Chapter 2 Homework Problem Solutions
ISyE 4311

Problem 2.7
Current bond price is $120\left[\frac{1-(1.07)^{-6}}{0.07}\right] + 1K/(1.07)^6 = 1,238.33$.
NB: Note how the bond price exceeds the face value of $1K$. The reason is that the bond’s
coupon rate is 12% (prevailing interest at time bond was issued), whereas the current interest
rate is now 7%. The bond is said to be selling at a **premium**. You should verify that the bond’s
price will be $1K$ if the discount rate is 12%.

Problem 2.8
PV of the cash outflow at time $t = 54$ is $100K\left[\frac{1-(1.05)^{-20}}{0.05}\right]/(1.05)^{10} = 765,072$.

PV of the cash deposits at time $t = 54$ is $500K/1.05 + D\left[\frac{1-(1.05)^{-10}}{0.05}\right] = 476,190 + 7.7217D$
$\implies D = 37,411$.

Problem 2.16
Price of the car at time $t = 18$ is $15K(1.02)^5 = 16,561$
Future value of deposits at time $t = 18$ is $1K\left[\frac{1.05^6-1}{0.05}\right] = 6,802$.
John will owe $9,759 = Y\left[\frac{1-(1.07)^{-7}}{0.07}\right] \implies Y = 1,811$.

Problem 2.17
Cash outflows are $20K$ at times $t = 4, 5$ and $60K$ at times $t = 15, \ldots, t = 34$.
PV of the cash outflow at time $t = 0$ is
$60K\left[\frac{1-(1.07)^{-20}}{0.07}\right]/(1.07)^{14} + 20K\left[\frac{1-(1.07)^{-2}}{0.07}\right]/(1.07)^3 = 246,512 + 29,518 = 276,030$.

Cash deposits of $D$ occur at times $t = 0, \ldots t = 3$ and $t = 6, \ldots, t = 14$.
PV of the cash deposits at time $t = 0$ is
$D\left(1 + \left[\frac{1-(1.07)^{-3}}{0.07}\right] \right) + D\left[\frac{1-(1.07)^{-9}}{0.07}\right]/(1.07)^5 = 3.6243D+4.6453D = 8.2696D \implies D = 33,379$.

Problem 2.18
The first group’s wage equals $8(1.04)(40K) = 332,800 := R$.
For this first group the company will have a cash outflow of $R$ at times $t = 2, \ldots, t = 21$.
The PV at time $t = 0$ of this cash outflow stream is $R\left[\frac{1-(1.1)^{-20}}{0.1}\right]/(1.1) = 2,575,740 := L_1$.
The second group’s cash outflow stream is the first group’s cash outflow stream shifted two
years to the right and multiplied by $1.04^2$.
The PV of this second group’s cash outflow stream at time $t = 2$ is therefore $L_1(1.04)^2$.
A similar argument shows that the PV’s of the third, fourth and fifth group’s cash outflow
stream at times $t = 4, 5, 6$ are $L_1(1.04)^4, L_1(1.04)^6, L_1(1.04)^8$, respectively.
Let $L$ denote the present value of the entire cash flow outflow stream and let $d = 1.04/1.1$. It follows that

\begin{align*}
L &= L_1(1 + d^2 + d^4 + d^6 + d^8) \\
d^2L &= L_1(d^2 + d^4 + d^6 + d^8 + d^{10}) \\
(1 - d^2)L &= L_1(1 - d^{10}) \implies L = 10,420,415.
\end{align*}

NB: One can solve for $L$ by using the growing annuity formula with $g = (1.04)^2 - 1 = 0.0816$ and $r = (1.1)^2 - 1 = 0.21$, as follows:

\begin{equation}
L = L_1 \left( 1 + \frac{(1.04)^2 \left[ 1 - (1.21/1.0816)^{-4} \right]}{0.21 - 0.0816} \right).
\end{equation}

**Problem 2.26**

The FV of $1$ in 40 years when $i = 4\%$ is \([(1.04)^{40} - 1]/0.04 = 95.0255\).