1. Consider a non-dividend paying stock whose price follows Geometric Brownian motion, 
\[ dS_t = \mu S_t \, dt + \sigma S_t \, dB_t \] 
with (annual) parameters \( \mu = 0.20, \sigma = 0.40, \) and \( S_0 = 100. \)

a. Determine the expected value for the stock price after 3 months (0.25 yr).

b. Determine the median value \( s^* \) for the stock price after 3 months. Recall that \( s^* \) is defined by the equation \( P[S(0.25) \geq s^*] = 0.50. \)

c. Determine the standard deviation of the stock price after 3 months.

d. Consider a 1-year binomial lattice representation of \( S \) with 1-month periods. What is the probability that a simulated stock price path will have its terminal stock price equal to 100?
ASSET PRICE DYNAMICS SAMPLE PROBLEM SOLUTIONS

1. a. \( E[S(0.25)] = S(0)\exp(0.25*0.20) = 105.127. \)
   b. \( \nu = 0.20 - (0.40)^2/2 = 0.12. \)
      \( \ln[S(0.25)/S(0)] \sim \text{Normal with mean} = 0.12(0.25) \text{ and variance} = 0.40(0.25). \) The mean and median values are the same for a normally distributed random variable, and so the median value of \( \ln[S(0.25)/S(0)] \) is 0.03, which implies the median value for \( S(0.25) \) is 100*\( \exp(0.03) = 103.045. \)
   c. \( \ln[S(t)/S(0)] := X(t) \sim N(\nu t, \sigma^2 t), \text{ where } \nu = \mu - 0.5*\sigma^2 = 0.12. \)
      \( S(t) = S(0)\exp[X(t)]. \)
      \( E[S(t)] = S(0)E[\exp(X(t))] = S(0)\exp[\nu t + 0.5 \sigma^2 t] \)
      \( = 100\exp[0.12(0.25) + 0.5(0.16)(0.25)] \text{ (when } t = 0.25) \)
      \( = 100\exp(0.05) = 105.127. \) (Same answer as part a.)
      \( E[S(t)^2] = S(0)^2E[\exp(2X(t))] = S(0)^2\exp[2\nu t + 2\sigma^2 t] \)
      \( = 10000\exp[0.06+0.08] = 11502.74 \text{ (when } t = 0.25). \)
      \( \text{Var}[S(0.25)] = 11502.74 - (105.127)^2 = 451.05. \)
      \( \text{Stdev}[S(0.25)] = 21.24. \)
   d. \( p = 0.5(1 + (0.12/0.40)) = 0.5433. \)
      For a simulated stock price at the end of the 12th month to have a value of 100, there must have been exactly 6 “up” and 6 “down” transitions. The probability of this occurrence is therefore \((12 \text{ choose } 6)^*(0.5433)^6(0.4567)^6 = 0.2156. \)