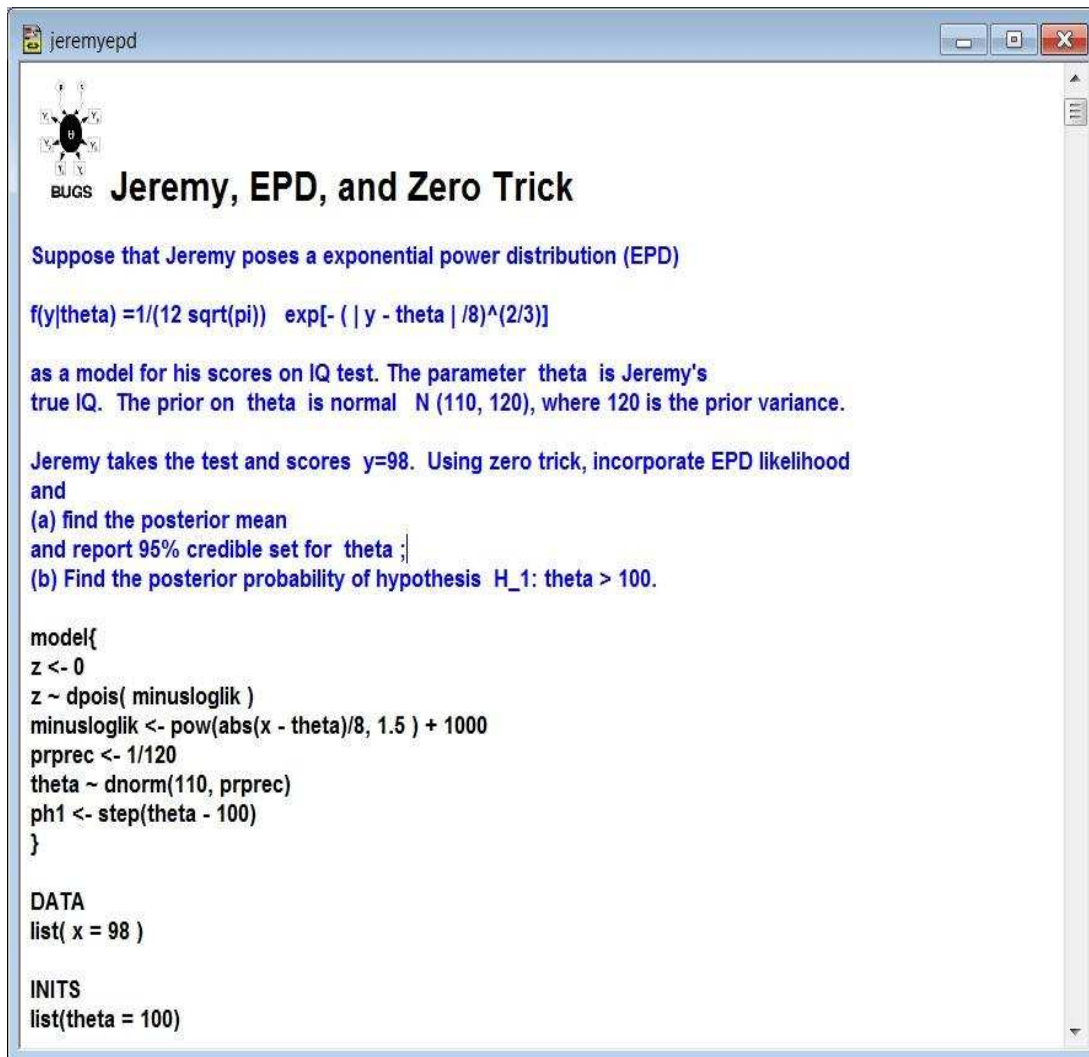
The screenshot shows the OpenBUGS software window. The title bar reads "OpenBUGS". The menu bar includes "File", "Edit", "Attributes", "Tools", "Info", "Model", "Inference", "Doodle", "Map", "PKBugs", "PKHelp", "Text", "Window", "Examples", and "Manuals". Below the menu bar is a "Help" button. The main window has a title bar "bisazerotrick". Inside, there is a BUGS logo and a diagram of a directed acyclic graph (DAG) with nodes θ , α , β , and Y_i . The text "Birnbaum-Saunders Model via Zero Trick" is displayed in blue. Below this, the probability density function is given as:

$$f(x) = \frac{1}{\sqrt{2\pi x^3}} \frac{(x + \beta)}{2\alpha\sqrt{\beta}} \exp\left\{-\frac{1}{2\alpha^2}\left(\frac{x}{\beta} + \frac{\beta}{x} - 2\right)\right\} \quad ; \quad x > 0, (\alpha, \beta) > 0$$

Below the formula, the BUGS model code is shown:

```
model {  
  for ( i in 1:n){  
    z[i] <- 0  
    z[i] ~ dpois(lambda[i])  
    lambda[i] <- 3/2 * log(x[i]) - log(x[i] + beta) + 1/(2 * alpha * alpha) * (x[i]/beta - 2 +  
    beta/x[i])+1000  
  }  
  alpha ~ dgamma(0.01, 0.01)  
  beta ~ dgamma(0.01, 0.01)  
}
```

Jeremy and Exponential Power Distribution.



BUGS **Jeremy, EPD, and Zero Trick**

Suppose that Jeremy poses a exponential power distribution (EPD)

$$f(y|\theta) = 1/(12 \sqrt{\pi}) \exp[-(|y - \theta|/8)^{2/3}]$$

as a model for his scores on IQ test. The parameter θ is Jeremy's true IQ. The prior on θ is normal $N(110, 120)$, where 120 is the prior variance.

Jeremy takes the test and scores $y=98$. Using zero trick, incorporate EPD likelihood and

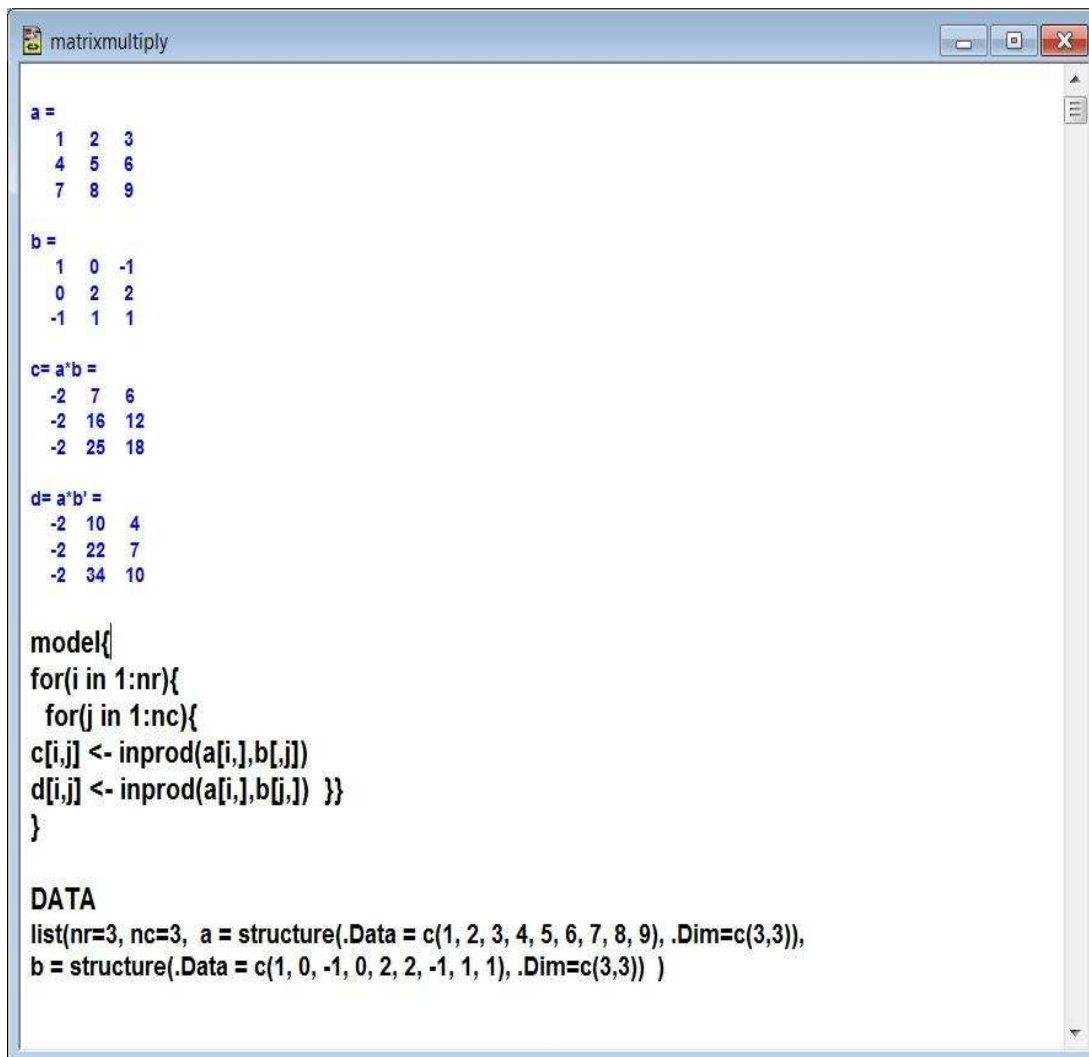
(a) find the posterior mean
and report 95% credible set for θ ;
(b) Find the posterior probability of hypothesis $H_1: \theta > 100$.

```
model{
  z <- 0
  z ~ dpois( minusloglik )
  minusloglik <- pow(abs(x - theta)/8, 1.5 ) + 1000
  prprec <- 1/120
  theta ~ dnorm(110, prprec)
  ph1 <- step(theta - 100)
}
```

DATA
list(x = 98)

INITS
list(theta = 100)

Multiply Matrices.



```
matrixmultiply

a =
  1 2 3
  4 5 6
  7 8 9

b =
  1 0 -1
  0 2 2
 -1 1 1

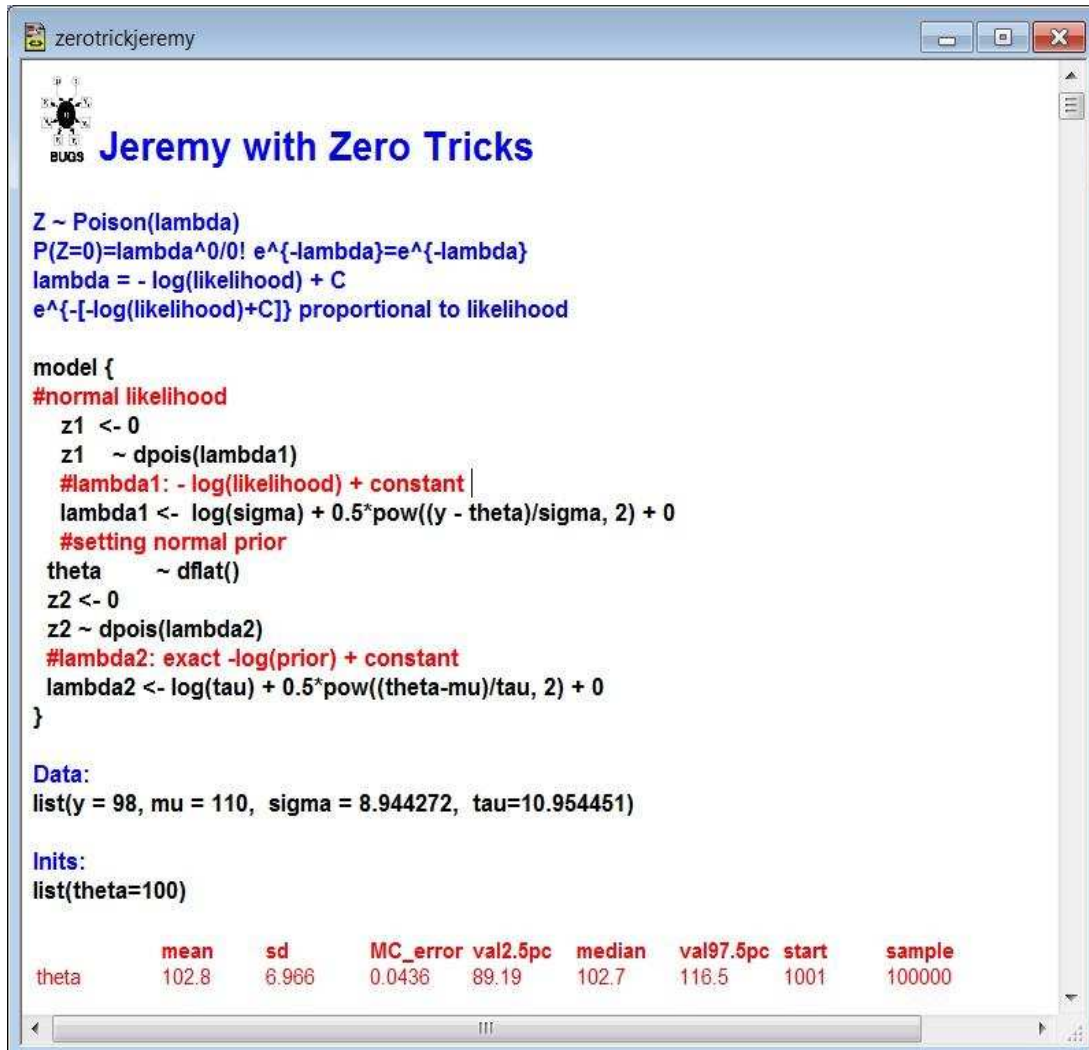
c= a*b =
 -2  7  6
 -2 16 12
 -2 25 18

d= a*b' =
 -2 10  4
 -2 22  7
 -2 34 10

model{
  for(i in 1:nr){
    for(j in 1:nc){
      c[i,j] <- inprod(a[i,],b[,j])
      d[i,j] <- inprod(a[i,],b[j,]) }}
}

DATA
list(nr=3, nc=3, a = structure(.Data = c(1, 2, 3, 4, 5, 6, 7, 8, 9), .Dim=c(3,3)),
b = structure(.Data = c(1, 0, -1, 0, 2, 2, -1, 1, 1), .Dim=c(3,3)) )
```

Jeremy via Zero Trick.



```
zerotrickjeremy

# BUGS

Jeremy with Zero Tricks

Z ~ Poison(lambda)
P(Z=0)=lambda^0/0! e^{-lambda}=e^{-lambda}
lambda = - log(likelihood) + C
e^{-[-log(likelihood)+C]} proportional to likelihood

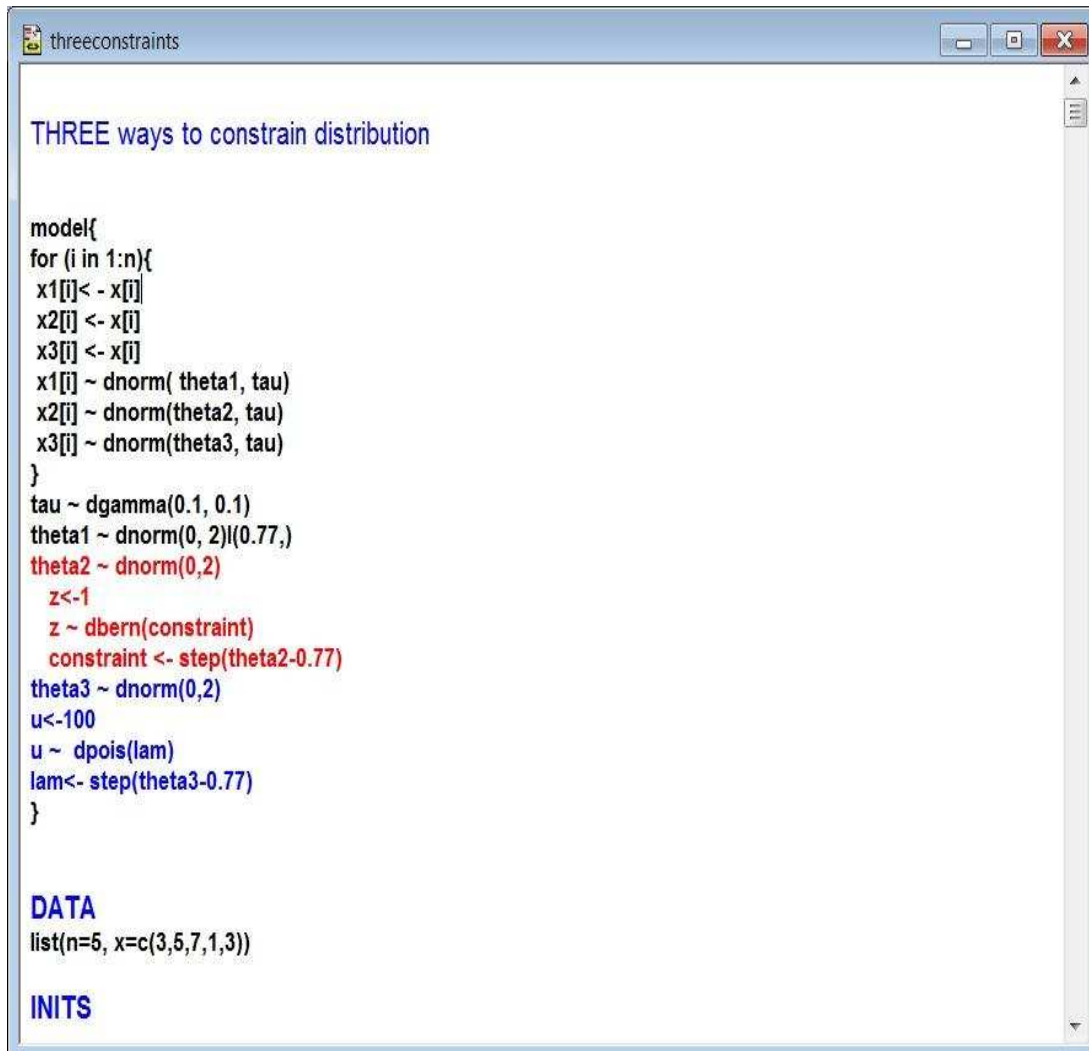
model {
  #normal likelihood
  z1 <- 0
  z1 ~ dpois(lambda1)
  #lambda1: - log(likelihood) + constant |
  lambda1 <- log(sigma) + 0.5*pow((y - theta)/sigma, 2) + 0
  #setting normal prior
  theta ~ dflat()
  z2 <- 0
  z2 ~ dpois(lambda2)
  #lambda2: exact -log(prior) + constant
  lambda2 <- log(tau) + 0.5*pow((theta-mu)/tau, 2) + 0
}

Data:
list(y = 98, mu = 110, sigma = 8.944272, tau=10.954451)

Inits:
list(theta=100)

theta      mean      sd      MC_error val2.5pc  median  val97.5pc start  sample
theta      102.8    6.966    0.0436   89.19    102.7   116.5    1001   100000
```

Restrict the Distribution.



```
threeconstraints

THREE ways to constrain distribution

model{
  for (i in 1:n){
    x1[i] <- - x[i]
    x2[i] <- x[i]
    x3[i] <- x[i]
    x1[i] ~ dnorm( theta1, tau)
    x2[i] ~ dnorm(theta2, tau)
    x3[i] ~ dnorm(theta3, tau)
  }
  tau ~ dgamma(0.1, 0.1)
  theta1 ~ dnorm(0, 2)|!(0.77,)
  theta2 ~ dnorm(0,2)
  z<-1
  z ~ dbern(constraint)
  constraint <- step(theta2-0.77)
  theta3 ~ dnorm(0,2)
  u<-100
  u ~ dpois(lam)
  lam<- step(theta3-0.77)
}

DATA
list(n=5, x=c(3,5,7,1,3))

INITS
```

Moon Illusion Make Doodle Plot that corresponds to the code bellow.

Kaufman and Rock (1962) concluded that the commonly observed fact that the moon near the horizon appears larger than does the moon at its zenith (highest point overhead) could be explained on the basis of the greater apparent distance of the moon when it is at the horizon. As part of a very complete series of experiments, the authors initially sought to estimate the moon horizon so as to match the size of a standard “moon” that appeared at its zenith, or vice versa. (In these measurements, they used not the actual moon but an artificial one created with special apparatus.) One of the first questions we might ask is whether there

really is a moon illusion - that is, whether a larger setting is required to match a horizon moon or a zenith moon. The following data for 10 subjects are taken from Kaufman and Rock's paper and represent the ratio of the diameter of the variable and standard moons. A ratio of 1.00 would indicate no illusion, whereas a ratio other than 1.00 would represent an illusion. (For example, a ratio of 1.50 would mean that the horizon moon appeared to have a diameter 1.50 times the diameter of the zenith moon.)

Evidence in support of an illusion would require that we reject $H_0 : \mu = 1.00$ in favor of $H_1 : \mu > 1.00$.

```
model{
  for (i in 1:n){
    X[i] ~ dnorm(mu, prec)
  }
  mu ~ dnorm(0, 0.00001)
  prec ~ dgamma(0.0001, 0.0001)
  sigma <- 1/sqrt(prec)
  #TEST
  prH1 <- step(mu - 1)
}
DATA
list(n=10, X=c(1.73, 1.06, 2.03, 1.40, 0.95, 1.13, 1.41, 1.73, 1.63, 1.56) )

INITS
list(mu = 0, prec=1)
```

Kaufman, L. and Rock, I. (1962). The moon illusion I. Science, 136, 953--961.