Chapter 4 Homework Problem Solutions
ISyE 4311

Problem 4.8
Purchase cost of the machine = 100K. With a terminal value of 20K and a life of 8 years, the
depreciation rate is 10K per year. Depreciation tax shield yields 4K per year. Value of this tax
shield at 15% is 17,949. With an estimated salvage value of 10K in year 8 and a book value of
20K, the after-tax cash flow in year 8 due to the asset sale is \(10K - 0.4(10K - 20K) = 14K\).
Value of the asset sale is 4,577. Net cost of the machine is therefore 100,000 - (17,949 + 4,577)
= 77,474. The machine will generate an after-tax cash flow of 9K per year for 8 years. The
value of this cash flow stream is 40,386. NPV = -37,088. Does not pay.

Problem 4.9
Each year the cash flows occur during the year. To compute the value of the cash flow stream,
we first compute the after-tax equivalent cash flow at the end of Dec for each year. The EAIR
is 12% and so the monthly interest rate is \((1.12)^{1/12} - 1\).
Installing the air conditioning system will generate a cash flow of 11K in the 4 months of June-
Sept and a cash flow of -5K in the month of Oct. The future value of this cash flow stream at
the end of Dec is
\[
11K \left[1 - \frac{(1.12)^{1/12} - 1}{(1.12)^{7/12} - 1}\right] (1.12)^{7/12} - 5K (1.12)^{2/12} = 40,817.
\]
(The calculation of the first term is the FV at the end of Dec of the PV at the beginning of
June.) There is a cash flow of 15K at the end of Dec due to depreciation.
The after-tax cash flow at the end of each Dec is therefore 0.65(40,817) + 0.35(15,000) = 31,781.
The PV of this cash flow stream is 31,781 \left[\frac{1 - (1.12)^{-10}}{0.12}\right] = 179,572. NPV = 29,572.

Problem 4.12
If the current machine will be kept it will yield a depreciation tax shield of 4K each year for 4
years. Value of this depreciation tax shield is 4K \left[\frac{1 - (1.15)^{-4}}{0.15}\right] = 11,420.
As for the new machine, its purchase cost is 100K. The sale of the current machine will yield a
cash flow of 20K - 0.4(20K - 40K) = 28K. Each year the new machine will yield a depreciation
tax shield of 8K for 5 years. The after-tax value of the asset sale in year 5 in projected to be
20K - 0.4(20K - 20K) = 20K. The net cost of the machine is therefore
\[
100K - 28K - 8K \left[\frac{1 - (1.15)^{-4}}{0.15}\right] - 20K/(1.15)^4 = 37,725.
\]
The new machine will yield an after-tax cash flow due to increased revenue of 24K per year for a
PV of 24K \left[\frac{1 - (1.15)^{-4}}{0.15}\right] = 68,519. So the NPV of the new machine is 68,519 - 37,725 = 30,794.
The value for acquiring the new machine is 30,794 - 11,420 = 19,374, so it pays to acquire it.
Problem 4.13
Purchase cost of the regular machine = 10K. With a terminal value of 0K and a life of 5 years, the depreciation rate is 2K per year. Depreciation tax shield yields 800 per year. Value of this tax shield at 11% is 2,957. No salvage value, so the net cost of the machine is 10K - 2,957 = 7,043. The machine will generate an additional after-tax cost of 900 per year. The value of this cost is 3,326. Total cost of the regular machine is therefore 10,369.
Since the cash flows for the color machine are three times that of the regular machine, the net cost of the color machine = 3(7,043) = 21,130. The color machine will generate a positive after-tax cash flow of 0.6(8,500 – 4,500) = 2,400 each year. The PV of this benefit at 11% is 8,870. Total cost of the color machine is therefore 12,260. Regular machine is best.

Problem 4.15
Since the revenues are identical to both alternatives, they will be excluded.
Purchase cost of the machine A is 4K. With a terminal value of 0K and a life of 5 years, the depreciation rate is 800 per year. Depreciation tax shield yields 280 per year. Value of this tax shield at 12% is 1,009. No salvage value, so the net cost of the machine is 4K - 1,009 = 2,991. The machine will generate an additional after-tax fixed cost of 195 per year. The value of this cost is 703. Total cost of machine A is therefore 3,694. After-tax unit variable cost per year for machine A is 0.78. PV of this unit variable cost is 2.812. So, given a production rate of n per year, the total cost for machine A is

\[3,694\lceil n/400K \rceil + 2.812n.\]

(Here, \([x]\) denotes the smallest integer at least as large as \(x\).)  
Purchase cost of the machine B is 10K. With a terminal value of 0K and a life of 5 years, the depreciation rate is 2K per year. Depreciation tax shield yields 700 per year. Value of this tax shield at 12% is 2,523. No salvage value, so the net cost of the machine is 10K - 2,523 = 7,477. The machine will generate an additional after-tax fixed cost of 136.5 per year. The value of this cost is 492. Total cost of machine B is therefore 7,969. After-tax unit variable cost per year for machine B is 0.52. PV of this unit variable cost is 1.874. So, given a production rate of n per year, the total cost for machine B is

\[7,969\lceil n/400K \rceil + 1.874n.\]

When \(n = 1M\), cost of machine A is 13.894M and the cost of machine B is 17.812M. Machine A is best.
NB: Machine A dominates machine B for all values of \(n\).

Problem 4.16
The wording of the problem suggests that the only decision is whether to replace both machines. It may turn out that the incremental benefit of replacing the second machine may not be worth the cost, as the benefit of the second machine is only realized for the 5 summer months. Purchase cost of a new machine is 20K. With a terminal value of 0K and a life of 5 years, the depreciation rate is 4K per year. Depreciation tax shield yields 1,600 per year. Value of this tax shield at 10% is 6,065. No salvage value. Net cost of the machine is 20K - 6,065 = 13,935.
If a new machine is acquired, the old one will be sold. Since the salvage value is 6K and the book value is 10K, the asset sale will yield an after-tax cash flow of 6K - 0.4(6K - 10K) = 7,600. When you sell the old machine you forfeit the value of its future depreciation tax shield. This is a cost. With a terminal value of 0K and a remaining life of 5 years, the depreciation rate is 2K per year. Depreciation tax shield yields 800 per year. Value of this tax shield at 10% is 3,033. So, the net cost of a new machine is 13,296 - 7,600 + 3,033 = 9,368.

To compute the after-tax yearly savings, we use the monthly interest rate is \( (1.1) \frac{1}{12} - 1 \).

Acquisition of the first machine generates an after-tax savings of 510 each month for a year-end value of 510 \( \left( \frac{(1+i)^{12}-1}{i} \right) \), which is 6,396 each year for the next 5 years. The PV of this cash flow stream is 6,396 \( \left( \frac{1}{1-(1.1)^{-5}} \right) = 24,246 \). The NPV of acquiring the first machine is therefore 24,246 - 9,368 = 14,878.

Acquisition of the second machine generates an after-tax savings of 510 each month for the 5-month period from May to Sept. The year-end value of this savings is 510 \( \left( \frac{(1+i)^{12}-1}{i} \right) (1 + i)^3 = 2,653 \) each year for the next 5 years. The PV of this cash flow stream is 2,653 \( \left( \frac{1-(1.1)^{-5}}{0.1} \right) = 10,057 \). The value of acquiring the second machine is therefore 10,057 - 9,368 = 689. It pays to acquire the second machine, too. Total value is 15,567.

Problem 4.17
Let \( \tau \) denote the tax rate. The net cost of the machine is

\[
1,000 \left( 1 - \frac{\tau}{10} \frac{1 - (1.08)^{-10}}{0.08} \right) = 1,000(1 - 0.671\tau).
\]

Value of the revenue cash flow stream is 300 \( \left( \frac{1-(1.08/0.05)^{-10}}{0.03} \right) = 2,455.07 \).

Value of the cost cash flow stream is 60 \( \left( \frac{1-(1.08)^{-10}}{0.08} \right) = 402.60 \).

The after-tax value of profit is \( (1 - \tau)(2,052.47) \). The project NPV is therefore

\[
(1 - \tau)(2,052.47) - 2,000(1 - 0.671\tau) = 52.47 - 710.47\tau.
\]

Project value is -303 when \( \tau = 50\% \). The break-even value of \( \tau \) is 7.39\%.

Problem 4.18
As a function of the annual revenue \( R \) and tax rate \( \tau \), the project value is

\[
(1 - \tau)(R - 40)(6.71) - 1000(1 - 0.671\tau).
\]

(See solution to Problem 17.) Project value is 315 when \( R = 300 \) and \( \tau = 0.4 \).

Problem 4.19
After-tax cost of R&D in years 1-4 is 120K each year. The investment in year 4 is 250K. The PV of these costs is 523,362.
Annual after-tax profit is $0.6[750K - (300K + 250K)] = 120K$. Depreciation tax shield yields $10K$ each year. So, the total cash flow in years 5-14 is $130K$. The PV of this cash flow stream is

$$130K \left[ \frac{1 - (1.12)^{-10}}{0.12} \right] / (1.12)^4 = 466,806.$$  

Project value is $-56,556$.

Let $L$ denote the loan amount. The loan cash flow is $L$ at time $t = 0$ and $-L/6$ at times $t = 1, \ldots, 6$. The PV of this loan cash flow is

$$L \left( 1 - \left[ \frac{1}{6} \frac{1 - (1.12)^{-6}}{0.12} \right] \right) = 0.31477L,$$

which must equal $56,556$ for the project to break-even.

The break-even value of $L$ is therefore $179,676$.

**Problem 4.20**

For the aggressive campaign, the PV of the first year’s cash flow is

$$20K \left[ 1 - \left( \frac{(1.07)^{1/12}}{1.1} \right)^{-12} \right] = 409,792.$$  

PV of the subsequent cash flows (in perpetuity) is

$$\frac{20K}{(1.07)^{1/12} - 1} \frac{1}{1.07} = 3,305,820.$$  

Aggressive campaign’s project value is therefore $409,792 + 3,305,820 - 400,000 = 3,315,612$.

For the regular campaign, the PV of the first year’s cash flow is

$$10K \left[ 1 - \left( \frac{(1.07)^{1/12}}{1.06} \right)^{-12} \right] = 162,028.$$  

PV of the subsequent cash flows (in perpetuity) is also $3,305,820$.

Regular campaign’s project value is therefore $162,028 + 3,305,820 - 150,000 = 3,317,848$.

Very close!