Chapter 3 Homework Problem Solutions
ISyE 4311

Problem 3.1
\[ 10,000 = \frac{11,100}{1 + EAIR} \implies EAIR = 11\%. \]

Problem 3.2
\[ 330 = 70 \left[ \frac{1 - (1 + i)^{-5}}{i} \right] \implies i = 1.994\% \implies EAIR = (1 + i)^{12} - 1 = 26.74\%. \]

Problem 3.3
\[ 90 = 10 \left[ \frac{1 - (1 + i)^{-11}}{i} \right] \implies i = 3.503\% \implies EAIR = (1 + i)^{12} - 1 = 51.16\%. \]

Problem 3.4
\[ 100K = Y \left[ \frac{1 - (1.01)^{-120}}{0.01} \right] \implies Y = 17,698.42. \]

Problem 3.5
\[ 100K = Y \left[ \frac{1 - (1.1)^{-20}}{0.1} \right] \implies Y = 11,745.96. \]
\[ 97.75K = Y \left[ \frac{1 - (1 + EAIR)^{-20}}{EAIR} \right] \implies EAIR = 10.34\%. \]

Problem 3.6

For 10 year loan:
\[ 100K = M \left[ \frac{1 - (1.01)^{-120}}{0.01} \right] \implies M = 1,434.71. \]
\[ 97.75K = M \left[ \frac{1 - (1 + i)^{-120}}{i} \right] \implies i = 1.047\% \implies EAIR = (1 + i)^{12} - 1 = 13.32\%. \]

NB: Total interest paid over life of loan = Total payment - Loan amount = 120(1434.71) - 97.75K = 74,415.

For 6 year loan:
\[ 100K = M \left[ \frac{1 - (1.01)^{-72}}{0.01} \right] \implies M = 1,955.02. \]
\[ 97.75K = M \left[ \frac{1 - (1 + i)^{-72}}{i} \right] \implies i = 1.072\% \implies EAIR = (1 + i)^{12} - 1 = 13.65\%. \]

NB: Total interest paid over life of loan = Total payment - Loan amount = 72(1955.02) - 97.75K = 43,011.
Problem 3.7

\[250K = M \left[1 - \frac{(1 + 0.08/12)^{-36}}{0.08/12}\right]\Rightarrow M = 7,834.09.\]

\[246K = M \left[1 - \frac{(1 + i)^{-36}}{i}\right]\Rightarrow i = 0.758\% \Rightarrow EAIR = (1 + i)^{12} - 1 = 9.49\%.

Problem 3.8

\[1000K = Y \left[1 - \frac{(1.01)^{-60}}{0.01}\right]\Rightarrow Y = 2,224.44.\]

\[97,750 = Y \left[1 - \frac{(1 + i)^{-60}}{i}\right]\Rightarrow i = 1.084\% \Rightarrow EAIR = (1 + i)^{12} - 1 = 13.81\%.

Problem 3.9

Purchase cost = 15K. Lease cost = 4K \left(1 + \frac{1 - (1.1)^{-3}}{0.1}\right) = 13,947. Lease.

Problem 3.10

Purchase cost = 2K. Lease cost = L \left(1 + \frac{1 - (1.15)^{-3}}{0.15}\right) \Rightarrow L = 609.16. Lease.

Problem 3.11

Purchase cost = 20K - 3K/(1.15)^3 = 18,027.

Lease cost = L \left(1 + \frac{1 - (1.15)^{-3}}{0.15}\right) \Rightarrow L = 5,491. Buy.

Problem 3.12

EAIR = 20\%. Monthly interest = (1.2)^{1/12} - 1.

Purchase cost = 2K - 500/(1.2)^2 = 1,653.

Lease cost = L \left(\frac{1 - (1.2)^{-2}}{(1.2)^{1/12} - 1}\right) \Rightarrow L = 82.81. Lease.

NB: Note that \((1.2)^{1/12})^2 = 1.2^2\).

Problem 3.13

EAIR = 7\%. Monthly interest = (1.07)^{1/12} - 1.

Purchase cost = 14,415 - 5,000/(1.07)^2 = 10,048.

Lease cost = 1,315 + 200 \left[\frac{1 - (1.07)^{-2}}{(1.07)^{1/12} - 1}\right] + (4,750 - 450)/(1.07)^2 = 9,548. Lease.

NB: Note that \((1.07)^{1/12})^2 = (1.07)^2\).

Remark: What is the break-even salvage value \(S^*\) that would make the lease and purchase options equivalent? Notice that as long as the salvage value is less than 9,750, changing it up or down will not change the difference of 10,048 - 9,548 = 500 between the PV’s calculated above. So \(S^* \geq 9,750\). Since 500(1.07)^2 = 575.45, it follows that \(S^* = 9,750 + 572.45 = 10,322.45\).

Problem 3.14

EAIR = 6\%. Monthly interest = (1.06)^{1/12} - 1.

Purchase cost = 45,415 - 35,000/(1.07)^2 = 17,692.

Lease cost = 1,315 + 400 \left[\frac{1 - (1.06)^{-4}}{(1.06)^{1/12} - 1}\right] - 450/(1.06)^4 = 18,044. Buy.

NB: Note that \((1.06)^{1/12})^4 = (1.06)^4\).
Remark: What is the break-even salvage value $S^\ast$ that would make the lease and purchase options equivalent? Notice that the salvage value will have to decrease to make up for the difference of $18,044 - 17,692 = 352$ between the PV’s calculated above. Since $352(1.06)^4 = 444.3$, it follows that the break-even salvage value $S^\ast = 35,000 - 444 = 34,556$.

Problem 3.15
Cost of the cash payment option is $90K$.
The appropriate discount rate to assess the cost of the zero-interest finance option is the cost of borrowing. Thus, the cost is $39K + \left(\frac{60K}{36}\right)\left[\frac{1-(1+0.10/12)^{36}}{0.10/12}\right] = 90,652$. Cash payment option is best; if need be, borrow the funds from the bank.
NB: You can finance the 39K from the bank, yielding a monthly payment of 1,258.42. The total payment of the zero-interest loan option would be 2,925. If you finance the 90K from the bank, the monthly payment would be 2,904. Of course, the difference in the monthly payment here is 21, whose PV equals 652, as it should.

Problem 3.16
$(1 + 0.19/12)^{12} = 20.75\%$. $(1 + 0.19/52)^{52} = 20.88\%$. $(1 + 0.189/365)^{365} = 20.80\%$.

Problem 3.17
$1K(1.12)^5 = 1,762.34$. $1K(1 + 0.11/2)^{10} = 1,708.14$. $1K(1 + 0.1/12)^{60} = 1,645.31$. $1Ke^{0.115(5)} = 1,777.13$.

Problem 3.18
$5,000. 10K/(1.025)^{10} = 7,811.98$. $9K/(1.025)^{8} = 7,386.72$. $450/[(1.025)^2 - 1] = 8,888.89$.

Problem 3.19
$10K(1.01)^{24} = 12,697.35$. $10K(1.125)^2 = 12,656.25$. $10K(1 + 0.115/365)^{730} = 12,585.54$. $10(1.1)(1.15) = 12,650.00$.

Problem 3.20
$$90K = M\left[\frac{1 - (1 + 0.0923/12)^{-300}}{0.0923/12}\right] \implies M = 769.50.$$  
He will still owe $4K$ on his credit card.
$$82K = M\left[\frac{1 - (1+i)^{-300}}{i}\right] \implies i = 0.868\% \implies EAIR = 10.93\%.$$ 

Problem 3.21
$(1 + 0.1999/365)^{365} = 1.2212$ or 22.12%.

Problem 3.22
Let $r^D_t, r^L_t$ denote the returns in year $t$ for the Dull and Lively funds, respectively.
A 1K investment in the Dull fund would grow to
$$1K(1 + r^D_1)(1 + r^D_2)\cdots(1 + r^D_5) = 132.44,$$
whereas a 1K investment in the Lively fund would grow to
$$1K(1 + r^L_1)(1 + r^L_2)\cdots(1 + r^L_5) = 118.13.$$

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EAIR for the Dull fund is \((1.3244)^{1/5} = 5.78\%\). EAIR for the Lively fund is \((1.1813)^{1/5} = 3.39\%\).

**Problem 3.23**
Let \(\nu^D, \nu^L\) denote the continuously compounded ex-post rate of returns for the respective funds. 
\[1K e^{5\nu^D} = 1,324.36 \implies \nu^D = 5.62\%. \quad 1K e^{5\nu^L} = 1,181.29 \implies \nu^L = 3.33\%.
\]
NB: The average return for the Dull fund is lower than the average return for the Lively fund. The continuously compounded rate of return is more relevant, however, and for a common stochastic model of returns, the expected continuously compounded rate of return is the expected return less one-half the standard deviation of ln returns. Here, the Lively fund’s variability is higher (ex-post), which contributes to its lower continuously compounded rate of return. We’ll learn more about this when we study asset price dynamics.

**Problem 3.24**
\[18.8K = 30(9.95) \left[1 - \frac{(1+i)^{-48}}{i}\right] + 8,995/(1 + i)^{48} \implies i = 0.661\% \implies EAIR = 8.224\%.
\]
\[18.8K = 298.5 \left[1 - \frac{(1.07)^{-4}}{(1.07)^{-72} - 1}\right] + B/(1.07)^4 \implies B = 8,235.07.
\]

**Problem 3.25**
\[150K = Y \left[1 - \frac{(1.1)^{-20}}{0.1}\right] \implies Y = 17,619.
\]
\[LB(10) = Y \left[1 - \frac{(1.1)^{-10}}{0.1}\right] \implies LB(10) = 108,261.
\]
\[108,261 = Y \left[1 - \frac{(1.08)^{-10}}{0.08}\right] \implies Y = 16,134.
\]
By taking out the new loan the couple will save 17,619 - 16,134 = 1,485 each year for 10 years. Even at zero percent interest, the value of this cash flow stream would be 14,850, which is less than the cost of 15K.