

Problems

1. **New Neighbors.** You just got new neighbors, a family with two elementary school children. You have not seen the children and are curious about their gender. Consider two scenarios.

(a) This morning you introduced the neighbor and asked whether he has any boys. He said: Yes. What is the probability that one child is a girl?

(b) This morning you saw one of the neighbors' children and that was a boy. What is the probability that one child is a girl?

You are either told (truth) that there is a boy in family (A) or you saw that there is a boy (B). Why then the probabilities in (A) and (B) differ? Assume that the probability of a boy or a girl is $1/2$, and that the genders of the two children are independent.

2. **New Neighbors with three children**

You just got new neighbors, a family with three elementary school children. You have not seen the children and are curious about their gender. Consider two scenarios.

(a) This morning you introduced the neighbor and asked whether he has any boys. He answers: Yes. What is the probability that one child is a girl?

(b) This morning you saw one of the neighbors' children and that was a boy. What is the probability that one child is a girl?

You are either told (truth) that there is a boy in family (A) or you saw that there is a boy (B). Why then the probabilities in (A) and (B) differ? Find the probabilities and explain. Assume that the probability of a boy or a girl is $1/2$, and that the genders of the three children are independent.

3. **Queen of Spades Revisited.** From the standard deck of 52 cards the 2 of diamonds, 3 of spades, 4 of clubs, and 5 of hearts are removed. From the remaining cards, one card is selected at random.

(a) Are the events A - the selected card is queen, and B - the selected card is a spade, independent?

(b) What is the answer to (a) if from the standard deck both red suits are eliminated, and prior to selecting a card, the deck contains only spades and clubs suits?

4. **Multiple Choice.** A student answers a multiple choice examination question that has 4 possible answers. Suppose that the probability that the student knows the answer to a question is 0.80 and the probability that the student guesses is 0.20. If the student guesses, the student is able to eliminate two choices as wrong and guess on the remaining two choices

in 40% of cases. In this case the probability of correct answer is $1/2$. The student is able to eliminate one choice as wrong and guess on the remaining three choices in 30%

of cases, and for 30% of questions the student is not able to eliminate any choice and probability a correct answer is $1/4$.

- (a) What is the probability that the randomly selected question is answered correctly?
- (b) If it is answered correctly, what is the probability that the student really knew the correct answer?
5. **Classifier.** In a machine learning classification procedure the items are classified as 1 or 0. Based on a training sample of size 120 in which there are 65 1's and 55 0's, the classifier predicts 70 1's and 50 0's. Out of 70 items predicted by the classifier as 1, 52 are correctly classified.
- (a) What are the sensitivity and specificity of the classifier?
- (b) From the population of items where the proportion of 0-labels is 99% (and 1-labels 1%), an item is selected at random. What is the probability that the item is of label 1, if the classifier says it was.
6. **Alzheimer's.** A medical research team wished to evaluate a proposed screening test for Alzheimer's disease. The test was given to a random sample of 450 patients with Alzheimer's disease and to an independent sample of 500 subjects without symptoms of the disease. The two samples were drawn from the population of subjects who are 65 years old or older. The results are as follows:

Test Result diagnosis	diagnosed Alzheimer, D	no Alzheimer's symptoms, D^c	Total
Positive Test T	436	5	441
Negative Test T^c	14	495	509
Total	450	500	950

(a) Using the numbers from the table estimate $P(T | D)$ and $P(T^c | D^c)$. Interpret these probabilities in terms of the problem, one sentence for each.

Probability of D (prevalence) is the rate of the disease in the relevant population (≥ 65 y.o.) and it is estimated to be 11.3¹

Find $P(D | T)$ (positive predicted value) using Bayes formula. You cannot find $P(D | T)$ using information from the table only - you need external info, such as prevalence.

7. **Mysterious Transfer.** Of two bags, one contains four white balls and three black balls and the other contains three white balls and five black balls. One ball is randomly selected from the first bag and placed unseen in the second bag.
- (a) What is the probability that a ball now drawn from the second bag will be black?
- (b) If the second ball is black, what is the probability that a black ball was transferred?

¹Evans, D. A. et al. (1990). Estimated Prevalence of Alzheimer's Disease in the United States. *Milbank Quarterly*, 68, 267-289.

8. **NIR and Raman in Parkinson's.** In a study by Schipper et al. (2008), 53 subjects, 21 with mild or moderate stages of Parkinson's disease and 32 age-matched controls, had whole blood samples analyzed using the near-infrared (NIR) spectroscopy and Raman spectroscopy methods. The data showed that the two independent biospectroscopy measurement techniques yielded similar and consistent results. In differentiating Parkinson's disease patients from the control group, Raman spectroscopy resulted in eight false positives and four false negatives. NIR spectroscopy resulted in four false positives and five false negatives.

For both methods, find the Positive Predicted Value, that is, the probability that a person who tested positive and was randomly selected from the same age group in the general population has the disease if no other clinical information is available.

9. **Twins.** Dizygotic (fraternal) twins have the same probability of each gender as in overall births, which is approximately 51% male, 49% female. Monozygotic (identical) twins must be of the same gender. Among all twin pregnancies, about 1/3 are monozygotic. Find the probability of two girls in

- (a) monozygotic pregnancy,
- (b) dizygotic pregnancy, and
- (c) dizygotic pregnancy given that we know that the gender of the babies is the same.
- (d) What is the probability of dizygotic pregnancy given that we know that the gender of the babies is the same?

If Mary is expecting twins, but no information about the type of pregnancy is available, what is the probability that the babies are

- (e) two girls,
- (f) of the same gender.
- (g) Find the probability that Mary's pregnancy is dizygotic if it is known that the babies are two girls. Retain four decimal places in your calculations.

Hint: (b) genders are independent; (c) since A is subset of B , $A \cap B = A$ and $P(A | B) = P(A)/P(B)$; (d, e) total probability; (f) Bayes' rule.

10. **Hexi.** There is a 10% chance that pure breed German shepherd Hexi is a carrier of canine hemophilia A. If she is a carrier, there is a 50 – 50 chance that she will pass the hemophiliac gene to a puppy.

Hexi has two male puppies and they are tested free of hemophilia. What is the probability that Hexi is a carrier, given this information about her puppies?

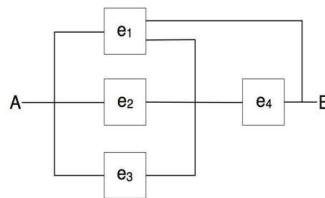
11. **Playing Dice with Casino.** You play the following game in a casino. You roll a pair of fair dice and then croupier rolls a pair of fair dice as well. If the sum on croupier's

dice is larger or equal than on yours, the casino wins. If the sum on your dice is strictly larger than 3 on croupier's, you win.

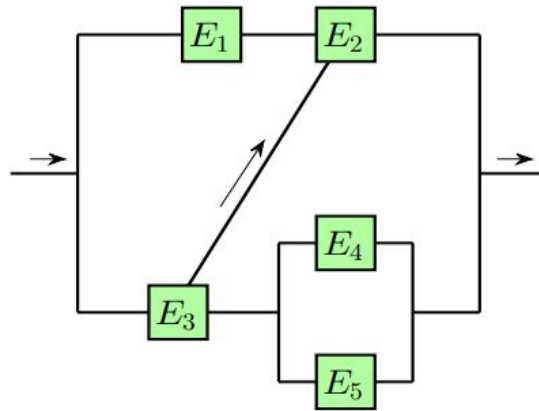
- (a) What is the probability that you win?
 - (b) If it is known that you won, what is the probability that croupier obtained a sum of 9?
 - (c) What is the expected croupier's sum if you won? Note that without any information on winners, croupier's (and your's) expected sum is 7.
12. **Redundant Wiring.** In a circuit shown in figure the electricity is to move from point A to point B . The four independently working elements in the circuit are operational (and the current goes through) with probabilities given in the table

Element	e_1	e_2	e_3	e_4
Operational with prob	0.5	0.2	0.3	0.8

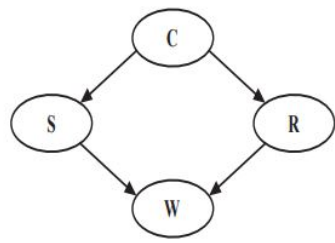
If an element fails, the current is not going through.



- (a) Is it possible to save on the wire that connects the elements without affecting functionality of the network? Explain which part of wiring can be removed.
 - (b) Find the probability that the electricity will flow from A to B .
13. **Cross-linked System.** Each of the five components in a cross-linked system is operational in a time interval $[0, T]$ with the probability of 0.6. The components are independent. Let E_i denote the event that i th component is operational at time T and E_i^c that it is not. Denote by A the event that the system is operational at time T .



- (a) Find the probabilities of events $H_1 = E_2^c E_3^c$, $H_2 = E_2^c E_3$, $H_3 = E_2 E_3^c$, and $H_4 = E_2 E_3$. Do these four probabilities sum up to 1 ?
 - (b) What is the probability of the system being operational if H_1 is true; that is, what is $\mathbb{P}(A | H_1)$? Find also $\mathbb{P}(A | H_2)$, $\mathbb{P}(A | H_3)$ and $\mathbb{P}(A | H_4)$.
 - (c) Using results in (a) and (b), find $\mathbb{P}(A)$.
14. **Sprinkler Bayes Net.** Suppose that a sprinkler (S) or rain (R) can make the grass in your yard wet (W). The probability that the sprinkler was on depends on whether the day was cloudy (C). The probability of rain also depends on whether the day was cloudy. The DAG for events C , S , R , and W is shown below:



The conditional probabilities of the nodes are given in the following tables:

C^c	C	S^c	S	Condition
0.5	0.5	0.50	0.50	C^c
		0.90	0.10	C

R^c	R	Condition	W^c	W	Condition
0.80	0.20	C^c	1	0	$S^c R^c$
0.20	0.80	C	0.10	0.90	$S^c R$
			0.10	0.90	SR^c
			0.01	0.99	SR

Using WinBUGS, approximate the probabilities (a) $\mathbb{P}(C | W)$, (b) $\mathbb{P}(S | W^c)$, and (c) $\mathbb{P}(C | R, W^c)$

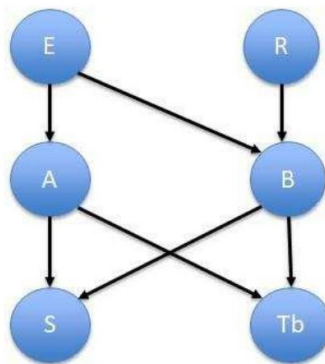
15. **Simple Diagnostic Bayes Network.** Incidences of diseases A and B depend on the exposure (E). Disease A is additionally influenced by risk factors (R). Both diseases lead to symptoms (S). Results of the test for disease B (T_B) are affected also by disease A . Positive test will be denoted as $T_B = 1$, negative as $T_B = 0$. The Bayes Network and needed conditional probabilities are shown in Figure.

The probabilities of nodes are as follows:

E	0	1	R	0	1	A	0	1
	0.8	0.2		0.7	0.3	E^c	0.9	0.1
						E	0.5	0.5

B	0	1	S	0	1	T_B	0	1
$E^c R^c$	0.95	0.05	$A^c B^c$	0.99	0.01	$A^c B^c$	0.99	0.01
$E^c R$	0.8	0.2	$A^c B$	0.5	0.5	$A^c B$	0.2	0.8
ER^c	0.7	0.3	AB^c	0.8	0.2	AB^c	0.95	0.05
ER	0.25	0.75	AB	0.4	0.6	AB	0.15	0.85

- (a) What is the probability of disease B , if disease A is not present, but symptoms S are present?



- (b) What is the probability of exposure E , if symptoms are present S and test for B is positive?

16. **Smart Alarm.** Your house has a "smart" alarm system that warns you against burglary with a long sound. The house is located in the seismically active area and the alarm system will emit a short sound if set off by an earthquake. The alarm can sound either way by error, or nor sound even in the case of earthquake or bulglary, You have two neighbors, Mary and John, who do not know each other. If they hear the alarm they call you, but this is not guaranteed. The likelihood of their call depends on type of sound if any. They also call you from time to time just to chat. The probabilities are:

E	0	1
	0.998	0.002

B	0	1
	0.999	0.001

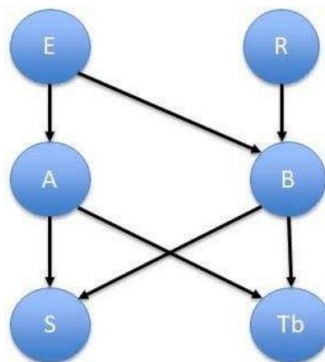
A	0	1	2
$E^c B^c$	0.998	0.001	0.001
$E^c B$	0.01	0.12	0.87
EB^c	0.01	0.75	0.24
EB	0.008	0.092	0.9

M	0	1
$A = 0$	0.99	0.01
$A = 1$	0.4	0.6
$A = 2$	0.3	0.7

J	0	1
$A = 0$	0.95	0.05
$A = 1$	0.1	0.9
$A = 2$	0.05	0.95

Write BUGS code to approximate conditional probabilities of nodes given the evidence.

- (a) Find the probability of a burglary if short sound was emitted by the alarm;
- (b) Find the probability of Mary calling you if John called you;
- (c) Find the probability of short sound if John called you and there was no earthquake;
- (d) Find the probability of any sound if there was no burglary and Mary did not call you; and
- (e) Find the probability of an earthquake if long sound was emitted by the alarm



Solutions and Hints

1. **New Neighbors.** (a) Before the information was obtained the sample space was $S = BB, BG, GB, GG$. All four outcomes were equally likely. I am told that there is a boy among children which excludes the outcome GG . Now the sample space becomes $S = BB, BG, GB$ and 2 out of 3 are favorable for the event 'Girl in the family'. The probability is $2/3$

(b) If a boy is seen, then the event 'Girl in the family' is equivalent to the event that the child that we did not see was a girl. This probability is $1/2$.

Remark. Why the difference? In scenario (B) the information is a result of random outcome. In scenario (A) the information obtained is not a result of a random outcome. Except for eliminating the outcome GG , the neighbor's statement does not provide any information about the remaining outcomes. The principle of insufficient reason prescribes that if there is no reason to think that one of the three remaining elementary outcomes BB, BG, GB is more probable than another, then one should assign equal probability to each.

Similar 'paradox' is Monty Hall problem https://en.wikipedia.org/wiki/Monty_Hall_problem, where switching doors improves the probability of a win. Monty does not open a random door, but the door that he chooses to open.

2. **New Neighbors with three children**

(a) $3/7$

(b) $1/2$

3. **Queen of Spades Revisited.**

(a) Events A and B are independent since $1/48 = P(A \cap B) = P(A) \cdot P(B) = 4/48 \cdot 12/48$.

(b) Independent, for $1/26 = 2/26 \cdot 13/26$

4. **Multiple Choice.** Let H_1 be the hypothesis that the student knows the question, H_2 is the event that the student guesses and was able to eliminate correctly two choices as wrong, H_3 is the event that the student guesses and eliminates correctly one choice as wrong, and H_4 is the event that student guesses and is not able to eliminate any of the choices as wrong. It is given that $\mathbb{P}(H_1) = 0.8, \mathbb{P}(H_2) = 0.2 \cdot 0.4 = 0.08, \mathbb{P}(H_3) = 0.2 \cdot 0.3 = 0.06$, and $\mathbb{P}(H_4) = 0.2 \cdot 0.3 = 0.06$.

Denote by A the event that the student answers the question correctly. Then, $\mathbb{P}(A | H_1) = 1, \mathbb{P}(A | H_2) = 1/2, \mathbb{P}(A | H_3) = 1/3$, and $\mathbb{P}(A | H_4) = 1/4$. Using the rule of total probability, the required probability in (A) is

$$\mathbb{P}(A) = \mathbb{P}(A | H_1) \mathbb{P}(H_1) + \mathbb{P}(A | H_2) \mathbb{P}(H_2) + \mathbb{P}(A | H_3) \mathbb{P}(H_3) + \mathbb{P}(A | H_4) \mathbb{P}(H_4) = 1 \cdot 0.8 + 0.08 \cdot 1/2 + 0.06 \cdot 1/3 + 0.06 \cdot 1/4 = 0.875.$$

In (B) we are interested in $\mathbb{P}(H_1 | A)$ and this can be found using Bayes' rule.

$$\mathbb{P}(H_1 | A) = \frac{\mathbb{P}(A | H_1)\mathbb{P}(H_1)}{\mathbb{P}(A)} = \frac{0.8}{0.875} = 0.9143$$

5. **Classifier.** Solution TBA (was a HW problem last semester)

6. **Alzheimer's.** $\mathbb{P}(T | D) = 436/450 = 0.9689$ and $\mathbb{P}(T^c | D^c) = 495/500 = 0.99$. The first is the probability that a patient who shows symptoms of Alzheimer's disease would test positive (sensitivity) and the second is the probability that a subject who does not have symptoms of Alzheimer would test negative (specificity). Note that $\mathbb{P}(T^c | D) = 1 - \mathbb{P}(T | D) = 0.0311$ and $\mathbb{P}(T | D^c) = 1 - \mathbb{P}(T^c | D^c) = 0.01$.

By Bayes' formula:

$$\begin{aligned} \mathbb{P}(D | T) &= \mathbb{P}(T | D)\mathbb{P}(D)/\mathbb{P}(T) \\ &= \frac{\mathbb{P}(T | D)\mathbb{P}(D)}{\mathbb{P}(T | D)\mathbb{P}(D) + \mathbb{P}(T | D^c)\mathbb{P}(D^c)} = \frac{0.9689 \times 0.113}{0.9689 \times 0.113 + 0.01 \times 0.887} = 0.9251 \end{aligned}$$

7. **Mysterious Transfer.** The solution requires using the rule of total probability, where the event of interest is A - a ball drawn from the second box is black, and the hypotheses are H_1 - transferred ball is white and H_2 - transferred ball is black. By accounting for the content of the first box, we find $\mathbb{P}(H_1) = 4/7$ and $\mathbb{P}(H_2) = 3/7$. The probability $\mathbb{P}(A | H_1) = 5/9$ since after the transfer there are 4 white and 5 black balls in the second box. Similarly, $\mathbb{P}(A | H_2) = 6/9$.

(a) The probability of selecting a black ball from the second box is

$$\mathbb{P}(A) = \mathbb{P}(A | H_1)\mathbb{P}(H_1) + \mathbb{P}(A | H_2)\mathbb{P}(H_2) = 5/9 \times 4/7 + 6/9 \times 3/7 = 38/63.$$

(b) By Bayes' rule:

$$\mathbb{P}(H_2 | A) = \mathbb{P}(A | H_2)\mathbb{P}(H_2)/\mathbb{P}(A) = (18/63)/(38/63) = 9/19.$$

8. **NIR and Raman in Parkinson's.** 21 is the number of true Parkinson's patients, and there are 32 people as control (without Parkinson's). We are told that for Raman spectroscopy, there are 8 FP and 4 FN. Using this info we create the following confusion matrix:

Raman Result	Predicted: Disease	Predicted: No Disease	Total
Actual: Disease	17	4	21
Actual: No Disease	8	24	32
Total	25	28	

for a Positive Predictive Value ($PPV = \frac{TP}{TP+FP}$) of 17/25

Raman Result	Predicted: Disease	Predicted: No Disease	Total
Actual: Disease	16	5	21
Actual: No Disease	4	28	32
Total	25	28	

for a Positive Predictive Value ($PPV = \frac{TP}{TP+FP}$) of 16/20

9. **Twins.** (a) 0.49;

(b) $0.49^2 = 0.2401$,

(c) Let S be event that sex is the same, and D -event of dizygotic pregnancy. $\mathbb{P}(GG | S, D) =$

$$\frac{\mathbb{P}(GG \cap S | D)}{\mathbb{P}(S | D)} = \frac{\mathbb{P}(GG | D)}{\mathbb{P}(S | D)} = \frac{0.49^2}{0.49^2 + 0.51^2} = 0.48$$

(d) This is Bayes formula. S - gender the same, hypotheses H_1 -monozygotic, H_2 -dizygotic $\mathbb{P}(H_1) = 1/3, \mathbb{P}(H_2) = 2/3, \mathbb{P}(S | H_1) = 1, \mathbb{P}(S | H_2) = 0.49^2 + 0.51^2 = 0.5002$.

$$\mathbb{P}(S) = \mathbb{P}(S | H_1) \mathbb{P}(H_1) + \mathbb{P}(S | H_2) \mathbb{P}(H_2) = 1/3 + 2/3 \cdot 0.5002 = 0.6668.$$

$$\mathbb{P}(H_2 | S) = 2/3 \cdot 0.5002 / 0.6668 = 0.5001.$$

(e) GG -two girls, $\mathbb{P}(GG | H_1) = 0.49; \mathbb{P}(GG | H_2) = 0.49^2$;

$$\mathbb{P}(GG) = 0.49 \cdot 1/3 + 0.49^2 \cdot 2/3 = 0.3234.$$

(f) Done in part (d) as denominator. (0.6668).

$$(g) \mathbb{P}(H_2 | GG) = \mathbb{P}(GG | H_2) \mathbb{P}(H_2) / \mathbb{P}(GG) = (0.49^2 \cdot 2/3) / 0.3234 = 0.4949$$

10. **Hexi.** Hint: Passing the hemophiliac gene is independent between the puppies. If the puppies are male then the only way they will get the hemophilia is from the mother carrier since hemophilia is X-chromosome-bound disorder.

Solution: Let H_1 denote the event that Hexi is a carrier, then $\mathbb{P}(H_1) = 0.1$. Let A be the event that the two male puppies are disease free. Then $\mathbb{P}(A | H_1^c) = 1$, that is, if Hexi is not a carrier, the puppies are disease free with probability 1.

$$\mathbb{P}(A) = \mathbb{P}(A | H_1) \mathbb{P}(H_1) + \mathbb{P}(A | H_1^c) \mathbb{P}(H_1^c) = 0.5 \cdot 0.5 \cdot 0.1 + 1 \cdot 0.9 = 0.925.$$

By Bayes formula,

$$\mathbb{P}(H_1 | A) = \frac{\mathbb{P}(A | H_1) \mathbb{P}(H_1)}{\mathbb{P}(A)} = 0.025 / 0.925 = 0.027.$$

11. Playing Dice with Casino.

(a)

$$\mathbb{P}(\text{Sum} = i) = \begin{cases} (i-1)/36, & 2 \leq i \leq 7 \\ (13-i)/36, & 7 \leq i \leq 12 \end{cases}$$

$$\begin{aligned} \mathbb{P}(\text{You won}) &= \frac{1}{36} \cdot 0 + \frac{2}{36} \times \frac{1}{36} + \frac{3}{36} \times \frac{1+2}{36} + \\ &\frac{4}{36} \times \frac{1+2+3}{36} + \frac{5}{36} \times \frac{10}{36} + \\ &\frac{6}{36} \times \frac{15}{36} + \frac{5}{36} \times \frac{21}{36} + \\ &\frac{4}{36} \times \frac{26}{36} + \frac{3}{36} \times \frac{30}{36} + \\ &\frac{2}{36} \times \frac{33}{36} + \frac{1}{36} \times \frac{35}{36} \\ &= 575/1296 = 0.44367 \end{aligned}$$

(b) Bayes formula:

$$\begin{aligned} \mathbb{P}(\text{Croupier} = 9 \mid \text{You won}) &= \frac{\mathbb{P}(\text{You won} \mid \text{Croupier} = 9) \times \mathbb{P}(\text{Croupier} = 9)}{\mathbb{P}(\text{You won})} \\ &= \frac{\frac{6}{36} \times \frac{4}{36}}{\frac{575}{1296}} \\ &= \frac{24}{575} = 0.04174 \end{aligned}$$

(c) Expectation of croupier's sum if you won is

$$E = \sum_{i=2}^{12} i \times \mathbb{P}(\text{Croupier} = i \mid \text{You won})$$

We already found that for $i = 9$, the probability is $24/575$. It is straightforward to find probabilities for other values of i and calculate the expectation. We can do it with the following Python script:

```
import numpy as np
casino=np.array([2,3,4,5,6,7,8,9,10,11,12])
prob=np.array([35,66,90,104,105,90,50,24,9,2,0])/575
np.dot(casino, prob) #5.453913043478261
```

If you won, we expect that croupier got the sum of $3136/575 = 5.453913043478261 \dots$. This is less than 7 – the expectation without any information.

```

import random
n=1000000
won=0
for x in range(n):
    you = random.randint(1,6) + random.randint(1,6)
    casino = random.randint(1,6) + random.randint(1,6)
    if casino < you:
        won+=1
print('Approx Prob of Winning =', won/n)
runfile('C:/Sandbox/highdice.py', wdir='C:/Sandbox')
Approx Prob of Winning = 0.442501
    
```

12. **Redundant Wiring.** *Hint:* Although this configuration can be analyzed directly, it is simpler to condition on the element e_1 and apply the Formula of Total Probability.

13. **Cross-linked System.** *Hint:* $\mathbb{P}(A | H_1) = 0$, $\mathbb{P}(A | H_2) = 1 - 0.4 \cdot 0.4$, $\mathbb{P}(A | H_3) = 0.6$, $\mathbb{P}(A | H_4) = 1$

Result: 0.7056

14. **Sprinkler Bayes Net** - OpenBUGS solution

```

model {
    cloudy ~ dcat(p.cloudy []);
    sprinkler ~ dcat(p.sprinkler [cloudy ,]);
    rain ~ dcat(p.rain [cloudy ,]);
    wetgrass ~ dcat(p.wetgrass [sprinkler , rain ,])
}
DATA
list(
#hard evidence , uncomment and instantiate...
# sprinkler = 1,
# cloudy = 1,
# rain = 1,
# wetgrass = 1,
#initial distributions
p.cloudy = c(0.5,0.5),
# conditionals
p.sprinkler = structure(.Data = c(0.50, 0.50,
0.90, 0.10), .Dim = c(2,2) ),
p.rain = structure(.Data = c(0.80, 0.20,
0.20, 0.80), .Dim = c(2,2) ),
p.wetgrass = structure(.Data = c(1., 0.0,
0.1, 0.9,
    
```

```
0.1, 0.9,
0.01, 0.99), .Dim = c(2,2,2) )
) #end list
```

For (a),

```
#hard evidence , uncomment and instantiate ...
# sprinkler = 1,
# cloudy = 1,
# rain = 1,
wetgrass = 2,
```

For (b)

```
#hard evidence , uncomment and instantiate ...
# sprinkler = 1,
# cloudy = 1,
# rain = 1,
wetgrass = 1,
```

For (c)

```
#hard evidence , uncomment and instantiate ...
# sprinkler = 1,
# cloudy = 1,
rain = 2,
wetgrass = 1,
```

15. **Simple Diagnostic Bayes Network.** (a) $P(db|da^c, sym) = 0.887$

(b) $P(exp|sym, tb) = 0.556$

Below is the OpenBUGS code. This is also included as an odc file for your convenience. However, we will only really need openbugs after the midterm, so don't worry about it too much just yet.

```
# Model
model {
  exp ~ dcat(p.exp []);
  risk ~ dcat(p.risk []);
  da ~ dcat(p.da[exp,]);
  db ~ dcat(p.db[exp, risk,]);
  sym ~ dcat(p.sym[da, db,]);
  tb ~ dcat(p.tb[da, db,]);
}

# Data
list(# da = 1,
```

```

# sym = 2,
# tb = 2
p.exp = c(0.8, 0.2),
p.risk = c(0.7, 0.3),
p.da = structure(.Data=c(0.90, 0.10, 0.50, 0.50),
  .Dim=c(2,2)),
p.db = structure(.Data=c(0.95, 0.05, 0.80, 0.20,
  0.70, 0.30, 0.25, 0.75),.Dim=c(2,2,2)),
p.sym = structure(.Data=c(0.99, 0.01, 0.50, 0.50,
  0.80, 0.20, 0.40, 0.60),.Dim=c(2,2,2)),
p.tb = structure(.Data=c(0.99, 0.01, 0.20, 0.80,
  0.95, 0.05, 0.15, 0.85),.Dim=c(2,2,2))

#Inits: none (gen inits instead)

```

16. **Smart Alarm.** Numerical solutions are as follows (results are from OpenBUGS):

(a) $P(\text{buglarly}|\text{alarm} = 2) = 0.046$

(b) $P(\text{MaryCalls}|\text{JohnCalls}) = 0.063$

(c) $P(\text{alarm} = \text{short_ound}|\text{JohnCalls}, \text{earthquake}^c) = 0.01884$

(d) $1 - P(\text{alarm} = \text{no_ound}|\text{buglary}^c, \text{MaryCalls}^c) = 0.0016$

(e) $P(\text{earthquake}|\text{alarm} = \text{long_ound}) = 0.205$

The openBUGS program is attached below and in an odc file.

```

#MODEL

model {
  earthquake ~ dcat(p.earthquake []);
  burglary ~ dcat(p.burglary []);
  alarm ~ dcat(p.alarm [earthquake, burglary,]);
  alarmno<- equals(alarm,1);
  alarmshort <- equals(alarm,2);
  alarmlong<- equals(alarm,3);
  Johncalls ~ dcat(p.Johncalls [alarm,])
  Marycalls ~ dcat(p.Marycalls [alarm,]);
}

#DATA IN:

list(
#hard evidence , uncomment and instantiate...

```

```
# earthquake = 1, #1 no, 2 yes
# burglary = 1, #1 no, 2 yes
# alarm = 1, #1 no, 2 short, 3 long
# Marycalls = 1, #1 no, 2 yes
# Johncalls = 1, #1 no, 2 yes

#initial distributions
p.earthquake = c(0.998,0.002),
p.burglary = c(0.999,0.001),

# conditionals
p.alarm = structure(.Data = c(0.998, 0.001, 0.001, #ec, bc
0.010, 0.120, 0.870, #ec, b
0.010, 0.750, 0.240, #e bc
0.008, 0.092, 0.900), .Dim = c(2,2,3)), #e,b

p.Marycalls = structure(.Data = c(0.99, 0.01,
0.40, 0.60,
0.30, 0.70), .Dim = c(3,2)),

p.Johncalls = structure(.Data = c(0.95, 0.05,
0.10, 0.90,
0.05, 0.95), .Dim = c(3,2)) )

#INITS NONE, just 'gen inits'
```