Utility Homework Problems

I. Lotteries and Certainty Equivalents

1. Consider an individual with zero initial wealth and a utility function $U(W) = 1 – \exp[-0.0001W]$. Find the certainty equivalent for each of the alternatives below.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>-10,000</th>
<th>0</th>
<th>10,000</th>
<th>20,000</th>
<th>30,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.3</td>
<td>0.4</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.15</td>
<td>0.65</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

2. Consider an individual with zero initial wealth and a utility function $U(W) = 1 – \exp[-0.0001W]$. The individual is choosing between a certain 20,000 and a lottery with probability $p$ of winning 30,000 or probability $(1-p)$ of winning 10,000. There is no cost for either alternative.

a. Find the probability $p$ so that the individual is indifferent if the initial wealth is 10,000.
b. Find the probability $p$ so that the individual is indifferent if the initial wealth is 20,000.

3. Consider an individual with initial wealth of 20,000 and a quadratic utility $U(W) = W – W^2/100,000$. The individual faces a lottery with probability 0.50 of winning 10,000 and a probability of 0.50 of winning 20,000. Find the certainty equivalent of the lottery.

4. Smith has an initial wealth of $125,000 and a utility function $U(W) = 3\ln(W) + 10$. Smith has the option of playing the following lottery: There is a 5% chance of losing 100,000, a 75% chance of winning 10,000, and a 20% chance of winning 500,000. There is no cost for the lottery. Determine the minimum amount of money Smith would have to be offered so that Smith would prefer to take this money and not play lottery A.

5. Smith has an initial wealth of $50,000 and a utility function $U(W) = \ln(W)$. Smith has the option of playing the following lottery A: \{ (0.10, -$40,000), (0.70, $10,000), (0.20, $100,000) \}. There is no cost for the lottery.

a. What is the minimum amount of money you would have to offer Smith so that Smith would prefer to take this money and not play lottery A?
b. Smith also has the option of playing lottery B: with probability 0.50 Smith will win $M$ and with probability 0.50 Smith will win nothing. There is no cost for the lottery. Determine the smallest value of $M$ for which Smith would prefer lottery B to lottery A.
II. Insurance Analysis

6. Consider a homeowner with utility function \( U(W) = 1 - \exp[-0.00001W] \) who is deciding on whether or not to buy fire insurance. There is a 1% chance of a fire in which case it will cost the homeowner 60,000. The insurance premium is 700 per year, and the deductible amount on the loss is \( D \). (In the case of a fire the homeowner will pay \( D \) in addition to the already paid premium of 700.) Determine the deductible amount that would make the homeowner indifferent between buying and not buying insurance. The homeowner’s initial wealth is 80,000.

7. Consider a homeowner with utility function \( U(W) = \ln(W) \) who is deciding on whether or not to buy fire insurance. There is a 1% chance of a fire in which case it will cost her 75,000. The homeowner’s initial wealth \( W_0 = 100,000 \). If she chooses to purchase an insurance policy, she will pay an insurance premium \( P \) up front, i.e., the cost of the policy, and she will pay an additional deductible amount \( D \) should a fire occur. She has spoken with two insurance companies, and each company has offered her a policy, as follows:

   Company 1: Premium \( P = 3000 \). Deductible \( D = 3,000 \).
   Company 2: Premium \( P = 2000 \). Deductible \( D = 8,000 \).

   a. Determine the best option for her.
   b. Determine the Certainty Equivalent and Risk Premium for the option you recommend in part (a).

8. Consider a homeowner with utility function \( U(W) = \ln(W) \) who is deciding on whether or not to buy fire insurance. The homeowner’s initial wealth \( W_0 = 200,000 \). There is a 1% chance of a fire in which case it will cost her 150,000. If she chooses to purchase an insurance policy, she will pay an insurance premium \( P \) up front, i.e., the cost of the policy, and she will pay an additional deductible amount \( D \) should a fire occur. She has spoken with two insurance companies, and each company has offered her a policy, as follows:

   Company 1: Premium \( P = 6000 \). Deductible \( D = 6,000 \).
   Company 2: Premium \( P = 4000 \). Deductible \( D = 16,000 \).

   a. Determine the best option for her.
   b. Determine the Certainty Equivalent and Risk Premium for the option you recommend in part (a).
Utility Homework Problem Solutions

1. \( U(W) = 1 - \exp[-0.0001W] \).

<table>
<thead>
<tr>
<th>Cash Amount</th>
<th>-10,000</th>
<th>0</th>
<th>10,000</th>
<th>20,000</th>
<th>30,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>-1.718</td>
<td>0</td>
<td>0.632</td>
<td>0.865</td>
<td>0.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative</th>
<th>( E[U(W)] )</th>
<th>( \text{CE} = U^{-1}[E(U(W))] = 10,000 \ln [1/(1-U)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3490</td>
<td>4,292</td>
</tr>
<tr>
<td>2</td>
<td>0.3723</td>
<td>4,657</td>
</tr>
<tr>
<td>3</td>
<td>0.5378</td>
<td>7,718</td>
</tr>
<tr>
<td>4</td>
<td>0.6008</td>
<td>9,183</td>
</tr>
<tr>
<td>5</td>
<td>-0.3840</td>
<td>-3,250</td>
</tr>
</tbody>
</table>

2. a. Initial wealth is 10,000. \( U(W) = 1 - \exp[-0.0001W] \).
    Seek the value of \( p \) for which \( U(30,000) = pU(40,000) + (1-p)U(20,000) \).
    Therefore: \( 0.9502 = 0.8647 + p[0.9817 - 0.8647] \) \( \Rightarrow \) \( p = 0.731 \)

b. Initial wealth is now 20,000. \( U(W) = 1 - \exp[-0.0001W] \).
    Seek the value of \( p \) for which \( U(40,000) = pU(50,000) + (1-p)U(30,000) \).
    Therefore: \( 0.9817 = 0.9502 + p[0.9533 - 0.9502] \) \( \Rightarrow \) \( p = 0.731 \) (as before)

3. \( U(W) = W - W^2/100,000 \). \( W_0 = 20,000 \).
    Begin by finding the value \( Y \) for which: \( 0.50U(30,000) + 0.50U(40,000) = U(Y) \).
    \( Y \) satisfies the quadratic equation \( Y^2/100,000 - Y + 22,500 = 0 \), whose roots are 34,189 and 65,811.
    Clearly, \( Y = 34,189 \). Now \( Y = W_0 + \text{CE} \), which means that \( \text{CE} = 14,189 \).

4. \( EU(W) = 0.05U(25,000) + 0.75U(135,000) + 0.20U(625,000) = 46.105616 \). Thus \( \ln(W) = 12.035205 \),
    which implies that \( W = 168,586.70 \), the certainty equivalent of the lottery. Since initial wealth \( W_0 = 125,000 \),
    Smith would have to be offered 43,586.70 not to play. Note that \( \ln(W) \) in lieu of \( 3\ln(W) + 10 \)
    would have worked just fine, too.

5. a. \( E[U(A)] = 0.10[\ln(10,000)] + 0.70[\ln(60,000)] + 0.20[\ln(150,000)] = 11.006182 = \ln(C) \).
    Thus \( C = 60,245 \), which means that break-even point for Smith is 10,245.

b. \( E[U(B)] = 0.50[\ln(50,000)] + 0.50[\ln(50,000 + M)] = E[U(A)] = 11.006182 \).
    Thus, the break-even point for Smith is \( M = 22,590 \).

6. We seek the deductible \( D \) for which the expected utility of not buying insurance equals the expected
    utility for buying insurance. \( U(W) = 1 - \exp[-0.00001W] \).

   If you do not buy insurance: \( E[U(W)] = 0.99U(80,000) + 0.01U(20,000) = .546977 \).
   If you do buy insurance: \( E[U(W)] = .99U(79,300) + .01U(79,300 - D) \)
   Therefore \( D = 11,228 \)
7. Option 1: \( E[U(W)] = 0.99 \ln(97,000) + 0.01 \ln(94,000) = 11.4821521. \)
   Option 2: \( E[U(W)] = 0.99 \ln(98,000) + 0.01 \ln(90,000) = 11.49187118. \)
   No Insurance: \( E[U(W)] = 0.99 \ln(100,000) + 0.01 \ln(25,000) = 11.49906252. \) Best option.

CE of no insurance option = \( \exp(11.49906252) = 98,623. \)
Expected wealth of no insurance option = \( 0.99(100,000) + 0.01(25,000) = 99,250. \)
Risk premium = \( 99,250 - 98,623 = 627. \)

8. Option 1: \( E[U(W)] = 0.99 \ln(194,000) + 0.01 \ln(188,000) = 12.17529928. \)
   Option 2: \( E[U(W)] = 0.99 \ln(196,000) + 0.01 \ln(180,000) = 12.18501836. \)
   No Insurance: \( E[U(W)] = 0.99 \ln(200,000) + 0.01 \ln(50,000) = 12.1922097. \) Best option.
CE of no insurance option = \( \exp(12.1922097) = 197,247. \)
Expected wealth of no insurance option = \( 0.99(200,000) + 0.01(50,000) = 198,500. \)
Risk premium = \( 198,500 - 197,247 = 1253. \)