Consider the infinite horizon gold mining problem when the profit flow $\pi(x, z)$ is given by $(x - z)z$ instead of $gz - 500z^2/x$. (Here, the symbol $x$ replaces the symbol $I$ for inventory.) Let $d$ denote the one-period discount factor. Let $V(x)$ denote the optimal value function, and let $z(x)$ denote the optimal production quantity.

1. Write down the Bellman equation.

2. Explain why $V(x)$ cannot be linear.

3. Determine $V(x)$ when $d = 0$.

4. Use the method of successive approximations to determine the functional form of $V(x)$ and $z(x)$.

5. Given the functional form for $V(x)$, determine the exact value function and optimal policy by finding the fixed point of the mapping $V \rightarrow T(V)$ associated with the Bellman equation. (This involves solving an algebraic equation.)

6. There is an alternative strategy to finding the fixed point:
   
   (a) Use the functional form for $z(x)$ to express the profit flow at each time $t$;
   (b) Determine $V(x)$ as the present value of this profit flow stream, which will be in terms of the unknown parameter(s) of $z(x)$; and
   (c) Optimize over the parameters to obtain the exact form of $V(x)$.

Carry out this alternative strategy.