SOLUTIONS
Answer the following questions (8 points each):

1. Discuss how a high uncertainty in the expected demand over a certain planning horizon can have an adversarial impact on the effort of modern companies to quote competitive prices. In your discussion, please, show clearly the logical dependencies that are invoked by your argument.

From the standpoint of the operating that we are considering in our course, a high uncertainty in the anticipated demand must be addressed by some "buffering" strategy, namely the buildup of considerable safety stock (in the case of production to stock) or of extra production capacity (in the case of production to order). Both of these buffering schemes come with a substantial cost that will eventually increase the final price of the considered product.

Some of you also remarked that a highly uncertain demand makes it difficult to price the product, in the first place.

As a more general remark, the field of revenue management that has been developed in the recent years tries to cope with this uncertainty through "dynamic pricing" schemes, that revise the product price as demand materializes, in an effort to (i) redefine prices that reflect better the observed (im)balances between supply and demand, and (ii) even influence the materializing demand so that it aligns better with the product availability.
2. What is the primary reason for the employment of an S-shaped (or "serpentine") layout for contemporary production lines? What is a typical problem that is caused by such a layout?

S-shaped layout is used in an effort to fit a long linear production line into the available production facility.

Frequently the compression of a lot of activity in a fairly small area creates a lot of cluttering, and the physical presence of the line also hinders the communication among the various parts of the facility. To address this last problem, sometimes the conveyors that support the material flow in such lines are mounted on the ceiling of the facility instead of the floor.
3. Briefly discuss how discrete-part, high-volume manufacturing companies (like automotive or computer manufacturers) try to support the prevailing market trend for "mass-customization". In particular, point out what are the key mechanisms and concepts that enable modern companies to support such customized production in an effective and efficient manner.

Currently, in many high-volume discrete part manufacturing industries, mass-customization is supported through the offering of a number of options for the various components of the end product. These options are easily interchangeable in the final assembly of the end product, something that is attained by modular design and pertinent standardization of certain key features of these items, involving their geometry, interfacing with the other component, etc.

Equally important in this process is the effective employment of the necessary IT infrastructure that

(a) enables an effective communication with the customer during the order-placement process that leads to a customized order, and also

(b) coordinates the overall production activity so that each produced unit matches properly the order specifications.

More specifically, in the considered operational regime, the end-product units are assembled to order, from prefabricated components that are produced to stock.

Operating as described above, companies manage to provide a very large range of end-product configurations, while maintaining stocks for a rather limited number of options in each component. (As explained in class, with k components and n options per component, we get n^k configurations in the end product.)
4. Consider a synchronous transfer line where the produced item moves from station to station in batches of \( n \) units per batch (e.g., these \( n \) units are placed in a carton and it is these cartons that move from station to station by means of the conveyor that integrates and coordinates the operation of the entire line). If we cut down in half, both, the aforementioned batch size \( n \) and the line cycle time \( c \), the line throughput \( TH \) will

i. double.

ii. be reduced to half its current value.

iii. remain the same.

iv. do neither of the above.

Please, briefly explain your answer.

a) The simplest way to explain this result is by remembering that in a synchronous transfer line, we get one "unit of product" at every \( c \) time unit (where \( c \) is the cycle time of the line, i.e., the time between two consecutive advancements of line).

In our case, the "unit of product" was a batch of \( n \) distinct product units, which in the new configuration is reduced to \( n/2 \). Hence,

\[
TH_{\text{old}} = \frac{n}{c}
\]

and

\[
TH_{\text{new}} = \frac{n/2}{c/2} = \frac{n}{c} = TH_{\text{old}}.
\]

b) If we want to address this problem by Little's Law, then, set \( K = \text{number of workstations in the line} \), and notice that:

(i) line \( WIP = K \cdot n \) (since at any point in time each workstation will have a box containing a batch)

(ii) line \( T = K \cdot c \) (since a box spends \( c \) time units at each workstation)

Hence,

\[
TH_{\text{old}} = \frac{K \cdot n}{K \cdot c} = \frac{n}{c} = \frac{n/2}{c/2} = \frac{K \cdot n/2}{K \cdot c} = TH_{\text{new}}.
\]
5. Consider an $M/M/1/c$ queue where the arrival rate $\lambda$ is equal to the processing rate $\mu$. Let $\text{TH}(c)$ denote the throughput of this queue, and compute the function $\Delta \text{TH}(c)$ defined by $\Delta \text{TH}(c) \equiv \text{TH}(c+1) - \text{TH}(c)$; i.e., $\Delta \text{TH}(c)$ denotes the change of the throughput of this queue if its buffering capacity is increased by one unit. Use your result from the above computation to investigate how this marginal change behaves as $c$ increases to larger and larger values.

Since $\lambda = \mu$, the steady-state distribution of this queue is uniform with probability $\frac{1}{c+1}$ in each state (remember that the state space of this queue has $c+1$ states, since we need to account for the empty state).

Hence, $\text{TH}(c) = 2 \left[ 1 - p^{(c)}_c \right] = 2 \left[ 1 - \frac{1}{c+1} \right]$.

For the queue with buffering capacity equal to $c+1$,

$\text{TH}(c+1) = 2 \left[ 1 - p^{(c+1)}_{c+1} \right] = 2 \left[ 1 - \frac{1}{c+2} \right]$.

And

$\Delta \text{TH}(c) = \text{TH}(c+1) - \text{TH}(c) = 2 \left[ 1 - \frac{1}{c+2} - 1 + \frac{1}{c+1} \right]$

$= 2 \frac{1}{(c+2)(c+1)} = \frac{2}{c^2 + 3c + 2}$.

From (4) we see that $\Delta \text{TH}(c)$ decreases as $c$ increases. We also see that $\Delta \text{TH}(c) > 0$. In other words, every extra unit of buffering capacity increases the queue throughput, but this increase becomes smaller and smaller (technically, this is known as an increase with diminishing returns).

$\Delta \text{TH}(c) \to 0$ as $c \to \infty$, but for $c \to \infty$ the line also becomes unstable since $\lambda = \mu$ and $c \to \infty$ essentially means an $M/M/1$ queue.
Problem 1 (20 points): Consider a synchronous transfer line with three workstations, $WS_1$, $WS_2$ and $WS_3$, that supports the assembly of a certain item. The sum of the processing times for all the tasks that are involved in this assembly is equal to 300 seconds. Also, the utilizations of the line workstations during any single cycle of this line are, respectively, 83.33%, 91.67% and 75.00%. Use this information in order to compute the line throughput $TH$. Please, express your result in terms of the number of units produced per hour.

*Hint:* Remember the existing relationship between the throughput $TH$ and the cycle time $c$ of a synchronous transfer line, and try to compute the former through the latter.

Let $c =$ the line cycle time (i.e., the time between two consecutive advancements by the line).

$x_i =$ amount of work assigned to $WS_i$, $i = 1, 2, 3$.

Then:

\[
\frac{x_1}{c} = 0.8333 \\
\frac{x_2}{c} = 0.9167 \\
\frac{x_3}{c} = 0.75
\]

\[
x_1 + x_2 + x_3 = 300
\]

\[
\Rightarrow \quad c \left(0.8333 + 0.9167 + 0.75\right) = 300 \Rightarrow
\]

\[
c = 120 \text{ sec.}
\]

So, $TH = \frac{1}{c} = \frac{1}{120} \text{ hr}^{-1}$ or in hourly terms:

\[
TH = \frac{1}{120} \times 3600 \text{ hr}^{-1} = 30 \text{ hr}^{-1}
\]
**Problem 2 (40 points):** Consider a queueing station with a single server and total buffering capacity of three jobs (including the buffering capacity of the server). The jobs processed by this station are of two types, 1 and 2. The jobs from job type \( i \) arrive at this station according to a Poisson process with rate \( \lambda_i \) and their processing times are exponentially distributed with rate \( \mu_i \). Furthermore, job type 1 has *preemptive priority* over job type 2; i.e., whenever there is a job of type 1 present in the station, the server will need to shift attention to this job type, possibly abandoning temporarily a job of type 2 that might be in service.

Please, answer the following questions:

i. (10 pts) Define an appropriate state for analyzing the operation of this queueing station as a continuous-time Markov chain (CTMC), and draw the state transition diagram for this chain.

ii. (10 pts) What is the stability condition for the chain that you defined in item #1 above?

iii. (10 pts) Use your results from items #1 and #2 above in order to determine when this queueing station reaches a "steady-state" regime, and characterize the station throughput \( TH_i \), \( i = 1, 2 \), for each of the two job types in this regime, by means of the corresponding steady-state probability distribution (you do not have to compute, or even characterize this last distribution; simply assume that it exists).

iv. (10 pts) Briefly explain how your results to the previous questions will change if the arriving jobs, from either type, are processed on a First-Come-First-Serve basis.

*Hint:* In your work, please, keep in mind that (a) the process that combines into a single stream two or more event streams generated according to a Poisson process is another Poisson process, and that (b) for events generated by a Poisson process, the probability of having two or more of these events taking place simultaneously is effectively zero.
(1) A concise representation of the state for this case is the vector \((x_1, x_2)\) where \(x_i\) reports the number of jobs in the queue of type \(i\), \(i = 1, 2\). This information is adequate to infer the distribution of these jobs in the queue, namely, if \(x_1 > 0\), then one of these type 1 jobs is in service and every other job is in the buffer. Also, the memoryless property of the exponential service times implies that a type 2 job that finds itself in the buffer after getting some service due to the arrival of a type 1 job, will be equivalent in terms of its processing needs to any other type 2 job that has not received any processing. The corresponding state space is as follows:

(11) Since the queue has a finite buffering capacity, it will always be stable. Arriving jobs to find the queue full are just lost.
(iii) For $i=1,2$

$$T_{H_i} = 2i \times \left[ 1 - \text{Prob (queue is full)} \right] =$$

$$= 2i \times \left[ 1 - P(0,3) - P(2,1) - P(5,2) - P(3,0) \right]$$

The above result is justified by the Poisson nature of each of the two arrival streams and PASTA.

(iv) In this case, the state must capture the sequence of the waiting jobs in the buffer. So, the state could be an ordered list, for instance an ordered list with its first element indicating the job in service, the second element indicating the first job in the buffer, and the third element the second job in the buffer. Hence, the states would look as follows:

- $\emptyset$: the empty state
- 1: a type-1 job in service
- 2: 
- 11: a type-1 job in service, and a type-1 job in the buffer
- 12: 
- etc.

The transitions among the states would be defined accordingly.

This new queue remains finite and therefore always stable. The throughput for each job type would still be defined as:

$$T_{H_i} = 2i \times \left[ 1 - \text{Prob (queue is full)} \right]$$

but now the Prob (queue is full) will be defined by the sum of the steady-state probabilities of the corresponding states in the new state transition diagram.

On the other hand, it is interesting to notice that

$$\frac{T_{H1}}{T_{H2}} = \frac{21}{2}$$

as in the previous case.