

6.5 EXERCISES

BMED6420

Brani Vidakovic; Fall 2018

Consult the class slides, hints, and cited literature for the solution of exercise problems.

1. Bayes' Net - Pancreatic Cancer.

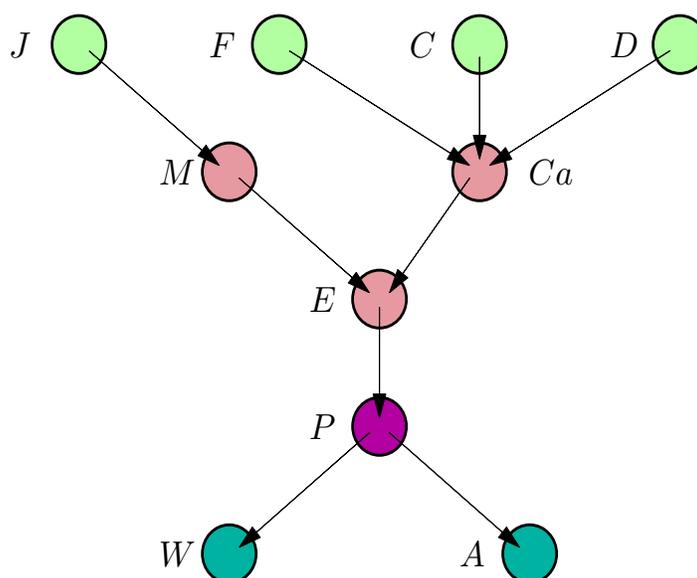
Project by BME Student Team: Claire Alpaugh, Jeff Bair, Catherine Gu, Sheng Jiang, Yeonghoon Joung, Saswat Panda, Sarah Reed, Jose Antonio Vasquez

Pancreatic Cancer 4th leading cause of cancer deaths in the US. Majority of diagnoses happen in late stage

1 Year survival rate: 25%

5 Year survival rate: 6%

Pancreatic Cancer (P)		
Risk Factor	Medical test	Symptoms
Diabetes (D)	EUS-FNA (E)	Abdominal Pain (A)
Family History (F)	Antigen CA 19-9 (Ca)	Weight Loss (W)
Chronic Pancreatitis (C)	MRI (M)	
Jaundice (J)		



```

model
{
jaundice ~ dcat(p.jaundice [ ] );
family ~ dcat(p.family [ ] );
pancreatitis ~ dcat(p.pancreatitis [ ] );
diabetes ~ dcat(p.diabetes [ ] );
mri ~ dcat(p.mri [ jaundice, ] );
ca19 ~ dcat(p.ca19 [ family, pancreatitis, diabetes, ] );
eus ~ dcat(p.eus [ mri, ca19, ] );
cancer ~ dcat(p.cancer [ eus, ] );
weight ~ dcat(p.weight [ cancer, ] );
abdominal ~ dcat( p.abdominal [cancer,]);
}

list(p.jaundice= c( 0.9996, 0.0004),
p.family= c(0.50,0.50),
p.pancreatitis= c(0.9981,0.0019),
p.diabetes=c(0.917,0.083),
p.mri= structure(.Data = c(0.99, 0.01,0.20, 0.80), .Dim = c(2,2) ),
p.ca19=structure(.Data = c(0.9999, 0.0001, 0.60, 0.40, 0.50,0.50,
0.25, 0.80, 0.35,0.65, 0.30, 0.70, 0.30, 0.70, 0.15, 0.85), .Dim =c(2,2,2,2) ),
p.eus= structure(.Data = c(0.999, 0.001, 0.20, 0.80, 0.40, 0.60, 0.10, 0.90),
.Dim = c(2,2,2) ),
p.cancer= structure(.Data = c(0.90, 0.10, 0.40, 0.60), .Dim =c(2,2) ),
p.weight= structure(.Data = c(0.50, 0.50, 0.05, 0.95), .Dim =c(2,2) ),
p.abdominal = structure(.Data = c(0.50, 0.50, 0.10, 0.90), .Dim =c(2,2) ),
family=(nhe, 1, or 2), pancreatitis =(nhe, 1, or 2), eus =(nhe, 1, or 2),
ca19 =(nhe, 1, or 2),mri =(nhe, 1, or 2), jaundice =(nhe, 1, or 2),
diabetes =(nhe, 1, or 2), abdominal =(nhe, 1, or 2))
#nhe= no hard evidence/just commented out

```

1. Sally is a healthy individual with no prominent risk factors or biomarkers for pancreatic cancer. What is the probability that she will get pancreatic cancer?

Scenario 1 (hard evidence): family = 1, pancreatitis = 1, eus = 1, mri = 1, jaundice = 1, diabetes = 1, ca19 = 1, weight = 1, abdominal = 1

Probability: 0.3%

2. Robert has diabetes and has a family history of pancreatic cancer. Because of these risk factors, he underwent a test for pancreatic scan and was found to have a positive MRI. What is the probability that he will get pancreatic cancer?

Scenario 2 (hard evidence): family = 2, pancreatitis = 1, mri = 2, jaundice = 1, diabetes = 2, weight = 1, abdominal = 1

Probability: 2.1%

3. Jennifer has a family history of pancreatic cancer as well as chronic pancreatitis. It is known that she has high levels of the biomarker CA 19-9 in the body. She has recently been experiencing abdominal pains. What is the probability that she will get pancreatic cancer?

Scenario 3 (hard evidence): family = 2, pancreatitis = 2, jaundice = 1, diabetes = 1, ca19 = 2, weight = 1, abdominal = 2

Probability: 15.7%

4. Sam has lost weight recently due to the fact that he has been diagnosed with jaundice. Because jaundice is a risk factor to pancreatic cancer he had a screening done. One of the results from his screening stated that he had elevated levels of CA 19-9. What is the probability that he has pancreatic cancer?

Scenario 4 (hard evidence): family = 1, pancreatitis = 1, jaundice = 2, diabetes = 1, ca19 = 2, weight = 2, abdominal = 1

Probability: 31.2%

5. Ava has been developing many health problems recently. She has been diagnosed with diabetes, chronic pancreatitis, and jaundice. She also has a family history of pancreatic cancer. She has recently been having abdominal pains as well as unexpected weight loss. When tested for pancreatic cancer the results show elevated levels for CA 19-9 and positive results with both EUS-FNA(biopsy) and MRI. What is the probability she has pancreatic cancer?

Scenario 5 (hard evidence): family = 2, pancreatitis = 2, eus = 2, mri = 2, jaundice = 2, diabetes = 2, ca19 = 2, weight = 2, abdominal = 2

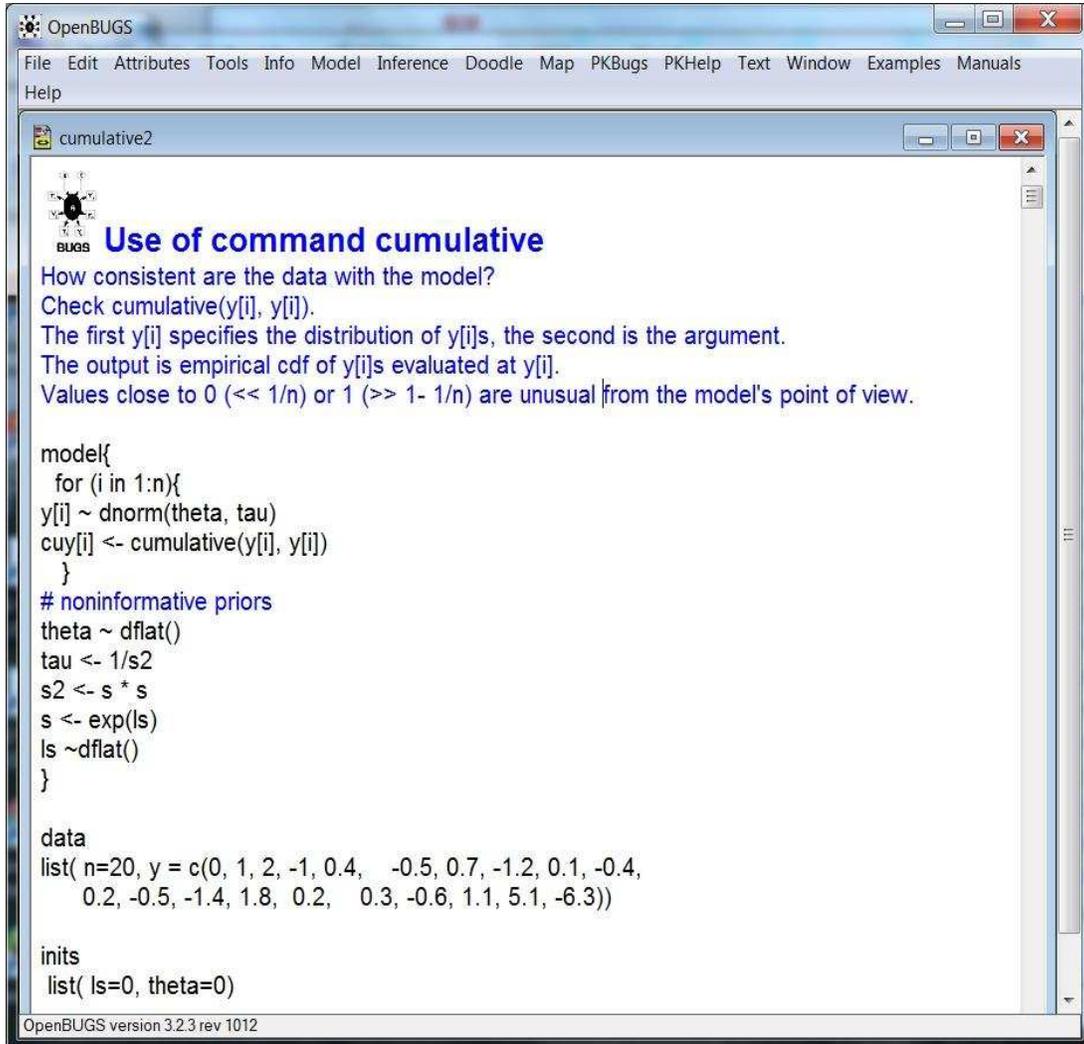
Probability: 83.9%

References

- Lee, Michael X., and Muhammad W. Siaf. "Screening for Early Pancreatic Ductal Adenocarcinoma: An Urgent Call!" *J. Pancreas* (online) 10.2 (2009): 104-08. 17 Mar. 2009. Web. 8 Apr. 2012.
- Permeth-Wey, Jennifer, and Kathleen M. Egan. "Family History Is a Significant Risk Factor for Pancreatic Cancer: Results from a Systematic Review and Meta-analysis." *Familial Cancer* 117th ser. 8.109 (2008). 2 Sept. 2008. Web. 8 Apr. 2012.
- Turowska, Aldona, Urszula Lebkowska, Bozena Kubas, Jacek R. Janica, Jerzy R. Ladny, and Kazimierz Kordecki. "The Role of Magnetic Resonance Imaging (MRI) with Magnetic Resonance Cholangiopancreatography (MRCP) in the Diagnosis and Assessment of Resectability of Pancreatic Tumors." *Med Sci Monit* 13: 90-97. 2007. Web. 8 Apr. 2012.
- Everhart, J. and D. Wright (1995). "Diabetes Mellitus as a Risk Factor for Pancreatic Cancer." *JAMA: The Journal of the American Medical Association* 273(20): 1605-1609.
- Fuchs, CS, GA Colditz, MJ Stampfer, EL Giovannucci, DJ Hunter, EB Rimm, WC Willett, and Fe Speizer. "A Prospective Study of Cigarette Smoking and the Risk of Pancreatic Cancer." *Archives of Internal Medicine* 156.19 (1996): 2255-266. Print.

Fernandez, E., C. La Vecchia, et al. (1995). "Pancreatitis and the risk of pancreatic cancer." *Pancreas* 11(2): 185-189.

Use of Command *cumulative*.



OpenBUGS

File Edit Attributes Tools Info Model Inference Doodle Map PKBugs PKHelp Text Window Examples Manuals Help

cumulative2

Use of command cumulative

How consistent are the data with the model?
Check `cumulative(y[i], y[i])`.
The first `y[i]` specifies the distribution of `y[i]`s, the second is the argument.
The output is empirical cdf of `y[i]`s evaluated at `y[i]`.
Values close to 0 ($\ll 1/n$) or 1 ($\gg 1 - 1/n$) are unusual from the model's point of view.

```
model{
  for (i in 1:n){
    y[i] ~ dnorm(theta, tau)
    cuy[i] <- cumulative(y[i], y[i])
  }
  # noninformative priors
  theta ~ dflat()
  tau <- 1/s2
  s2 <- s * s
  s <- exp(ls)
  ls ~dflat()
}
```

data

```
list( n=20, y = c(0, 1, 2, -1, 0.4, -0.5, 0.7, -1.2, 0.1, -0.4,
  0.2, -0.5, -1.4, 1.8, 0.2, 0.3, -0.6, 1.1, 5.1, -6.3))
```

inits

```
list( ls=0, theta=0)
```

OpenBUGS version 3.2.3 rev 1012

How Many Trick.



“HOW MANY” TRICK

Suppose the cost of a repair is gamma distributed with mean \$100 and standard deviation \$50.
The alotted amount for repairs is \$1000 dollars. How many repairs can you have until the alotment is depleted.
 $100=EY= a/b$; $50=\sigma(Y)= \sqrt{a}/b \rightarrow a=4, b=1/25=0.04$

Start with N large -- say 20, so 1000/100 is much smaller.

The BUGS Book Page 26

```
model {
  for (i in 1:20) {Y[i] ~ dgamma(4, 0.04)}
  cum[1]      <- Y[1]
  for (i in 2:20) {
    cum[i]    <- cum[i - 1] + Y[i]
  }
  for (i in 1:20) {
    cum.step[i] <- i*step(1000 - cum[i])
  }
  number <- ranked(cum.step[], 20) # maximum number in cum.step
  check <- equals(cum.step[20], 0) # always 1 if N=20 big enough
}
```

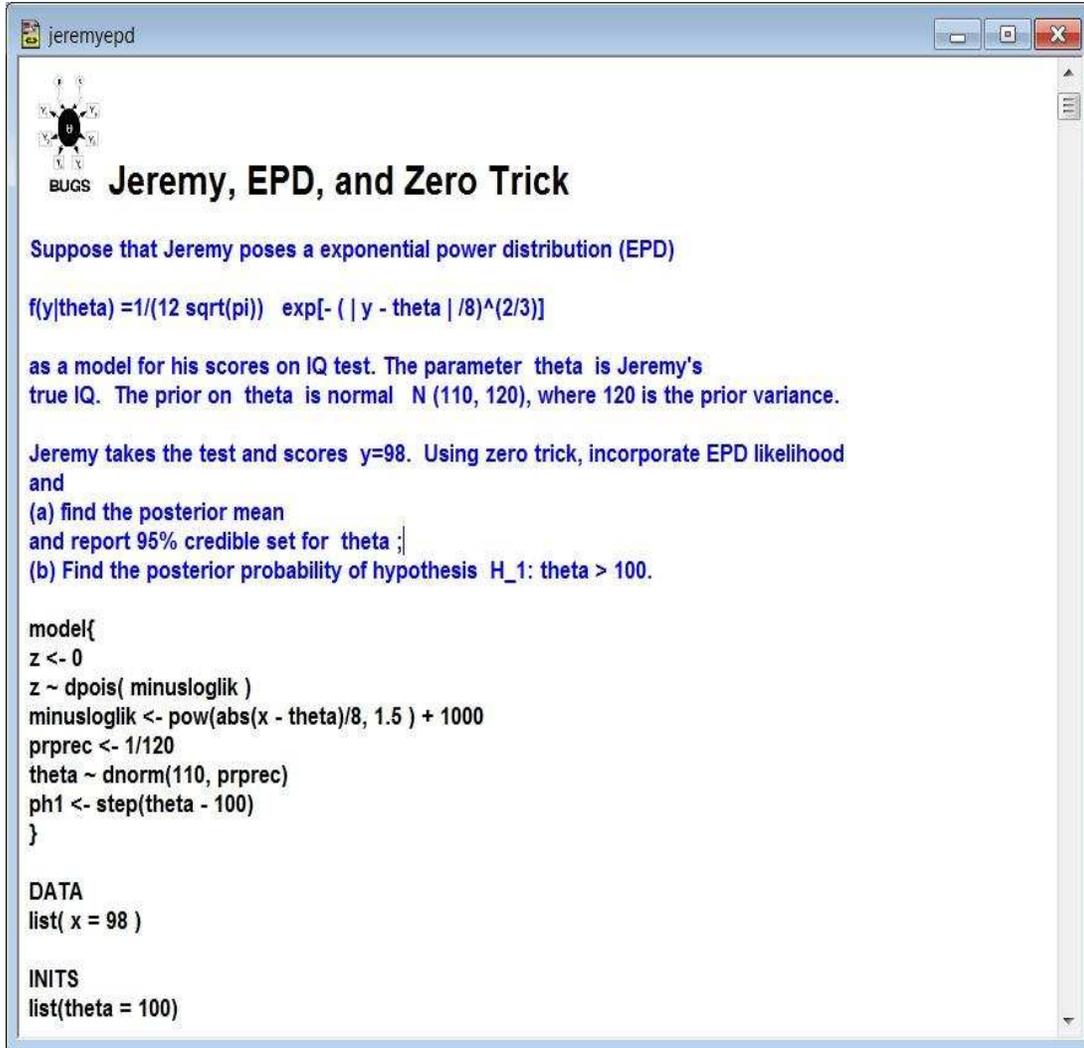
Birnbaum-Saunders Distribution.

The screenshot shows the OpenBUGS interface. The main window contains a diagram of the model structure with nodes μ , σ , θ , Y_1 , Y_2 , Y_3 , Y_4 , and Y_5 . Below the diagram is the title "Birnbaum-Saunders Model via Zero Trick".

$$f(x) = \frac{1}{\sqrt{2\pi x^3}} \frac{(x+\beta)}{2\alpha\sqrt{\beta}} \exp\left\{-\frac{1}{2\alpha^2}\left(\frac{x}{\beta} + \frac{\beta}{x} - 2\right)\right\} ; \quad x > 0, (\alpha, \beta) > 0$$

```
model {  
  for ( i in 1:n){  
    z[i] <- 0  
    z[i] ~ dpois(lambda[i])  
    lambda[i] <- 3/2 * log(x[i]) - log(x[i] + beta) + 1/(2 * alpha * alpha) * (x[i]/beta - 2 +  
    beta/x[i])+1000  
  }  
  alpha ~ dgamma(0.01, 0.01)  
  beta ~ dgamma(0.01, 0.01)  
}
```

Jeremy and Exponential Power Distribution.



The screenshot shows a window titled "jeremyepd" with a BUGS logo and the title "Jeremy, EPD, and Zero Trick". The text describes a Bayesian model for Jeremy's IQ scores. The likelihood function is given as $f(y|\theta) = 1/(12 \sqrt{\pi}) \exp[-(|y - \theta|/8)^{2/3}]$. The prior for θ is normal with mean 110 and variance 120. The observed data is $y=98$. The model is defined by the following code:

```
model{
z <- 0
z ~ dpois( minusloglik )
minusloglik <- pow(abs(x - theta)/8, 1.5 ) + 1000
prprec <- 1/120
theta ~ dnorm(110, prprec)
ph1 <- step(theta - 100)
}
```

DATA

```
list( x = 98 )
```

INITS

```
list(theta = 100)
```

Multiply Matrices.

```
matrixmultiply

a =
  1  2  3
  4  5  6
  7  8  9

b =
  1  0 -1
  0  2  2
 -1  1  1

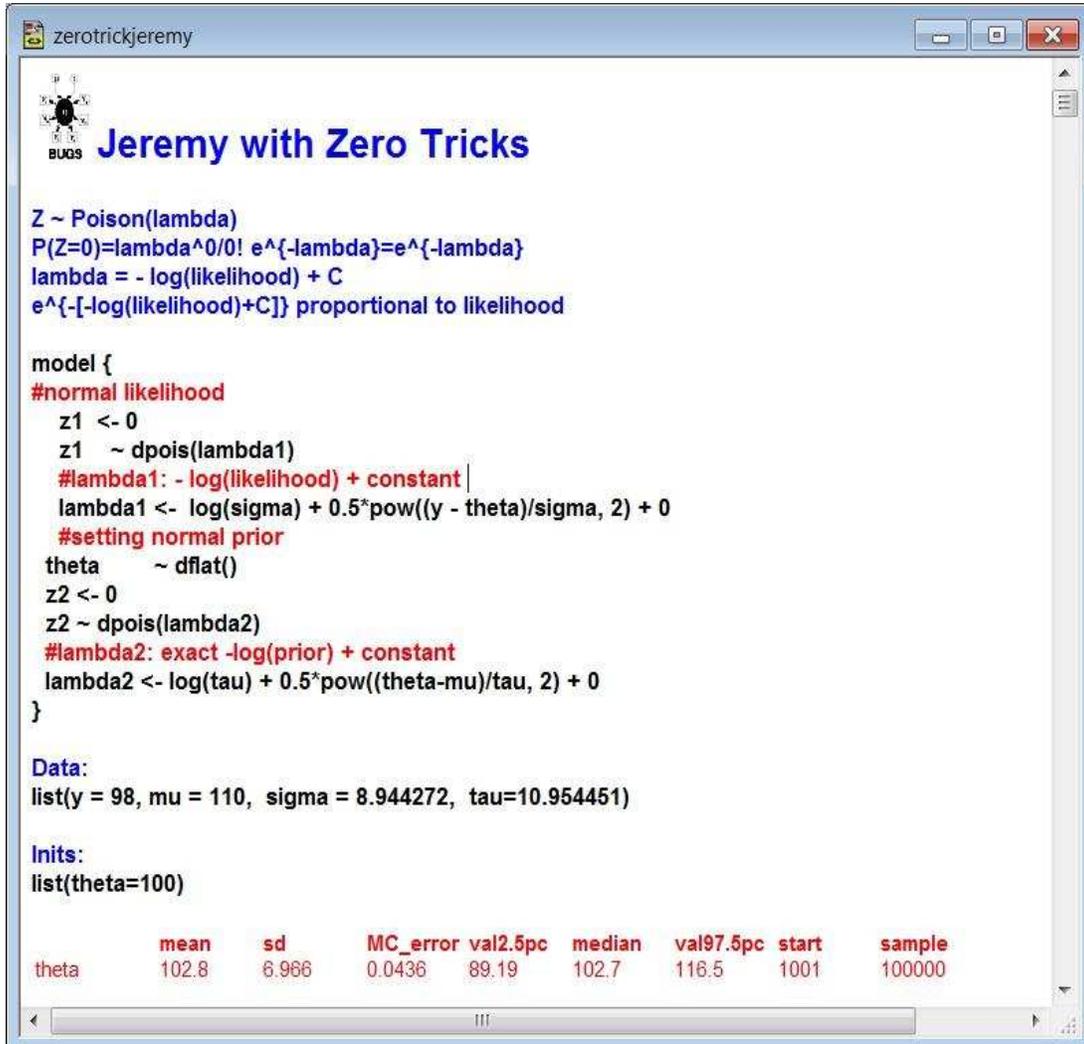
c= a*b =
 -2  7  6
 -2 16 12
 -2 25 18

d= a*b' =
 -2 10  4
 -2 22  7
 -2 34 10

model{
  for(i in 1:nr){
    for(j in 1:nc){
      c[i,j] <- inprod(a[i,],b[,j])
      d[i,j] <- inprod(a[i,],b[j,]) }}
}

DATA
list(nr=3, nc=3, a = structure(.Data = c(1, 2, 3, 4, 5, 6, 7, 8, 9), .Dim=c(3,3)),
b = structure(.Data = c(1, 0, -1, 0, 2, 2, -1, 1, 1), .Dim=c(3,3)) )
```

Jeremy via Zero Trick.



```
zerotrickjeremy
```

 **Jeremy with Zero Tricks**

$Z \sim \text{Poisson}(\lambda)$
 $P(Z=0) = \lambda^0 / 0! e^{-\lambda} = e^{-\lambda}$
 $\lambda = -\log(\text{likelihood}) + C$
 $e^{-[-\log(\text{likelihood}) + C]}$ proportional to likelihood

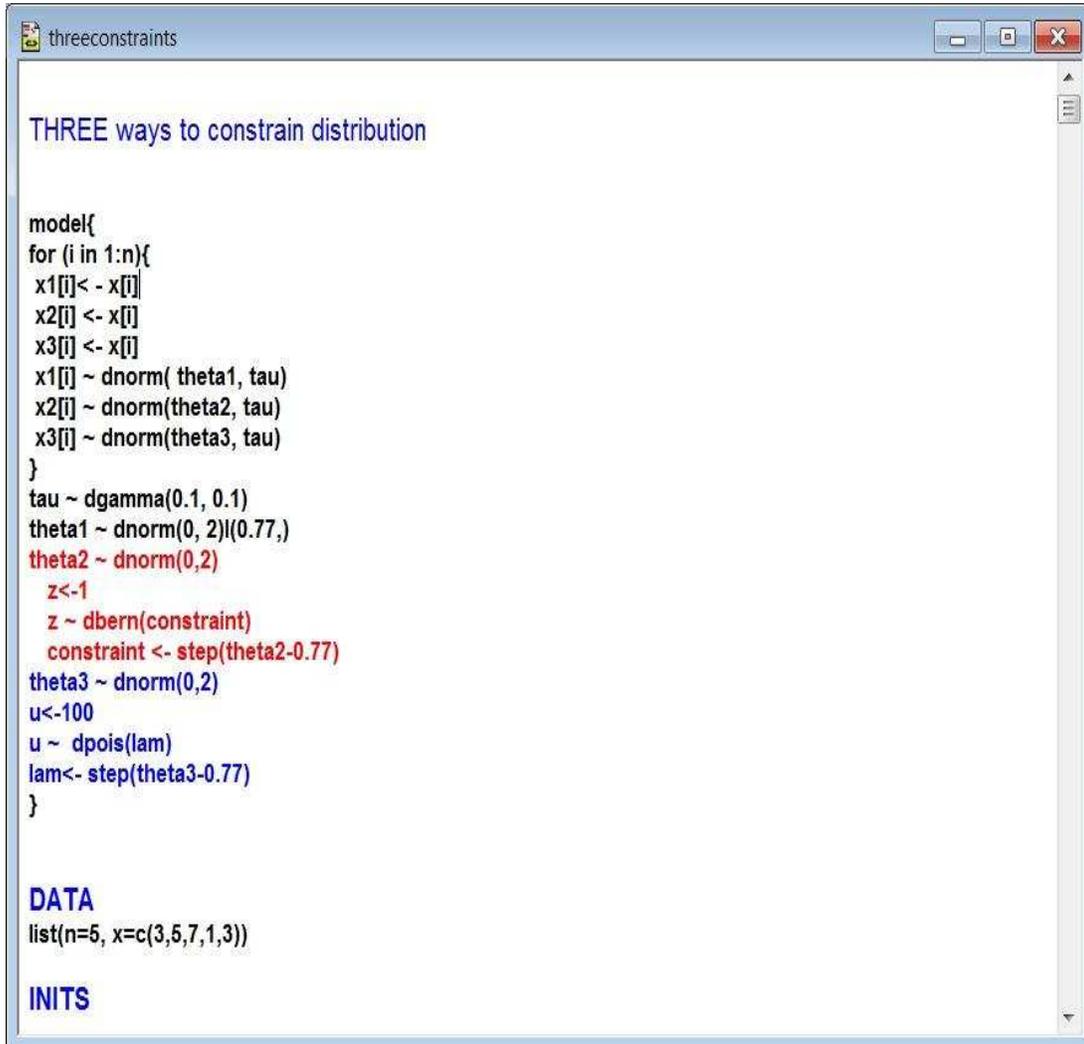
```
model {  
  #normal likelihood  
  z1 <- 0  
  z1 ~ dpois(lambda1)  
  #lambda1: - log(likelihood) + constant |  
  lambda1 <- log(sigma) + 0.5*pow((y - theta)/sigma, 2) + 0  
  #setting normal prior  
  theta ~ dflat()  
  z2 <- 0  
  z2 ~ dpois(lambda2)  
  #lambda2: exact -log(prior) + constant  
  lambda2 <- log(tau) + 0.5*pow((theta-mu)/tau, 2) + 0  
}
```

Data:
list(y = 98, mu = 110, sigma = 8.944272, tau=10.954451)

Inits:
list(theta=100)

	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
theta	102.8	6.966	0.0436	89.19	102.7	116.5	1001	100000

Restrict the Distribution.



```
threeconstraints

THREE ways to constrain distribution

model{
  for (i in 1:n){
    x1[i] <- x[i]
    x2[i] <- x[i]
    x3[i] <- x[i]
    x1[i] ~ dnorm(theta1, tau)
    x2[i] ~ dnorm(theta2, tau)
    x3[i] ~ dnorm(theta3, tau)
  }
  tau ~ dgamma(0.1, 0.1)
  theta1 ~ dnorm(0, 2)|(0.77,)
  theta2 ~ dnorm(0,2)
  z <- 1
  z ~ dbern(constraint)
  constraint <- step(theta2-0.77)
  theta3 ~ dnorm(0,2)
  u <- 100
  u ~ dpois(lam)
  lam <- step(theta3-0.77)
}

DATA
list(n=5, x=c(3,5,7,1,3))

INITS
```

Moon Illusion Make Doodle Plot that corresponds to the code bellow.

Kaufman and Rock (1962) concluded that the commonly observed fact that the moon near the horizon appears larger than does the moon at its zenith (highest point overhead) could be explained on the basis of the greater apparent distance of the moon when it is at the horizon. As part of a very complete series of experiments, the authors initially sought to estimate the moon horizon so as to match the size of a standard “moon” that appeared at its zenith, or vice versa. (In these measurements, they used not the actual moon but an artificial one created with special apparatus.) One of the first questions we might ask is whether there

really is a moon illusion - that is, whether a larger setting is required to match a horizon moon or a zenith moon. The following data for 10 subjects are taken from Kaufman and Rock's paper and represent the ratio of the diameter of the variable and standard moons. A ratio of 1.00 would indicate no illusion, whereas a ratio other than 1.00 would represent an illusion. (For example, a ratio of 1.50 would mean that the horizon moon appeared to have a diameter 1.50 times the diameter of the zenith moon.)

Evidence in support of an illusion would require that we reject $H_0 : \mu = 1.00$ in favor of $H_1 : \mu > 1.00$.

```
model{
for (i in 1:n){
X[i] ~ dnorm(mu, prec)
}
mu ~ dnorm(0, 0.00001)
prec ~ dgamma(0.0001, 0.0001)
sigma <- 1/sqrt(prec)
#TEST
prH1 <- step(mu - 1)
}
DATA
list(n=10, X=c(1.73, 1.06, 2.03, 1.40, 0.95, 1.13, 1.41, 1.73, 1.63, 1.56) )

INITS
list(mu = 0, prec=1)
```

Kaufman, L. and Rock, I. (1962). The moon illusion I. *Science*, 136, 953--961.