We develop a structural model to investigate the effects of asymmetric beliefs and agency conflicts on dynamic principal–agent relationships. Optimism has a first-order effect on incentives, investments, and output, which could reconcile the private equity puzzle. Asymmetric beliefs cause optimal contracts to have features consistent with observed venture capital and research and development (R&D) contracts. We derive testable implications for the effects of project characteristics on contractual features. We calibrate our model to data on pharmaceutical R&D projects and show that optimism indeed significantly influences project values. Permanent and transitory components of risk have opposing effects on project values and durations. (JEL D83, D86, G30)

Real-world principal–agent settings such as venture capital (VC) and research and development (R&D) are characterized by imperfect information and asymmetric beliefs about project payoffs (Malmendier and Tate 2005; Baker, Ruback, and Wurgler 2007; Ben-David, Graham, and Harvey 2008). Further, long-term principal–agent relationships feature dynamic actions by both parties. For example, VC projects have salient features such as dynamic staging of the VC’s investments, progressive vesting of the entrepreneur’s stake, and the presence of intertemporal milestones or performance targets that must be met for the project to continue. Observed contracts also feature complex combinations of different types of financial securities with debt- and equity-like
features as well as investment schedules that could increase over time, decrease over time, and sometimes vary nonmonotonically (e.g., Grabowski, Vernon, and DiMasi 2002; Kaplan and Stromberg 2003; Cochrane 2005).

Motivated by the above evidence, we develop a dynamic principal–agent model that differs from previous models by incorporating three key features in a unified framework: (i) asymmetric beliefs and risk attitudes; (ii) dynamic actions by both parties; and (iii) endogenous termination of the relationship. We show how the intertemporal trade-off between the beneficial effects of agent optimism and the detrimental effects of the costs of risk-sharing affects the value generated by the principal–agent relationship and the characteristics of optimal dynamic contracts. The main contributions of our analysis are as follows.

1. We show that optimism has a first-order effect on incentives, investment, and output that could explain why venture capitalists and entrepreneurs invest in innovative ventures despite their high failure rates.

2. Consistent with observed contracts in environments such as VC and R&D, the optimal contracts predicted by our analysis feature (i) debt- and equity-like components; (ii) staged investment; (iii) investment schedules (increasing, decreasing, and nonmonotonic) that depend on the relative magnitudes of the agent’s optimism and the costs of risk sharing; (iv) progressive vesting of the agent’s stake; and (v) the presence of performance targets that must be met for the project to continue. In the benchmark scenario with symmetric beliefs, investments and incentive intensities increase over time. The presence of asymmetric beliefs therefore plays a key role in generating the widely different paths of investments and incentives in observed contracts.

3. Our unified framework also generates new, potentially testable implications for the effects of underlying project characteristics on the time paths of investment and compensation. In contrast with traditional principal–agent models with symmetric beliefs in which risk negatively affects investment, we show that significant levels of optimism could lead to a positive relation between risk and investment.

4. We show that managerial optimism is, indeed, a key driver of R&D project values by calibrating our structural model to data on pharmaceutical R&D projects. Using the calibrated model, we demonstrate that permanent and transitory components of a project’s risk have differing effects on its value and duration.

In our multiperiod framework, a cash-constrained, risk-averse agent with a project approaches a risk-neutral principal for financing. The project generates value through physical capital investments by the principal and effort by the agent. As in the study by Holmström and Milgrom (1987), we model the project’s contractible termination payoff or “outside value.” The variance of the change in the termination payoff is the project’s intrinsic risk, which is
invariant through time. The expected change of the termination payoff has two components: a fixed component that represents the project’s *core output* and a discretionary component that is affected by the principal’s investment and the agent’s effort. The principal and the agent have imperfect information about the project’s core output and have differing priors. They “agree to disagree” about their respective mean assessments. The difference of their mean assessments is the *degree of agent optimism*.

The common variance of the principal’s and the agent’s assessments of the project’s core output is its *transient risk*. The transient risk is resolved over time as intermediate observations of the project’s termination payoff enable the players to update their assessments of the project’s core output in a Bayesian manner. The principal chooses her dynamic investment policy, the agent’s compensation, and the termination time to maximize her expected payoffs. The agent dynamically chooses his effort to maximize his expected utility.

We derive the incentive-efficient contracts between the principal and the agent. The optimal contracts predicted by our analysis have a number of features consistent with observed contracts in VC and R&D, the canonical environments in which our framework is applicable. They feature debt- and equity-like components, staged investment, progressive vesting of the agent’s stake, and the presence of intertemporal milestones or performance targets to ensure continuation of the project.

We first derive “static” properties of the equilibrium contracts that hold in each period. In each period, the evolution of the agent’s stake has a performance-sensitive component that depends on the change in the project’s termination payoff and a performance-invariant component that does not. The characteristics of optimal contracts—the principal’s investments, the agent’s compensation and effort, and the termination time—are determined by the agent’s pay-performance sensitivities, that is, the sensitivities of the change in the agent’s stake in each period to the change in the termination payoff. Conditional on the project’s continuation, the agent’s pay-performance sensitivities and effort as well as the principal’s investments are deterministic functions of time. The performance-invariant component of the change in the agent’s stake in each period is, however, stochastic. It depends on the project’s termination payoff history through its effect on the principal’s and agent’s updated assessments of the project’s core output.

In any period, the agent’s pay-performance sensitivity increases with the degree of agent optimism and declines with the *price of risk* (the product of the agent’s risk aversion and the project’s total risk). When the agent is optimistic, he overvalues the expected change in the project’s termination payoff relative to the principal. The principal exploits the agent’s optimism by increasing the performance-sensitive component of the agent’s compensation relative to the performance-invariant component. The agent’s pay-performance sensitivity therefore increases with his optimism even though optimism does not directly
affect the agent’s effort. In dynamic principal–agent models with symmetric beliefs (e.g., Holmström and Milgrom 1987; Gibbons and Murphy 1992), incentives are affected by only the risk of the project. Because the principal and agent have differing expectations of the project’s core output, their beliefs have a first-order effect on incentives even though neither party directly affects the project’s core output.

The effects of the agent’s optimism and the price of risk on the principal’s investment and the agent’s effort depend on their relative magnitudes. If the degree of agent optimism is less than the price of risk, the principal’s investment and the agent’s effort increase (decrease) with the degree of agent optimism (price of risk). If the degree of agent optimism exceeds the price of risk, however, the principal’s investment decreases (increases) with the degree of agent optimism (price of risk), while the agent’s effort varies nonmonotonically.

In traditional principal–agent models with symmetric beliefs, the negative effects of the costs of risk sharing arising from the agent’s risk aversion always lead to underinvestment of effort relative to the first best scenario in which the principal and agent are risk neutral. In our framework, however, optimism significantly mitigates the detrimental effects of the costs of risk sharing. In fact, if the degree of agent optimism is sufficiently high, it could even cause the agent to overinvest effort relative to the benchmark scenario with symmetric beliefs and universal risk-neutrality. Further, the principal finds it beneficial to exploit agent optimism by significantly increasing her investments in projects relative to the scenario in which beliefs are symmetric. The first-order effect of optimism on investment explains why venture capitalists and entrepreneurs continue to invest in innovative ventures despite their high failure rates.

A body of literature modifies the traditional principal–agent model by introducing nonpecuniary private benefits to explain such investments. Recent evidence, however, shows that these private benefits must be implausibly high relative to typical entrepreneurial incomes to justify observed investment levels, which leads to a “private equity puzzle” (Moskowitz and Vissing-Jorgensen 2002). Our analysis shows that the presence of entrepreneurial optimism, and its rational exploitation by venture capitalists, potentially reconciles the private equity puzzle. Importantly, the beneficial effects of optimism survive without exogenously restricting the set of permissible contracts.

The dynamics of equilibrium contracts are influenced by the principal and agent rationally updating their assessments of the project’s core output over time. The passage of time causes transient risk to be resolved, thereby lowering the price of risk. However, the passage of time also lowers the degree of agent optimism (and, therefore, the rents the principal can extract) as the agent revises his assessments of core output. The process of Bayesian learning leads to different equilibrium dynamics depending on the relative rates of decline of the degree of agent optimism and the price of risk.
If the initial degree of agent optimism is below a threshold relative to the price of risk (the agent is “moderately optimistic”), the beneficial effect of time on the costs of risk sharing dominates its negative effect on the degree of agent optimism. Hence, the agent’s pay-performance sensitivity and effort as well as the principal’s investments increase over time. If the initial degree of agent optimism is above a threshold relative to the price of risk (the agent is “exuberant”), the negative effect of time on the agent’s optimism dominates. The agent’s pay-performance sensitivity, therefore, declines over time. Because the agent is initially exuberant, the degree of agent optimism exceeds the price of risk in early periods. In these periods, the principal’s investments increase, while the agent’s effort varies nonmonotonically. In later periods, when the degree of agent optimism falls below the price of risk, investment and effort both decline over time.

Therefore, depending on the level of agent optimism, his compensation could become more or less sensitive to performance over time, while the principal’s investment path could increase, decrease, or even vary nonmonotonically. In the benchmark scenario with symmetric beliefs, the principal’s investments, the agent’s pay-performance sensitivity, and his effort increase over time. In reality, however, VC and R&D projects feature investment schedules that could be increasing, decreasing, or nonmonotonic over time (e.g., Grabowski et al. 2002; Cochrane 2005). Our analysis shows that the presence of asymmetric beliefs plays a central role in generating different time paths of investments and incentives.

Our results directly lead to potentially testable implications for how changes in agent optimism or the price of risk affect contractual dynamics that we describe in detail in Section 4. The effects of the agent’s optimism and the price of risk on investment and compensation paths depend on whether the agent is moderately optimistic or exuberant. In particular, we show that the presence of optimism could lead to a positive relation between risk and investment, which sharply contrasts with traditional principal–agent models with symmetric beliefs in which risk negatively affects investment.

We complement prior literature in behavioral corporate finance by developing a fully specified structural model that can be calibrated to data. As an illustrative application of the model, we apply it to an R&D setting. The “principal” represents the shareholders of a firm and the “agent” represents the R&D manager who controls and executes R&D activities. We calibrate the model to data on the distributions of investments and returns for a sample of pharmaceutical R&D projects reported in Grabowski et al. (2002). In addition to the other parameters, we calibrate the shareholders’ mean assessment of the core output of an R&D project, the manager’s mean assessment, as well as the true mean core output. Our calibration shows that the average R&D manager is significantly optimistic about the value of an R&D project in absolute terms as well as relative to shareholders, whereas shareholders are actually pessimistic in absolute terms. Consistent with the intuition gleaned from our theoretical
results, the analysis of the calibrated model shows that managerial optimism mitigates the agency costs of risk sharing between shareholders and managers by over 30%.

The numerical analysis of the calibrated model leads to additional testable implications that link a project’s transient and intrinsic risks to its value and duration. Transient risk potentially has conflicting effects on the project’s value and duration. The variance of the evolution of the shareholders’ and manager’s mean assessments of core output increases with the initial transient risk because intermediate signals about core output are relatively more informative (the “signal to noise” ratio increases). As a result, the real option value of continuing the relationship increases, which positively affects the project’s value and duration. An increase in transient risk, however, also increases the costs of risk sharing. In the calibrated model, the positive effect dominates so that the project’s value and duration increase with the initial transient risk. An increase in the project’s intrinsic risk lowers the variance of the evolution of the mean assessments of core output and also increases the costs of risk sharing. Consequently, the project’s value and duration decline with intrinsic risk. Our results, therefore, imply that permanent and transitory components of the risks of projects have differing effects on their values and durations.1

The positive effect of transient risk on project value sharply differs from the predictions of traditional principal–agent models with symmetric beliefs and exogenous termination in which an increase in uncertainty unambiguously has a negative effect on project value because costs of risk sharing increase. As the intuition above shows, the interactive effects of asymmetric beliefs and endogenous termination cause transient risk to positively affect value.

In a dynamic principal–agent model with symmetric beliefs, Gibbons and Murphy (1992) show that the agent’s compensation becomes more sensitive to performance over time. We show that the presence of asymmetric beliefs significantly alters the dynamics of compensation. The agent’s compensation becomes more or less sensitive to performance over time depending on whether he is moderately optimistic or exuberant. Furthermore, depending on the level of agent optimism, the principal’s investment policy could become more aggressive over time or less aggressive over time or vary nonmonotonically.2

Landier and Thesmar (2009) analyze the effects of optimism in a two-period model in which the principal and agent are risk neutral and the contractual space is exogenously restricted to debt contracts. They show that optimistic entrepreneurs are more likely to choose short-term debt. Adrian and Westerfield (2009) analyze a continuous-time principal–agent model with heterogeneous beliefs. They demonstrate that the interaction between

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1 In a dynamic, single-agent framework, Berk, Green, and Naik (2004) show that permanent and transitory components of a project’s risk have complex interactive effects on its value.

incentive provision and learning creates an intertemporal source of “disagreement risk” that alters optimal risk sharing. We complement their study in several respects. First, both the principal and the agent take productive actions in our model. We derive new implications for the effects of heterogeneous beliefs, agency conflicts, and project characteristics on the principal’s dynamic investment policy and the agent’s compensation. Second, both the principal and the agent rationally update their assessments of the project’s core output in a Bayesian manner. Third, the equilibrium termination time of the project in our model is a stopping time, which leads to predictions for the effects of the project’s intrinsic/transient risks, and the degree of agent optimism on the duration and value of the project. Fourth, we calibrate our structural model and obtain quantitative insights into the level and effects of optimism.

Gervais, Heaton, and Odean (2007) develop a two-period principal–agent model and show that managerial overconfidence can increase firm value. Our focus is on the effects of optimism, rather than overconfidence, in a dynamic principal–agent setting. A different stream of the literature examines the effects of asymmetric beliefs on financial intermediation. Allen and Gale (1999) analyze a two-period binomial model with asymmetric beliefs and show that financial markets perform better than intermediaries in the financing of new technologies when there is significant diversity of opinion among investors. Coval and Thakor (2005) show that financial intermediaries arise endogenously when entrepreneurs are optimistic relative to investors.

The plan for the article is as follows. In Section 1, we develop a static, single-period model to illustrate the intuition underlying the effects of optimism on incentives. We present the dynamic model in Section 2. We derive the equilibrium and its properties in Sections 3 and 4. In Section 5, we present the calibration of the model and its numerical analysis. Section 6 concludes. We provide all supplementary material in Appendixes A, B, and C.

1. A Single-Period Model

We begin our investigation of the effects of asymmetric beliefs on principal–agent relationships in a static, single-period model. The properties of the equilibrium in the dynamic model that we analyze in subsequent sections depend on those of the equilibrium in the static model.

1.1 The project’s payoff

We consider a single-period framework with dates 0, 1. An agent with a project approaches a principal for financing at date zero. If the principal agrees to invest in the project, she provides an initial amount of “seed” capital, $V_0$. The project’s payoff, which occurs at date 1, depends on the principal’s incremental capital investment $c_0$ over the period and the agent’s effort $\eta_0$. The project’s
payoff, $V_1$, satisfies

\[
V_1 - V_0 = \Theta - l_0 + S_1 + Ac_0^\alpha \eta_0^\beta .
\]

(1)

*Discretionary output* is the portion of the project’s incremental payoff, $V_1 - V_0$, that depends on the principal’s physical capital and the agent’s human capital (effort) investments, and is represented by a Cobb-Douglas production function. *Base output* represents the portion of the payoff that is independent of the principal’s and agent’s actions.

The first component of the base output, $\Theta$, represents the project’s *core output*. The principal and agent have imperfect information about $\Theta$ and could also differ in their beliefs about its value. The principal’s and agent’s respective priors on $\Theta$ are normally distributed with

\[
\Theta \sim N(\mu_{Pr}^0, \sigma_{Pr}^2), \quad \Theta \sim N(\mu_{Ag}^0, \sigma_{Ag}^2), \quad \text{and} \quad \mu_{Ag}^0 \geq \mu_{Pr}^0 .
\]

(2)

The agent is initially more optimistic than the principal; however, we do not make any assumptions about how the agent’s and principal’s mean assessments of core output relate to its true mean.

The principal’s and agent’s respective beliefs are common knowledge, that is, they agree to disagree.\(^3\) This assumption is supported by theoretical arguments as well as empirical evidence. Morris (1995) argues that there is nothing in Bayesian decision theory or standard theories of rationality that requires agents to have the same priors. Allen and Gale (1999) discuss the long tradition in economics of allowing for differences in prior beliefs. In fact, the requirement that agents have a common prior can lead to counterfactual predictions, such as the impossibility of trade in common value assets. Kandel and Pearson (1995) provide evidence that trading around earnings announcements is due to differences in priors. The common prior assumption is especially questionable in new industries and technologies. As discussed by Allen and Gale, casual empiricism suggests that there is a wide variation in views on the effectiveness and value of an innovation immediately after the innovation has occurred. Since data based on actual experience with new products or technologies are nonexistent or small, there is diversity of opinion and people agree to disagree.

The second component of the base output, $l_0$, represents “operating costs,” which could include wages to salaried employees, depreciation expenses, decline in revenues due to competition, fixed costs arising from increases in the scale of the project, etc. These costs are deterministic.

---

\(^3\) While the principal and agent disagree on the mean of the project’s core output, they agree on its variance, $\sigma_0^2$. The behavioral economics literature (Baker et al. 2007) distinguishes between *optimism* and *overconfidence*. An agent is “optimistic” if his assessment of the mean (the first moment) of the distribution is higher than that of the principal, while he is “overconfident” if his assessment of the variance (the second moment) of the distribution is lower than that of the principal. The agent is, therefore, optimistic but not overconfident in our framework.
The third component of the base output, $S_1$, is a normal random variable with mean 0 and variance $s^2$. The random variable $S_1$ represents the project’s intrinsic risk, which is the uncertainty in the project’s payoff from the perspective of a hypothetical “omnisicient” agent who knows the project’s true core output $\Theta$.

### 1.2 Principal–agent contracting

The risk-neutral principal offers the agent a contract that specifies the agent’s payoff at date 1, $P_1$; the principal’s capital investment, $c_0$; and the agent’s effort, $\eta_0$. The agent is risk averse with Constant Absolute Risk Aversion (CARA) preferences. His total expected utility (including disutility of effort) is

$$E^{Ag}\left[-\exp(-\lambda(P_1 - k\hat{y}_0))\right],$$

where the superscript on the expectation denotes that it is with respect to the agent’s beliefs.

The allocation of bargaining power between the principal and the agent is determined by the “certainty equivalent” reservation utility or promised payoff, $P_0$, that the agent must be guaranteed to participate in the contract. We consider all possible allocations of bargaining power between the principal and the agent that are indexed by different values of $P_0$. A contract is feasible if and only if it is incentive compatible for the agent to exert the specified effort, $\eta_0$, i.e.,

$$\eta_0 = \arg \max_\eta E^{Ag}\left[-\exp(-\lambda(P_1 - k\hat{y}_0))\right] \text{ (incentive compatibility)},$$

and the contract meets the agent’s participation constraint at date zero, i.e.,

$$E^{Ag}\left[-\exp(-\lambda(P_1 - k\hat{y}_0))\right] \geq -\exp(-\lambda P_0) \text{ (agent’s participation)}.$$

As we discuss later in Section 3, it suffices to restrict consideration to affine contracts in which the agent’s promised payoff at date 1 satisfies

$$P_1 - P_0 = a_0 + b_0(V_1 - V_0).$$

In Equation (6), $a_0$ determines the performance-invariant or “cash” component of the agent’s compensation, while $b_0(V_1 - V_0)$ determines the performance-sensitive component. We refer to the parameter $b_0$ as the agent’s pay-performance sensitivity.

**Remark 1.** The agent’s compensation can be rewritten as $P_1 = (P_0 + a_0 - b_0V_0) + b_0V_1$, which implies that the agent’s contract can be implemented through a cash payment, $P_0 + a_0 - b_0V_0$, plus an equity stake, $b_0$. As we will see shortly, the static model actually represents a single period in our dynamic model in which all payoffs occur at the termination time. Unlike the static model, the agent’s compensation contract cannot be implemented through a
simple equity stake, so that the pay-performance sensitivity should not be interpreted as an equity share (see Remark 3).

1.3 The optimal contract
We assume the following condition on the parameters $\alpha, \beta,$ and $\gamma$ for the remainder of the article:

**Assumption 1.** $(1 - \alpha)\gamma/\beta > 2$.

This condition implies that (i) the agent faces decreasing returns to scale from the provision of effort; and (ii) the agent’s disutility from effort is sufficiently pronounced relative to its positive contribution to output so that an optimal contract between the principal and the agent exists.

Fix the agent’s contractual parameters $a_0, b_0,$ and the principal’s investment $c_0$. A feasible contract must satisfy the incentive compatibility condition (4).

Substituting Equations (6) and (1) into Equation (4), and using the fact that $\Theta + S \sim N(\mu_0^{Ag}, \sigma_0^2 + s^2)$ under the agent’s beliefs,

$$
\eta_0 = \arg \max_{\hat{\eta}} E^{Ag}\{- \exp\{- \lambda(\mathbf{P}_0 + a_0 + b_0(\Theta - l_0 + S_1 + Ac_0^\alpha \hat{\eta}^\beta) - k \hat{\eta}^\gamma)\}\}
$$

$$= \arg \max_{\hat{\eta}} - \exp\{- \lambda(\mathbf{P}_0 + a_0 + b_0(\mu_0^{Ag} - l_0 + Ac_0^\alpha \hat{\eta}^\beta) - k \hat{\eta}^\gamma
- 0.5\lambda b_0^2(\sigma_0^2 + s^2))\}\}.
$$

(7)

Assumption 1 guarantees that a unique solution to Equation (7) exists. We conclude that the effort level specified by the contract is incentive compatible for the agent if and only if

$$
\eta_0 = \eta(b_0, c_0) := \left(\frac{A\beta c_0^\alpha b_0}{\gamma k}\right)^{\frac{1}{\gamma - \beta}}.
$$

(8)

The arguments of the agent’s effort indicate its dependence on the pay-performance sensitivity $b_0$ and the principal’s investment $c_0$. (The effort does not depend on the contractual parameter $a_0$.) We henceforth refer to $\eta(\cdot, \cdot)$ as the agent’s optimal effort function.

A feasible contract must also satisfy the agent’s participation constraint (5). The principal chooses the contract so that the agent’s participation or “promise keeping” constraint is satisfied at equality. Consequently, the contractual parameter $a_0$ can be expressed in terms of the contractual parameters $b_0$ and $c_0$ via the identity

$$
a_0(b_0, c_0) := 0.5\lambda b_0^2(\sigma_0^2 + s^2) + k\eta_0(b_0, c_0)^\gamma - b_0(Ac_0^\alpha \eta_0(b_0, c_0)^\beta - l_0 + \mu_0^{Ag}).
$$

(9)

We henceforth refer to $a_0(\cdot, \cdot)$ as the agent’s performance-invariant compensation function.
The principal is risk neutral. She chooses the contractual parameters $b_0$ and $c_0$ to maximize her expected payoff. Because $V_0$ and $P_0$ are fixed, the contractual parameters maximize the principal’s expected incremental payoff, which is the expected change in her stake $(V_1 - P_1) - (V_0 - P_0)$ less her investment $c_0$. The expectation is under the principal’s beliefs. Fix contractual parameters $b_0$ and $c_0$. The risk-neutral principal’s expected incremental payoff is

$$\Lambda(b_0, c_0) := E_{Pr}^0[(V_1 - P_1 - c_0) - (V_0 - P_0)]$$

$$= E_{Pr}^0[(1 - b_0)(V_1 - V_0) - a_0(b_0, c_0) - c_0].$$

Substituting the agent’s effort (8) into Equation (10) and using the fact that $E_{Pr}^0[V_1 - V_0] = E_{Pr}^0[\lambda(c_0^\alpha - l)] = (A c_0^\alpha - l_0 + \mu_{Pr}^0)$, the principal’s expected incremental payoff simplifies to

$$\Lambda(b_0, c_0) = \Delta_0 b_0 - 0.5 p_0 b_0^2 + \phi(b_0 c_0^\alpha) - c_0 + \mu_{Pr}^0 - l_0. (11)$$

In Equation (11),

$$\Delta_0 := \mu_{Ag}^0 - \mu_{Pr}^0$$

represents the degree of agent optimism,

$$p_0 := \lambda(\sigma_0^2 + s^2)$$

represents the price of risk, and

$$\phi(b) := A^{\gamma} \left( \frac{1}{k} \right) \frac{\beta}{1 - \frac{\beta b}{\gamma}} \left( \frac{\beta b}{\gamma} \right)^{\frac{\alpha}{\gamma}} \left( 1 - \frac{\beta b}{\gamma} \right).$$

The principal’s capital investment and the agent’s pay-performance sensitivity maximize the principal’s expected incremental payoff (11). Fix the pay-performance sensitivity $b_0$. As we show later, the optimal investment is zero whenever $b_0 \geq \gamma/\beta$, which occurs if the degree of agent optimism $\Delta_0$ is sufficiently high. We impose an upper bound on $\Delta_0$ that guarantees that the investment is positive if the project continues. (The precise upper bound is specified later in Assumption 2.) Then fix a value of $b_0 \in (0, \gamma/\beta)$. Assumption 1 guarantees that the function $\Lambda(b_0, \cdot)$ is strictly concave in the investment $c_0$ (the exponent on $c_0$ is guaranteed to be less than 1). Setting the partial derivative of $\Lambda(b_0, \cdot)$ with respect to $c_0$ equal to zero yields

$$c(b_0) := \left[ \frac{\alpha \gamma}{\gamma - \beta} \phi(b_0) \right]^{\gamma - \beta}. (15)$$

---

4 The principal invests in the project only if her expected payoff net of her investments is nonnegative. We set up the principal’s objective this way in anticipation of the analysis of the equilibrium in the dynamic model.
We refer to $c(\cdot)$ as the optimal investment function. As a function of the pay-performance sensitivity $b_0$, the principal’s expected incremental payoff can now be expressed as

$$\Lambda(b_0, c(b_0)) = \Delta_0 b_0 - 0.5 p_0 b_0^2 + K c(b_0) + \left(\mu_0^{Pr} - l_0\right),$$

(16)

where $K = \frac{\gamma - \beta - ay}{ay} > 0$ by Assumption 1. Ignoring the constant term $(\mu_0^{Pr} - l_0)$, the principal chooses the agent’s pay-performance sensitivity to solve

$$b_0^* := \arg \max_{b_0 \geq 0} \Delta_0 b_0 - 0.5 p_0 b_0^2 + K c(b_0).$$

(17)

The optimal contract is completely specified by $b_0^*$; the optimal investment $c_0^*$ is $c(b_0^*)$, the agent’s optimal effort $\eta_0^*$ is $\eta_0(b_0^*, c_0^*)$, and the performance-invariant parameter $\alpha_0^*$ is $a_0(b_0^*, c_0^*)$.

1.4 The optimal investment function

It follows directly from Equations (16) and (17) that the properties of the optimal contract critically depend on the properties of the optimal investment function $c(\cdot)$. The next proposition establishes properties of this function that play a key role in our subsequent analysis. The proof of this result and all subsequent ones are in Appendix A.

**Proposition 1.1** (Optimal Investment Function). The optimal investment function $c(\cdot)$ is

(i) strictly quasi-concave on $[0, \frac{\gamma}{\beta}]$ and achieves its maximum at $b = 1$, and

(ii) strictly concave on $[0, b^M]$ and strictly convex on $[b^M, \frac{\gamma}{\beta}]$, where $b^M \in (1, \frac{\gamma}{\beta})$ is the unique minimum of the marginal optimal investment function $c'(\cdot)$.

(iii) Further, $c'(0) = \infty$.

Figure 1 illustrates the structure of the optimal investment function. The intuition for the nonmonotonicity of the function is that an increase in the agent’s pay-performance sensitivity affects the principal’s investment in two distinct but opposite ways. On the positive side, the agent increases his effort. Because investment and effort are complementary, the increase in the agent’s effort provides an incentive for the principal to increase her investment. On the negative side, since the agent’s disutility of effort increases, the principal’s cost to maintain the agent’s participation also increases. For lower values of the pay-performance sensitivity, the complementarity of investment and effort causes the benefits of increased output to dominate. Hence, the principal finds it beneficial to increase her investment. However, beyond a threshold level of pay-performance sensitivity, the costs of inducing high effort from the agent...
are so high that the principal lowers her investment. In other words, it is optimal for the principal to allow output to be dominated by the agent’s effort.

1.5 The principal’s periodic flow function

By Equation (17), the properties of the optimal contract depend on the properties of the principal’s periodic flow function that is defined as follows:

\[ F_0(b_0) := \Delta_0 b_0 - 0.5 p_0 b_0^2 + K c(b_0). \] (18)

It consists of three components: the economic rent from the agent’s optimism, which the principal extracts from the agent by exploiting his optimism about the project’s core output (we elaborate on this later); the costs of risk sharing, between the risk-neutral principal and the risk-averse agent; and the return on investment, which reflects the principal’s expected return as a result of her investment and the agent’s effort. By Equation (17), the optimal pay-performance sensitivity, \( b_0^* \), maximizes the principal’s periodic flow function.

The ratio of the degree of the agent’s optimism to the price of risk, \( \Delta_0/p_0 \), provides an a priori bound on the value of \( b_0^* \).

**Proposition 1.2.** The agent’s optimal pay-performance sensitivity is bounded above by \( \max\{1, \Delta_0/p_0\} \).

We assume that the initial degree of agent optimism, \( \Delta_0 \), is below a threshold relative to the price of risk, \( p_0 \), defined in Equation (13).

**Assumption 2.** \( \Delta_0/p_0 \leq b^M \), where \( b^M \) is defined in Proposition 1.1.

By Proposition 1.2 and Assumption 2, the agent’s optimal pay-performance sensitivity must lie in the interval \( [0, b^M] \). By Proposition 1.1, the optimal
investment function \( c(\cdot) \) is strictly concave on the interval \([0, b^M]\). It then follows that the principal’s periodic flow function (18) is also strictly concave on \([0, b^M]\). Consequently, there exists a unique solution \( b^*_0 \) to Equation (17). Moreover, \( b^*_0 \) must be strictly positive, since the derivative of the periodic flow function at zero is infinite.

1.6 Properties of the optimal contract
We now explore the effects of the degree of agent optimism, \( \Delta_0 = \mu^A_0 - \mu^P_0 \), and the price of risk, \( p_0 = \lambda(\sigma^2_0 + s^2) \), on the optimal contract.

**Proposition 1.3** (Effects of Optimism on Optimal Contract). The degree of agent optimism has the following effects on the optimal contractual parameters.

(i) The agent’s optimal pay-performance sensitivity, \( b^*_0 \), increases with the degree of agent optimism, \( \Delta_0 \). If \( \Delta_0 < p_0 \), \( b^*_0 < 1 \); if \( \Delta_0 = p_0 \), \( b^*_0 = 1 \); and if \( \Delta_0 > p_0 \), \( b^*_0 > 1 \).

(ii) The principal’s optimal investment, \( c^*_0 \), increases with \( \Delta_0 \) for \( \Delta_0 < p_0 \) (that is, when \( b^*_0 < 1 \)), and decreases with \( \Delta_0 \) for \( \Delta_0 > p_0 \) (that is, when \( b^*_0 > 1 \)).

(iii) The agent’s optimal effort, \( \eta^*_0 \), increases with \( \Delta_0 \) for \( \Delta_0 < p_0 \) (that is, when \( b^*_0 < 1 \)), and varies nonmonotonically with \( \Delta_0 \) for \( \Delta_0 > p_0 \) (that is, when \( b^*_0 > 1 \)).

When the agent is optimistic, he overvalues the project’s expected payoff relative to the principal. To exploit the agent’s optimism, the principal increases the performance-sensitive component of the agent’s compensation (because the agent overvalues this component) and decreases the performance-invariant or cash component. As the degree of agent optimism increases, the extent to which the agent overvalues the project’s payoff increases, which implies his pay-performance sensitivity increases. Note that the agent’s pay-performance sensitivity increases with the degree of agent optimism even though optimism does not directly affect the agent’s effort.

As long as the degree of agent optimism is less than the price of risk, the agent’s effort increases with the degree of optimism because his pay-performance sensitivity increases. The agent’s optimism, therefore, indirectly affects his effort through the sensitivity of his contractual compensation to the project’s payoff. Because investment and effort are complementary, the principal also increases her investment as the agent’s optimism increases.

If the degree of agent optimism exceeds the price of risk, however, the agent’s pay-performance sensitivity exceeds one. In this region, it follows from the intuition for the nonmonotonicity of the optimal investment function \( c(\cdot) \) (see the discussion following Proposition 1.1) that it is too costly for the principal to encourage the agent to exert high effort by increasing her investment. In other words, when the agent’s optimism is above a threshold, it
is optimal for the principal to allow the project’s output to be dominated by the agent’s effort relative to her investment. The principal’s investment, therefore, declines with the degree of agent optimism in this region. The increase in the agent’s pay-performance sensitivity with optimism has a positive effect on his effort, while the decline in the principal’s investment has a negative effect. The interaction between these opposing effects causes the agent’s effort to vary nonmonotonically in general.

The first-order effect of the agent’s optimism on optimal incentives sharply differs from the predictions of traditional principal–agent models such as Holmström and Milgrom’s (1987) in which incentives are affected by only the risk of the project. The fact that the principal and agent have differing expectations of the project’s core output causes incentives to depend on their beliefs even though neither the principal nor the agent directly affects the project’s core output.

Our results and the intuition underlying them imply that optimism could significantly mitigate the detrimental effects of the costs of risk sharing. In fact, if the degree of agent optimism is sufficiently high, it could even cause the agent to overinvest effort relative to the benchmark scenario with symmetric beliefs and universal risk-neutrality. Further, the principal finds it beneficial to exploit this optimism. In contrast, in traditional principal–agent models with symmetric beliefs, the negative effects of the costs of risk sharing arising from the agent’s risk aversion always lead to underinvestment of effort relative to the first best scenario in which the principal and agent are risk neutral. The first-order effect of optimism on investment accords well with evidence that venture capitalists and entrepreneurs continue to invest in highly innovative ventures even though the chances of failure are extremely high.

**Proposition 1.4 (Effects of Risk on Optimal Contract).** Risk has the following effects on the optimal contractual parameters.

(i) The agent’s optimal pay-performance sensitivity, \(b_0^*\), decreases with the price of risk, \(p_0\).

(ii) The principal’s optimal investment, \(c_0^*\), decreases with \(p_0\) for \(\Delta_0 < p_0\) (that is, when \(b_0^* < 1\)), and increases with \(p_0\) for \(\Delta_0 > p_0\) (that is, when \(b_0^* > 1\)).

(iii) The agent’s optimal effort, \(\eta_0^*\), decreases with \(p_0\) for \(\Delta_0 < p_0\) (that is, when \(b_0^* < 1\)) and varies nonmonotonically with \(p_0\) for \(\Delta_0 > p_0\) (that is, when \(b_0^* > 1\)).

(The proof of this proposition is omitted, since it mirrors the proof for Proposition 1.3.)

An increase in the price of risk increases the costs of risk sharing between the principal and the agent, which lowers the “power of incentives” that can be provided to the risk-averse agent as measured by his pay-performance.
sensitivity. The nonmonotonic variation of the principal’s optimal investment with the agent’s pay-performance sensitivity, however, implies that the price of risk does not always have a negative effect on the principal’s investment. When the degree of agent optimism is less than the price of risk, investment and effort decline with the price of risk. When the degree of agent optimism is greater than the price of risk, investment increases with the price of risk and the agent’s effort varies nonmonotonically. The presence of significant optimism could, therefore, lead to a positive relation between risk and investment.

**Remark 2.** The pay-performance sensitivity could exceed one when the principal is significantly pessimistic relative to the agent and therefore requires additional insurance (or “collateral”) against low project realizations to be induced to invest in the project (see Propositions 1.3 and 1.4). The standard empirical proxy for the pay-performance sensitivity is the total “delta” of the agent’s stake (see Core, Guay, and Larcker 2003), and it could be close to one or even exceed it. For example, bank debt financing is common for small startup firms. The “principal” holds a pure debt contract. Because the agent holds all the equity, the delta of his stake either equals one (if the debt is risk free) or could be close to one. Further, in private debt financing, the agent is often required to post additional assets as collateral that could be seized by the principal in the event of default. One possible implementation of such a financing arrangement is for the principal to hold debt plus a security that provides insurance against bad states such as a put option. In addition to equity, the agent is therefore short a put option. Since a put option has a negative delta, the total “delta” of the agent’s stake exceeds one. As we further discuss in Remark 3, however, we again emphasize that the agent’s pay-performance sensitivity in each period in the dynamic model should not be interpreted as an equity stake.

2. The Dynamic Model

The dynamic model is a multiperiod extension of the static model. It introduces new economic forces that are absent in the static setting, namely, the effects of long-term contracting and learning. The principal and agent can update their assessments of the project’s core output based on intermediate signals. The process of Bayesian learning leads to different equilibrium dynamics depending on the relative magnitudes of the degree of agent optimism and the costs of risk sharing between the principal and the agent, both of which decline over time (though at different rates). Furthermore, the contractual relationship could be terminated at an intermediate random date based on posterior assessments of the project’s quality.

We consider a finite horizon framework with equally spaced dates 0, 1, 2, . . . , T. As in the single-period model, the principal provides an initial seed capital $V_0$ at date zero. She offers the agent a long-term contract that describes her subsequent investments in the project, the agent’s compensation,
and the termination date of the project. Investments are made at the beginning of each period. The termination date could, in general, be contingent on the project’s history.

2.1 The termination payoff process

The key state variable in the model is the project’s termination payoff $V_i$, which is the total payoff if the project is terminated at date $i$. The termination payoff process is contractible.\(^5\) For simplicity, we assume that there are no intermediate cash flows, so that all payoffs occur upon termination.

The initial termination payoff of the project equals the seed capital investment $V_0$. As in the static model, the incremental termination payoff or the change in the termination payoff over period $[i, i+1]$, $V_{i+1} - V_i$, is the sum of the base output, a stochastic component that is unaffected by the actions of the principal and agent, and the discretionary output, a deterministic component that depends on the physical capital investment by the principal and the human capital investment (effort) by the agent. It is given by

$$V_{i+1} - V_i = \Theta - l_i + S_{i+1} + Ac_i^a \eta_i^\beta.$$  \(19\)

As in the static model, $\Theta$ represents the project’s core output. The principal and agent have imperfect information about $\Theta$. Their respective initial priors on $\Theta$ are described exactly as in Equation (2) of the static model and are common knowledge. The $l_i$ represents “operating costs,” which are deterministic. The third component of the base output, $S_{i+1}$, is a random variable that represents the “intrinsic” component of the project’s risk in period $[i, i+1]$. The variables, $\{S_{i+1}\}$, are i.i.d. normal random variables with mean $0$ and variance $s^2$ that are independent of $\Theta$. We refer to the parameter $s$ as the project’s intrinsic risk.

The discretionary output is observable to the principal and the agent. However, as in the literature on incomplete contracting, the discretionary output is nonverifiable and, therefore, noncontractible. Consequently, the agent’s contract can be explicitly contingent only on the termination payoff process.

Define the random variable,

$$\xi_{i+1} := V_{i+1} - V_i - Ac_i^a \eta_i^\beta + l_i = \Theta + S_{i+1}, \quad i = 1, 2, \ldots, T - 1.$$  \(20\)

---

\(^5\) The termination payoff is the value of the project “outside the principal–agent relationship,” that is, it is the present value of the project’s future earnings from the perspective of “outside” investors at date $i$. As in studies such as Kiyotaki and Moore (1997), the principal and agent possess project-specific skills that are not transferable, so that the termination payoff (the “outside” value) is, in general, lower than its “inside” value. We can alter the model so that the principal and agent observe only unbiased “noisy” signals of the project’s termination payoff. More precisely, $V_i = V_i^{\text{true}} + \epsilon_i$, where $V_i^{\text{true}}$ is the actual termination payoff, $\epsilon_i \sim N(0, \sigma_i^2)$, and the $\{\epsilon_i\}$ are i.i.d. across time and are independent of $\{V_i\}$. Our results remain qualitatively unaltered.
It follows from well-known formulas (DeGroot 1970) that the posterior distribution on $\Theta$ for each date $i; i \geq 1$ is $N(\mu_{iPr}, \sigma_i^2)$, $N(\mu_{iAg}, \sigma_i^2)$, where

$$\sigma_i^2 = \frac{s^2 \sigma_{i-1}^2}{s^2 + \sigma_i^2} = \frac{s^2 \sigma_0^2}{s^2 + i \sigma_0^2},$$ (21)

$$\mu_i^j = \frac{s^2 \mu_{i-1}^j + \sigma_i^2 \xi_i}{s^2 + \sigma_i^2} = \frac{s^2 \mu_0^j + \sigma_0^2 (\sum_{t=1}^i \xi_t)}{s^2 + i \sigma_0^2}, \quad j = Pr, Ag.$$ (22)

Note that $E[\mu_i^j | \mu_{i-1}^j] = \mu_{i-1}^j$, since $E[\xi_i | \mu_{i-1}^j] = \mu_{i-1}^j$, and that $\sigma_i$ tends to zero as $i \to \infty$. We refer to the parameter $\sigma_i$ as the project’s \textit{transient} risk.

Define

$$\Delta_i := \mu_{iAg} - \mu_{iPr} = \frac{s^2 \Delta_0}{s^2 + i \sigma_0^2} = \frac{\sigma_i^2}{\sigma_0^2} \Delta_0, \quad i = 0, 1, 2, \ldots.$$ (23)

We henceforth refer to $\Delta_i$ as the \textit{degree of agent optimism} at date $i$. It follows from Equation (23) that the degree of agent optimism declines deterministically over time as the principal and agent update their priors on $\Theta$ in a Bayesian manner based on observations of the termination payoff process.

2.2 Principal–agent contracting

The contract describes the principal’s incremental capital investments over time, the agent’s effort, and the payoffs to be received by both parties upon termination. Let $\{\mathcal{F}_i\}$ denote the information filtration generated by the history of termination payoffs, the principal’s investments, and the project’s discretionary outputs. A feasible contract is described by the quadruple $(P_\tau, c, \eta, \tau)$, where $c$ and $\eta$ are $\{\mathcal{F}_i\}$-adapted stochastic processes, $\tau$ is an $\{\mathcal{F}_i\}$-stopping time, and $P_\tau$ is a nonnegative $\{\mathcal{F}_i\}$-measurable random variable. $P_\tau$ is the agent’s contractually promised payoff and $V_\tau - P_\tau$ is the principal’s payoff at the termination time $\tau$ of the contractual relationship. $c_i$ is the principal’s investment and $\eta_i$ is the agent’s effort in period $[i, i+1]$. The termination time $\tau$ is, in general, a \textit{stopping} time that is contingent on the project’s performance history. The contract must be \textit{incentive compatible} for the agent. That is, given the termination date $\tau$, the agent’s contractually promised payoff $P_\tau$, and the principal’s investments $c$, it is optimal for the agent to choose the effort levels $\eta$ specified by the contract.

The principal is risk neutral, while the agent is risk averse with intertemporal CARA preferences. The principal and the agent have a common time discount rate that we normalize to zero in our theoretical analysis to simplify notation. The agent’s expected utility at date zero from a contract $(P_\tau, c, \eta, \tau)$ is

$$E_0^{Ag} \left[ -\exp \left\{ -\lambda \left( P_\tau - \sum_{i=0}^{\tau-1} k \eta_i^\gamma \right) \right\} \right].$$ (24)
In Equation (24), $E_{0}^{Ag}$ denotes the expectation with respect to the agent’s beliefs at time 0. We define the agent’s certainty equivalent expected future utility, $P_i$, from the contract at any date $i$ as

$$
\exp(-\lambda P_i) = E_{i}^{Ag} \exp \left( -\lambda \left( P_\tau - \sum_{j=i}^{\tau-1} k_\gamma j \right) \right),
$$

where the notation $E_{i}^{Ag}$ denotes the agent’s expectation conditioned on the information available at date $i$, that is, the $\sigma$-field $\mathcal{F}_i$. The agent’s certainty equivalent future expected utility at the contractual termination date $\tau$ is his contractually promised terminal payoff $P_\tau$. We refer to the agent’s certainty equivalent expected future utility $\{P_i, i \geq 0\}$ as his promised payoff.

We define the agent’s continuation utility ratio at date $i$, CUR$(i)$, as the ratio of his expected utility from continuing the relationship to his utility if the relationship were terminated at date $i$ and he received his promised payoff $P_i$. Since the agent has a negative exponential utility function, the continuation utility ratio is

$$
CUR_i := E_{i}^{Ag} \left[ \exp \left\{ -\lambda \left( P_\tau - P_i - \sum_{j=i}^{\tau-1} k_\gamma j \right) \right\} \right].
$$

It follows from the definition of the agent’s promised payoff process in Equation (25) that the agent’s continuation utility ratio as defined above must be equal to one at all dates and states. In other words, the promise keeping constraint must be satisfied in each state of nature, that is, the contractually promised payoff of the agent must actually be delivered by the contract.

As in the static model, we consider all possible allocations of bargaining power between the principal and the agent that are indexed by different values of the agent’s initial promised payoff $P_0$. A contract $(P_\tau, c, \eta, \tau)$ is feasible if and only if it is incentive compatible for the agent and meets his participation constraint at date zero. The risk-neutral principal’s optimal contract choice is a feasible contract that maximizes her expected payoff net of her investments. More precisely, a contract $(P_\tau, c, \eta, \tau)$ is optimal if and only if it solves the optimization problem

$$
(P_\tau, c, \eta, \tau) = \arg \max_{(P'_\tau, c', \eta', \tau')} E_{0}^{Pr} \left[ V_{\tau'} - P_{\tau'} - \sum_{j=0}^{\tau'-1} c'_j \right],
$$

where $E_{0}^{Pr}$ denotes the expectation with respect to the principal’s beliefs at time 0 and the maximization is over all feasible contracts.
3. Dynamic Equilibrium

By Equation (24), the agent has multiplicatively separable CARA preferences and the termination payoff evolves as a Gaussian process as in the studies by Holmström and Milgrom (1987) and Gibbons and Murphy (1992). Following Gibbons and Murphy, therefore, we restrict consideration to contracts in which the agent’s promised payoff process has the affine form

\[ P_{i+1} - P_i = a_i + b_i(V_{i+1} - V_i), \quad i = 0, 2, \ldots, T - 1. \]  

(28)

In Equation (28), the contractual parameters \( a_i, b_i \) are \( \mathcal{F}_i \) measurable.\(^6\) It follows easily from Equation (28) that the agent’s contractual payoff \( P_\tau \) at the termination date \( \tau \) is

\[ P_\tau = P_0 + \sum_{i=0}^{\tau-1} [a_i + b_i(V_{i+1} - V_i)]. \]  

(29)

As discussed in Section 2.2, the agent’s initial promised payoff \( P_0 \) determines the allocation of bargaining power between the principal and the agent. Extending the interpretation of the contractual parameters in the static model, the parameter \( a_i \) is the “performance invariant” component of the agent’s incremental promised payoff, while \( b_i \) is the agent’s pay-performance sensitivity in period \([i, i + 1]\). A feasible contract is characterized by \( P_0, a, b, c, \eta, \) and \( \tau \), where \( P_0 \) is the agent’s initial promised payoff, \( a, b \) determine the evolution of the agent’s promised payoff (see Equation (28)), \( c \) is the principal’s investment process, \( \eta \) is the agent’s effort process, and \( \tau \) is the termination date.

The dynamic equilibrium is a “periodic” repetition of the equilibrium of the static model but incorporates the fact that the principal’s and agent’s posterior assessments of the project’s core output, \( \Theta \), evolve over time due to Bayesian learning as described in Equations (21)–(23). Let

\[ p_i := \lambda(\sigma_i^2 + s^2) \]  

(30)

represent the price of risk at date \( i \). In direct analogy with Equations (8), (9), and (18) in the static model, define the agent’s optimal effort function,

\[ \eta(b, c) := \left( \frac{A\beta c^\alpha b}{\gamma k} \right)^{1/\gamma - \beta}, \]  

(31)

the agent’s performance-invariant compensation function at date \( i \),

\[ a_i(b, c) := 0.5 p_i b^2 + k \eta^\gamma - b \left( A c^\alpha \eta^\beta - l_i + \mu_i^A g \right). \]  

(32)

\(^6\) As in Holmström and Milgrom (1987), we can reformulate our model in continuous time and prove that the optimal contract must have the affine structure (28) (details available).
and the principal’s periodic flow function at date i,

\[ F_i(b) := \Delta_i b - 0.5 p_i b^2 + Kc(b). \]  

**Theorem 3.1** (Dynamic equilibrium). Conditional on the project not being terminated before date \( i \leq T - 1 \), we have the following:

(a) The agent’s pay-performance sensitivity \( b_i^* = \arg \max_{b \geq 0} F_i(b) \).
(b) The principal’s investment is \( c_i^* = c(b_i^*) \).
(c) The agent’s effort is \( \eta_i^* = \eta(b_i^*, c_i^*) \).
(d) The agent’s performance-invariant compensation is \( a_i^* = a_i(b_i^*, c_i^*) \).
(e) The termination time of the contract solves the following optimal stopping problem:

\[ \tau^* = \arg \max_{\tau' \leq T} E_pr \left[ \sum_{j=0}^{\tau'-1} F_j(b_j^*) + (\mu_j^{Pr} - l_j) \right], \]  

where the maximization is overall \( \{F_i\} \)-stoping times \( \tau' \leq T \).

The optimal contractual parameters, \( (a_i^*, b_i^*, c_i^*, \eta_i^*) \), in period \([i, i + 1]\) are given by their corresponding values in the static model (see Section 1.3) with the principal’s and agent’s assessments of the project’s core output set to \( N(\mu_i^{Pr}, \sigma_i^2) \) and \( N(\mu_i^{Ag}, \sigma_i^2) \), respectively, that is, their respective assessments at the beginning of period \([i, i + 1]\) in the dynamic model. By Theorem 3.1, the equilibrium values for the pay-performance sensitivity, investment and effort at each point in time (conditional upon continuation) are positive and deterministic. The two components of the contract that are stochastic are the component \( a_i^* \) of the agent’s compensation, which is adjusted based on realizations of the termination payoff \( V_i \) of the project, and, of course, the termination time \( \tau \).

4. Properties of the Equilibrium

4.1 Static properties

The agent’s pay-performance sensitivity in period \([i, i + 1]\) maximizes the principal’s periodic flow function \( F_i(\cdot) \). Since \( F_i(\cdot) \) is identical in form to \( F_0(\cdot) \), the sensitivities of the contractual parameters to changes in the degree of agent optimism, \( \Delta_i \), or the price of risk, \( p_i \), in period \([i, i + 1]\) are described by Propositions 1.3 and 1.4, respectively, with \( \Delta_i \) replacing \( \Delta_0 \) and \( p_i \) replacing \( p_0 \). It can be shown that \( \Delta_i/p_i < \Delta_0/p_0 \). Consequently, Assumption 2 holds for each period, too.
4.2 Dynamic properties

The dynamic properties of the equilibrium arise from the fact that the principal’s and agent’s posterior assessments of the project’s core output evolve over time due to Bayesian learning. Let

\[ F(b) := -0.5 \lambda \sigma^2 b^2 + K c(b) \]  

(35)

denote the principal’s periodic flow function in the benchmark scenario in which there is perfect information about the project’s core output, that is, when \( \sigma_i = 0 \) and \( \Delta_i = 0 \). Let \((b^*_p, c^*_p, \eta^*_p)\) denote the optimal pay-performance sensitivity, investment, and effort in this benchmark scenario. Using Equations (21) and (23), the principal’s period flow function at date \( i \) can be expressed as

\[ F_i(b) = \left( \frac{\Delta_0}{\sigma_0^2} b - 0.5 \lambda b^2 \right) \sigma_i^2 + F(b). \]

(36)

Since \( \sigma_i \to 0 \) as \( i \to \infty \), it follows from the Theorem of the Maximum that \( b^*_i \to b^*_p \), and thus \((c^*_i, \eta^*_i) \to (c^*_p, \eta^*_p)\) by continuity. We now characterize the dynamics of these economic variables.

**Theorem 4.1** (The Dynamics of Compensation, Investment, and Effort). Conditional on the project surviving beyond date \( i \):

(a) If \( \Delta_0 < \lambda \sigma_0^2 b^*_p \) (that is, the agent is “moderately optimistic”), then \( b^*_i \), \( c^*_i \), \( \eta^*_i \) increase monotonically toward \( b^*_p \), \( c^*_p \), and \( \eta^*_p \), respectively.

(b) If \( \Delta_0 = \lambda \sigma_0^2 b^*_p \), then \( b^*_i \), \( c^*_i \), and \( \eta^*_i \) are constant and equal \( b^*_p \), \( c^*_p \), and \( \eta^*_p \), respectively.

(c) Suppose \( \Delta_0 > \lambda \sigma_0^2 b^*_p \) (that is, the agent is “exuberant”):

(i) \( b^*_i \) decrease monotonically toward \( b^*_p \);

(ii) if \( \Delta_0 \leq p_0 \), that is, the initial degree of agent optimism is less than or equal to the price of risk, then \( c^*_i \) decrease monotonically toward \( c^*_p \); otherwise, if \( \Delta_0 > p_0 \), the \( c^*_i \) initially increase until \( i \leq i^* := (\Delta_0 - p_0)/\lambda \sigma_0^2 \) and then decrease monotonically thereafter toward \( c^*_p \);

(iii) for \( i \geq p_0 \), \( \eta^*_i \) decrease monotonically toward \( \eta^*_p \).

At time \( i^* \) the agent’s pay-performance sensitivity equals one and the principal’s investment attains its maximum possible value.

Figure 2 illustrates the results of Theorem 4.1. It shows the variations of the agent’s pay-performance sensitivity and the principal’s investment with the initial degree of agent optimism, \( \Delta_0 \). By Theorem 3.1, the contractual parameters in any period are determined by the relative magnitudes of the degree of agent optimism and the price of risk. The passage of time causes transient risk to be resolved, thereby lowering the price of risk and the costs.
of risk sharing. However, the passage of time also lowers the degree of agent optimism and, therefore, the rents that the principal can extract by exploiting the agent’s optimism. The time paths of the agent’s pay-performance sensitivity, the principal’s investment, and the agent’s effort depend on the relative rates of decline of the rents from the agent’s optimism and the costs of risk sharing.

If the agent is moderately optimistic, then the beneficial effect of time on the costs of risk sharing dominates its negative effect on the degree of agent optimism. Hence, the agent’s pay-performance sensitivity and effort both increase over time. Because investment and effort are complementary, and the degree of agent optimism is low relative to the price of risk, the principal’s investments also increase over time (see Sections 1.6 and 4.1).

If the agent is exuberant, the negative effect of the evolution of time on the degree of agent optimism dominates its positive effect on the costs of risk sharing. The agent’s pay-performance sensitivity, therefore, declines over time. The dynamics of effort and investment are more complicated due to the nonmonotonicity of the optimal investment function (see Proposition 1.1 and Figure 1). If the degree of agent optimism is lower than the price of risk in each period, the agent’s pay-performance sensitivity is less than one. The intuition underlying the static properties of the contract (see Sections 1.6 and 4.1) implies that the agent’s effort and the principal’s investment decline over time. If the initial degree of agent optimism is high enough, however, the degree of agent optimism exceeds the price of risk in early periods so that the agent’s pay-performance sensitivity is greater than one. In these periods, the intuition underlying the static properties of the contract implies that the principal’s investment increases, while the agent’s effort varies nonmonotonically. In later periods when the degree of agent optimism falls below the price of risk, investment and effort both decline over time.
At the threshold level of the initial degree of the agent’s optimism (which separates the two regions described above), the positive effects of the resolution of transient risk and the negative effects of the decline in the agent’s optimism balance each other exactly so that pay-performance sensitivity, investment, and effort are constant over time. In the benchmark scenarios in which either there is no uncertainty about the project’s core output (as in Holmström and Milgrom 1987) or there is uncertainty, but beliefs are symmetric, Theorem 4.1 implies that the principal’s investments and the agent’s pay-performance sensitivity and his effort are either constant or increase over time. In reality, however, VC and R&D projects feature investment schedules that could be increasing, decreasing, or nonmonotonic over time (e.g., see Cochrane 2005). Our results show that the presence of asymmetric beliefs plays a central role in generating different time paths of investments and incentives.

In traditional dynamic principal–agent models such as Holmström and Milgrom (1987), the optimal contract can be implemented through a simple combination of equity and zero-coupon debt. The varying dynamics of optimal contracts predicted by Theorem 4.1 imply that in the presence of asymmetric beliefs, optimal contracts can be implemented only through combinations of several financial securities, which is consistent with evidence that VC contracts feature complex combinations of various financial securities (Kaplan and Stromberg 2003).

**Remark 3.** As shown in Theorem 4.1, the pay-performance sensitivities could exceed one if the agent is initially exuberant. Keep in mind, however, that all payoffs occur at the termination time $\tau$. By Equation (29), the agent’s final payoff as a proportion of the project’s total payoff is

$$\frac{P_i}{V_\tau} = \frac{P_0 + \sum_{i=0}^{\tau-1}[a_i + b_i(V_{i+1} - V_i)]}{V_\tau}. \quad (37)$$

Because the $a_i$ are stochastic and the $b_i$ vary over time, the agent’s compensation cannot simply be expressed as a combination of cash and an equity stake as in the static model. In other words, the pay-performance sensitivities should not be interpreted as equity stakes.

The pay-performance sensitivity exceeds one only for periods $i \leq i^* := (\Delta_0 - p_0)/\lambda \sigma_0^2$. When the value of $\sigma_0$ is high relative to $\Delta_0$, as might be expected for highly innovative projects, the value of $i^*$ is low. Consequently, an agent’s initial exuberance dissipates quickly. Agent exuberance is likely to be economically relevant in the early stages of ventures. Given the paucity of detailed data on contracts in early stage ventures—Kaplan and Stromberg (2003) include only descriptive data on types of financial securities used in VC contracts but not their precise composition—we allow for the possibility of agent exuberance in early periods because the implications could be empirically relevant when detailed data become available.
4.3 Sensitivity of equilibrium dynamics

We now investigate how the equilibrium dynamics vary with the degree of agent optimism $\Delta_0$, the agent’s risk aversion $\lambda$, the project’s initial transient risk $\sigma_0$, and the intrinsic risk $s$.

**Theorem 4.2** (The effects of risk). *The effects of risk on the contractual dynamics are as follows.*

(a) The agent’s optimal pay-performance sensitivity path is a pointwise decreasing function of the agent’s risk aversion, $\lambda$, and the initial transient risk, $\sigma_0$.

(b) If $\Delta_0 < \lambda\sigma_0^2 b_p^*$ (the agent is “moderately optimistic”), then the principal’s optimal investment is a pointwise decreasing function of the agent’s risk aversion, $\lambda$, and the initial transient risk, $\sigma_0$.

(c) If $\Delta_0 > \lambda\sigma_0^2 b_p^*$ (the agent is “exuberant”), the principal’s investment path changes as depicted in Figure 3(a) as a result of an increase in the agent’s risk aversion or the initial transient risk—the time-path of investment shifts “to the left” if $\lambda$ or $\sigma_0$ increases. More precisely, let $\lambda_1 < \lambda_2$ and $\sigma_1^0 < \sigma_2^0$ be two possible values of the agent’s risk aversion and initial transient risk, respectively. There exist $t^*(\lambda_1, \lambda_2)$ and $t^{**}(\sigma_1^0, \sigma_2^0)$ such that the principal’s investments when the agent’s risk aversion is $\lambda_1$ (the initial transient risk is $\sigma_1^0$) are higher than her investments when the agent’s risk aversion is $\lambda_2$ (the initial transient risk is $\sigma_2^0$) for $i < t^*(\lambda_1, \lambda_2)$ ($i < t^{**}(\sigma_1^0, \sigma_2^0)$) and lower for $i > t^*(\lambda_1, \lambda_2)$ ($i > t^{**}(\sigma_1^0, \sigma_2^0)$).

---

8 Since $b_p^* < 1, \lambda\sigma_0^2 b_p^* < \lambda(\sigma_0^2 + s^2) = p_0$. The agent, therefore, can be exuberant even when his degree of optimism is less than the price of risk. In this case, regardless of the values of $\lambda$ or $\sigma_0$, the principal’s investment path decreases over time—see Theorem 4.1c(ii). The investment paths depicted in this figure, therefore, do not have an “increasing region,” and the time path of investment shifts downward.
Table 1
Effects of agent optimism and price of risk on contractual dynamics

<table>
<thead>
<tr>
<th></th>
<th>Pay-performance sensitivities</th>
<th>Investments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Early periods</td>
<td>Late periods</td>
</tr>
<tr>
<td>Price of risk</td>
<td>Moderately optimistic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>exuberant</td>
<td></td>
</tr>
<tr>
<td>Agent optimism</td>
<td>Moderately optimistic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>exuberant</td>
<td></td>
</tr>
</tbody>
</table>

The table displays the effects of the price of risk and initial agent optimism on the agent’s pay-performance sensitivities and the principal’s investments. “+” and “−” denote a positive and negative pointwise relation, respectively.

(d) If $\Delta_0 \leq 2p_0$, then properties (a)–(c) hold for the intrinsic risk, $s$. If $\Delta_0 > 2p_0$, intrinsic risk has more complex, nonmonotonic effects on the pay-performance sensitivity and investment paths.

The agent’s pay-performance sensitivity, $b^*$, in any period $[i, i + 1]$ declines with his risk aversion, the initial transient risk, and the intrinsic risk, as an increase in any of these parameters increases the costs of risk sharing between the principal and the agent. The effects of risk aversion and intrinsic and transient risk on the principal’s investment path are, however, more subtle due to the presence of optimism. If the agent is moderately optimistic, the costs of risk sharing outweigh the benefits of the agent’s optimism. The principal’s equilibrium investment path, therefore, declines pointwise with the agent’s risk aversion and the project’s intrinsic and transient risks in this region. If the agent is exuberant, an increase in risk increases the costs of risk sharing. Because the agent’s pay-performance sensitivity is greater than one in this region, however, it follows from the static properties of the contract (see Sections 4.1 and 1.6) that the principal optimally increases her investment. In later periods, the degree of agent optimism falls below the price of risk so that the agent’s pay-performance sensitivity is less than one. It follows from the static properties of the contract that the principal’s investment declines. The first two rows of Table 1 summarize the effects of the price of risk on the agent’s pay-performance sensitivity and the principal’s investment paths.

The results of Theorem 4.2 extend the results of traditional “real options” analyses of the effects of risk on investment (see Dixit and Pindyck 1994). Theorem 4.2 shows the effects of the interplay between optimism and agency conflicts, which are not considered in traditional real options models, on the relation between risk and investment. In particular, the relation between risk and investment could be positive or negative depending on the degree of agent optimism.

By the intuition above, the effects of agent optimism, $\Delta_0$, on the agent’s pay-performance sensitivities and the principal’s investments are opposite to those of the price of risk. The last two rows of Table 1 summarize the effects of agent optimism.
Theorem 4.3 (The effects of optimism). The effects of optimism on the contractual dynamics are as follows.

(a) The agent’s optimal pay-performance sensitivity path is a pointwise increasing function of the initial degree of agent optimism, \( \Delta_0 \).

(b) If \( \Delta_0 \leq \lambda \sigma_0^2 b^*_p \) (the agent is “moderately optimistic”), the principal’s optimal investment path is a pointwise increasing function of the initial degree of agent optimism, \( \Delta_0 \).

(c) If \( \Delta_0 \geq \lambda \sigma_0^2 b^*_p \) (the agent is “exuberant”), the principal’s investment path changes as depicted in Figure 3(b) as a result of an increase in the agent’s optimism—the time path of investment shifts “to the right” if \( \Delta_0 \) increases. More precisely, let \( \lambda \sigma_0^2 b^*_p < \Delta_0^1 < \Delta_0^2 \) be two possible values of the initial degree of agent optimism, respectively. There exists \( t^*(\Delta_0^1, \Delta_0^2) \) such that the principal’s investments, when the initial degree of agent optimism is \( \Delta_0^1 \), are lower than her investments when the initial degree of agent optimism is \( \Delta_0^2 \) for \( i < t^*(\Delta_0^1, \Delta_0^2) \) and higher for \( i > t^*(\Delta_0^1, \Delta_0^2) \).

4.4 Project duration

The optimal termination time of the contract is characterized in the following proposition.

Proposition 4.4 (Optimal termination policy). There exist \( \mu^*_i, 0 \leq i \leq T - 1 \), such that the principal terminates the project at date \( i \) if and only if \( \mu^{pr}_i < \mu^*_i \).

From Equation (22), we can show that \( \mu^{pr}_i \geq \mu^*_i \) if and only if \( V_i \geq V^*_i \), where

\[
V^*_i := V_0 + \sum_{t=0}^{i-1} \left( Ac_t \alpha \eta_t \beta - l_t \right) + \frac{(s^2 + i \sigma_0^2) \mu^*_i - s^2 \mu_0}{\sigma_0^2}.
\]

The \( V^*_i \) may be thought of as performance targets the firm must reach at each date or else it will terminate the project. Thus, either the \( \mu^*_i \) or the \( V^*_i \) may be used to define the trigger policy; the performance targets are more commonly used in practice.

The following result describes the effect of the agent’s initial assessment of core output, his risk aversion, and his cost of effort on the duration of the project.

Proposition 4.5 (Effects of optimism, risk aversion, and cost of effort on duration). The project duration \( \tau \) (a) increases with the agent’s initial degree of optimism, \( \Delta_0 \), and (b) decreases with his risk aversion, \( \lambda \), and his cost of effort, \( k \).
As discussed earlier, an increase in the agent’s initial degree of optimism about core output increases the rents that the principal is able to extract by exploiting the agent’s optimism, thereby increasing her expected continuation value in every period. Hence, it is optimal for the principal to prolong the project’s duration. An increase in the agent’s risk aversion or cost of effort, however, increases the costs of risk sharing for the principal, thereby lowering her continuation value in every period. Hence, the principal terminates the project earlier.

5. An Application to Firm R&D

As an illustrative application of the model, we adapt it to investigate the effects of asymmetric beliefs and agency conflicts on firm R&D. In this application, the “principal” represents the shareholders of a firm, while the “agent” is the R&D manager who controls the execution of R&D projects. Our application of the model to study firm R&D permits a quantitative investigation of the effects of optimism and agency conflicts on the characteristics of R&D projects—the value they generate, the time paths of investments in R&D, and the structure of optimal dynamic incentive contracts for R&D managers. Our numerical analysis also leads to additional testable implications of the model, specifically the effects of intrinsic risk, transient risk, and optimism on project values and durations.

5.1 Numerical implementation and calibration

We describe the numerical implementation of the model in Appendix B. In our numerical implementation, we incorporate a nonzero discount rate, \( r \), for the principal and the agent. Since the principal is risk neutral, \( r \) is the risk-free rate, which we set to 4% in our analysis. We assume a competitive market for capital provision by shareholders so that the manager captures the surplus from the project. The initial promised payoff, \( P_0 \), of the manager is, therefore, determined by the condition that shareholders earn competitive returns on their investments (see Appendix B).

We determine the baseline parameter values by calibrating the model to data on the distributions of investments and returns for a sample of pharmaceutical R&D projects reported by Grabowski et al. (2002). They examine time-series data on the investments and cash flows associated with 118 new chemical entities (NCEs) introduced by the U.S. pharmaceutical industry between 1990 and 1994. For each NCE project, they estimate the project value (net present value [NPV] of future cash flows) at the terminal date of the R&D process. R&D investment expenditures for each NCE project occur prior to the terminal date. They thereby obtain a distribution of project values at the terminal date of the R&D process. Hereafter, we refer to the project value at the terminal date as the “project terminal value.” They also report the mean project terminal value (see row 2, column 2, of Table 2) as well as the mean project terminal value...
### Table 2

**Observed and predicted statistics**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
<th>117/118</th>
<th>R&amp;D Exp</th>
<th>% &gt; Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>525.0</td>
<td>0.0</td>
<td>9.0</td>
<td>30.0</td>
<td>56.0</td>
<td>126.0</td>
<td>235.0</td>
<td>433.0</td>
<td>629.0</td>
<td>1015.0</td>
<td>2722.0</td>
<td>479.0</td>
<td>480.0</td>
<td>0.34</td>
</tr>
<tr>
<td>Predicted</td>
<td>539.0</td>
<td>0.0</td>
<td>0.4</td>
<td>39.4</td>
<td>92.8</td>
<td>139.5</td>
<td>223.9</td>
<td>383.2</td>
<td>679.7</td>
<td>1198.7</td>
<td>2492.4</td>
<td>492.3</td>
<td>441.6</td>
<td>0.32</td>
</tr>
<tr>
<td>Stdev</td>
<td>72.69</td>
<td>0.0</td>
<td>6.1</td>
<td>19.5</td>
<td>18.9</td>
<td>22.6</td>
<td>38.1</td>
<td>60.6</td>
<td>82.4</td>
<td>95.2</td>
<td>204.1</td>
<td>6.7</td>
<td>43.0</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The table displays the observed values of the statistics used to calibrate the model, their predicted values from the model, and the standard deviations of the statistics obtained by generating bootstrapped samples using the model as described in Appendix C. R&D Exp denotes the mean R&D expenditure (in millions of dollars) for the sample of 118 projects of Grabowski et al. (2002); 1st, 2nd, . . . , 10th denote the mean project terminal value (in millions of dollars) in each decile of the distribution of project terminal values; Mean denotes the mean project terminal value; 117/118 denotes the mean project terminal value for the remaining 117 projects after the top project is deleted from the sample; and % > Cost denotes the proportion of the projects whose terminal values exceeded the mean R&D expenditure across projects in the sample.
in each decile of the distribution of project terminal values in their sample (see row 2, columns 3–12, of Table 2).

In line with Grabowski et al. (2002), we assume a planning horizon of \( T = 12 \) periods and set the length of each period to one year. The loss function \( l_t \) in Equation (19) is assumed to have the functional form \( l_t = l_1 i_t^2 \). We calibrate the true mean, \( \mu_0^{\text{true}} \), of the core output distribution in addition to the principal’s, \( \mu_0^{\text{Pr}} \), and agent’s, \( \mu_0^{\text{Ag}} \), assessments. In particular, we do not assume that the agent is optimistic a priori so that \( \mu_0^{\text{Ag}} \) could be greater than or less than \( \mu_0^{\text{Pr}} \). The true mean of the core output distribution affects the distribution of observed returns of R&D projects to which we calibrate the model. The calibration of the model therefore provides information on the absolute levels of optimism (or pessimism) of investors (the principal) in R&D projects and their managers (the agent).

The thirteen calibrated parameters of the model are grouped into “technology,” “preference,” and “belief” categories (see Table 3). As explained in Appendix A, all observable economic variables depend only on the ratio \( \gamma / \beta \). Hence, only the ratio \( \gamma / \beta \) (and not the parameters \( \beta \) and \( \gamma \) separately) is identified by the data.

We use the simulated method of moments to estimate the baseline parameters (see Adda and Cooper 2003). For a candidate vector of parameter values, we simulate forty thousand sample paths and compute the predicted values of the statistics reported by Grabowski et al. (2002). The baseline values of the parameters are those that minimize the distance between the observed values of the statistics. The observed and predicted values of the statistics are shown in the second and third rows of Table 2. The baseline values of the parameters are shown in Table 3. Because we do not have access to the actual data used by Grabowski et al. (2002), we use parametric bootstrapping to determine the standard errors of the parameter estimates and the statistics (see Davison and Hinkley 1997 and Appendix C). The standard errors of the predicted statistics are shown in the fourth row of Table 2. The model is able to match the observed statistics reasonably well. Consistent with the fact that R&D is risky and uncertain, the baseline values of the intrinsic risk, \( s \), and transient risk, \( \sigma_0 \), are high (relative to \( \mu_0^{\text{true}} \)). We define the

\[
\text{Project Value} := E_0^{\text{true}} \left[ e^{-r \tau} V_\tau - \sum_{i=0}^{\tau-1} e^{-r_i} c_i^a \right] \quad (38)
\]

\( \text{Project Value} \) to be the expectation (under the true beliefs) at date 0 of the present value of the termination payoff less the present value of the investments made until termination. The initial seed capital, \( V_0 = 192.3 \) million, could be interpreted as the project’s book value at date zero. The estimated values of the mean assessments of core output by the principal and the agent are \$12.5\) million and \$77.8\) million per year, respectively. The true mean of the core output
Table 3
Baseline parameter values

<table>
<thead>
<tr>
<th>Technology parameters</th>
<th>Belief parameters</th>
<th>Preference parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>$A$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>192.3</td>
<td>22.63</td>
<td>0.29</td>
</tr>
<tr>
<td>(26.6)</td>
<td>(5.73)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>$\mu_0^{true}$</td>
<td>$\mu_0^{Pr}$</td>
<td>$\Delta_0$</td>
</tr>
<tr>
<td>24.36</td>
<td>12.46</td>
<td>65.31</td>
</tr>
<tr>
<td>(3.64)</td>
<td>(1.17)</td>
<td>(6.58)</td>
</tr>
<tr>
<td>$s$</td>
<td>$\sigma_0$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>171.86</td>
<td>153.54</td>
<td>0.0027</td>
</tr>
<tr>
<td>(19.89)</td>
<td>(8.20)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$l_1$</td>
<td></td>
<td>$k$</td>
</tr>
<tr>
<td>13.52</td>
<td></td>
<td>12.46</td>
</tr>
<tr>
<td>(2.57)</td>
<td></td>
<td>(1.19)</td>
</tr>
<tr>
<td>$l_2$</td>
<td></td>
<td>$\gamma/\beta$</td>
</tr>
<tr>
<td>1.24</td>
<td></td>
<td>7.94</td>
</tr>
<tr>
<td>(0.14)</td>
<td></td>
<td>(0.78)</td>
</tr>
</tbody>
</table>

The table displays the baseline parameter values obtained by the model calibration. The point estimates and their standard errors are shown in the second and third rows, respectively. The standard errors are obtained by generating bootstrapped samples using the model as described in Appendix C. The point estimates for the initial seed capital, $V_0$; the means of the product quality distribution, $\mu_0^{true}$ and $\mu_0^{Pr}$; degree of agent optimism, $\Delta_0$; and the standard deviations that define the intrinsic and transient risks, $s$ and $\sigma_0$, are recorded in millions of dollars. The remaining parameters estimated are the production function parameters, $A$ and $\alpha$; the parameters associated with the loss function, $l_1$ and $l_2$; the agent’s risk aversion parameter, $\lambda$; and the disutility of effort parameters, $k$ and $\gamma/\beta$. 
distribution is $24.4 million per year. In the actual scenario in which the principal’s beliefs, the agent’s beliefs, and the true core output differ from each other, the project value is $343.9 million. In the hypothetical scenarios in which the true distribution of core output coincides with the principal’s and agent’s beliefs, the project values are $316.2 million and $489.0 million, respectively.

The model calibration, therefore, suggests that on average, managers (the agent) not only are optimistic relative to investors, but are also significantly optimistic relative to the true distribution of core output. (Recall that we did not assume that managers are more optimistic a priori.) Investors know that they are pessimistic relative to managers but do not know that they are pessimistic with respect to the true distribution of core output. The significant relative optimism of R&D managers implies that they can be provided with powerful incentives. The increased output generated by managers’ optimism more than offsets investors’ relative pessimism, which illustrates the potentially significant beneficial effects of optimism.

5.2 Numerical analysis
We compare the actual scenario with two benchmark scenarios: (i) the no agency scenario in which beliefs are symmetric and the agent is risk neutral; and (ii) the symmetric beliefs scenario in which beliefs are symmetric, but the agent is risk averse. We compute the following variables in each scenario:

\[
\text{Project Surplus} := E_0^{\text{true}}[e^{-r \tau} V_{\tau}] - V_0 - E_0^{\text{true}} \left[ \sum_{t=0}^{\tau-1} e^{-r \tau} c_t^* \right].
\] (39)

\[
\text{Agent’s Expected Payoff (Agent Payoff)} := E_0^{\text{true}}[e^{-r \tau} P_{\tau}].
\] (40)

\[
\text{Expected R&D Expenditures (RDE)} := V_0 + E_0^{\text{true}} \left[ \sum_{t=0}^{\tau-1} e^{-r \tau} c_t^* + e^{-r \tau} P_{\tau} \right].
\] (41)

The project NPV or surplus equals the project value as defined in Equation (38) less the initial seed capital, \(V_0\). In computing the above quantities in the actual and benchmark scenarios, the termination payoff process evolves as in Equation (19) with the contractual parameters \((a^*, b^*, c^*)\), the agent’s effort \(\eta^*\), and the performance targets \(V^*\), set to their equilibrium values for the particular scenario (no agency, symmetric beliefs, or actual) being analyzed. Note that the expectations in Equations (39)–(41) are with respect to the true distribution of project quality.

**Baseline analysis.** Table 4 records the three economic variables, the average project duration, the pay-performance sensitivities (the \(b^*_t\)), and the principal’s investments (the \(c^*_t\)) in the first four periods for each of the three scenarios when the parameters take their baseline values. The agency costs are substantial. In the actual scenario, the project surplus and the agent’s payoff are lower than their values in the benchmark “no agency” scenario by, respectively, $13.4 million and $23.1 million. Further, R&D expenditures decline by $26.9 million. The difference between the values of the project surplus, the agent’s payoff, and
Table 4
Baseline outputs in actual and benchmark scenarios

<table>
<thead>
<tr>
<th>Agency scenario</th>
<th>Project surplus</th>
<th>Agent payoff</th>
<th>RDE</th>
<th>$E_{T_i}^{\text{true}}$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>151.5</td>
<td>131.6</td>
<td>360.4</td>
<td>3.73</td>
<td>49%</td>
<td>38%</td>
<td>33%</td>
<td>31%</td>
<td>11.34</td>
<td>10.99</td>
<td>10.77</td>
<td>10.63</td>
</tr>
<tr>
<td>Symmetric</td>
<td>144.6</td>
<td>121.3</td>
<td>345.3</td>
<td>3.62</td>
<td>16%</td>
<td>18%</td>
<td>18%</td>
<td>19%</td>
<td>9.48</td>
<td>9.68</td>
<td>9.76</td>
<td>9.81</td>
</tr>
<tr>
<td>No agency</td>
<td>164.9</td>
<td>154.7</td>
<td>387.3</td>
<td>3.76</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>11.89</td>
<td>11.89</td>
<td>11.89</td>
<td>11.89</td>
</tr>
</tbody>
</table>

The table displays the baseline outputs obtained for each of the three agency scenarios when the parameters take their baseline values. These include the Project Surplus, Agent Payoff, and the R&D expenditures, Equations (39)–(41), as well as the pay-performance sensitivities and the principal’s investment in the first four periods.

R&D expenditures in the no agency and symmetric scenarios (the agency costs of risk sharing) are $20.3$ million, $33.1$ million, and $42.0$ million, respectively. We see that the agent’s optimism significantly mitigates the agency costs of risk sharing.

The effects of transient risk and intrinsic risk. Figure 4(a) displays the variations of the project surplus, the agent’s payoff, R&D expenditures, and project duration with the initial transient risk, $\sigma_0$. All four economic variables increase with the transient risk. By Equations (21) and (22), the variance of the evolution of $\mu_i$ at date $i$ is

$$\left(\sigma_i^\mu\right)^2 = \frac{s^2}{[(s/\sigma_0)^2 + i][s/\sigma_0]^2 + i]} ,$$

which increases monotonically with the initial transient risk $\sigma_0$. Ceteris paribus, an increase in the transient risk $\sigma_0$ implies that the intermediate signals about core output are relatively more informative (the “signal to noise” ratio is high). Hence, the updated assessments of core output are more responsive to signals. Because the variance of the evolution of the principal’s assessment of the project’s quality increases with transient risk, the principal’s future “option value” of continuing the relationship increases. The positive effect of transient risk on the option value of continuation dominates its negative effects on the costs of risk sharing. Hence, the project surplus, the agent’s payoff, R&D expenditures, and project duration increase with transient risk.

Figure 4(b) displays the variations of project surplus, the agent’s payoff, R&D expenditures, and project duration with the intrinsic risk, $s$. From Equation (42), the variance of the evolution of the assessment of core output decreases with intrinsic risk above a threshold. Hence, the option value of continuing the relationship in any period also declines. An increase in the intrinsic risk also increases the costs of risk sharing. Hence, all four economic variables decline with intrinsic risk.

The results of Figure 4(a) and (b) together demonstrate that different components of risk—transient and intrinsic—have opposite effects on project values and durations. The intuition for the findings implies that imperfect information,
optimism, and agency conflicts play key roles in generating the opposite effects of intrinsic and transient risk.

6. Conclusions

We develop a dynamic structural model to examine the effects of asymmetric beliefs and agency conflicts on dynamic principal–agent relationships. Our model differs from previous models by incorporating three key features in
a unified framework: (i) asymmetric beliefs and risk attitudes; (ii) dynamic actions by both parties; and (iii) endogenous termination. The optimal contracts predicted by our analysis have novel features that reflect the intertemporal trade-off between the beneficial effects of the agent’s optimism and the detrimental effects of the costs of risk sharing.

Optimism could significantly mitigate the agency costs of risk sharing and enhance project values, which potentially explains why venture capitalists and entrepreneurs continue to engage in projects with high failure rates. The optimal contracts predicted by our analysis have many features in common with observed contracts in environments such as VC and R&D. Our unified framework also leads to potentially testable implications that link underlying project characteristics to the principal’s investments, the agent’s compensation, and the project’s duration.

Finally, we provide quantitative guidance on the effects of asymmetric beliefs by calibrating our structural model to data on pharmaceutical R&D projects. Our analysis shows that managerial optimism could be a significant driver of R&D project values. Permanent and transitory components of a project’s risk have contrasting effects on its value and duration.

Appendix A: Proofs

**Proof of Proposition 1.1.** (i) and (iii) The marginal (optimal) investment is

\[
    c'(b) \propto \left( \frac{1}{k} \right)^{\frac{1}{1-\alpha \gamma - \beta}} b'\left(\gamma/\beta - b\right)^t(1-b),
\]

where

\[
    t := \frac{2 - (1-\alpha)(\gamma/\beta)}{(1-\alpha)(\gamma/\beta) - 1} \quad \text{and} \quad s := \frac{\alpha \gamma}{(1-\alpha)\gamma - \beta},
\]

and where the symbol \(\propto\) means “equal up to a positive multiplicative constant.” Under Assumption 1, the parameter \(s\) is positive and the parameter \(t\) is negative. Since \(\gamma > 1\) (Assumption 1), the strict quasi-concavity of \(c(\cdot)\) easily follows from Equation (A1). Since \(c(0) = c(\gamma) = 0\) and \(c'(0) = +\infty\), it also follows from Equation (A1) that \(c(\cdot)\) achieves its maximum at \(b = 1\).

(ii) The second derivative is

\[
    c''(b) \propto b'^{-1}(\gamma/\beta - b)^{-1}[t(\gamma/\beta - b)(1-b) - sb(1-b) - b(\gamma/\beta - b)].
\]

The expression inside the brackets is a strictly convex quadratic function whose value at 1 is negative, whose value at \(\gamma/\beta > 1\) is positive, and whose value at 0 is negative, since \(t < 0\). Consequently, there is exactly one root \(b^M\) of the quadratic in the interval \((1, \gamma/\beta)\) such that \(c''(b^M) = 0\). At \(b^M\) the marginal investment is at its minimum. Moreover, since \(c''(\cdot)\) is negative on \([0, b^M]\) and is positive on \((b^M, \gamma/\beta)\), the optimal investment function is strictly concave on \([0, b^M]\) and strictly convex on \([b^M, \gamma/\beta]\). 

\[\blacksquare\]
The remaining results will be proved for the general (dynamic) setting. The ratio of the degree of the agent’s optimism, \( \Delta_i \), to the price of risk, \( p_i = \lambda(s^2 + \sigma_i^2) \), in period \( i \) may be expressed as

\[
\frac{\Delta_i}{p_i} = \frac{\Delta_0}{\lambda(s^2 + (i + 1)\sigma_0^2)}. \tag{A4}
\]

Note that the right-hand side of Equation (A4) is strictly less than \( \Delta_0/p_0 \), which by Assumption 1 is less than \( b_M \). For our subsequent analysis, it is convenient to use Equation (A4) to express the derivative of the principal’s periodic flow function \( F_i(\cdot) \) as

\[
F_i'(b) = \Delta_i - p_i b + Kc'(b) = p_i \left[ \frac{\Delta_i}{p_i} - b \right] + Kc'(b) = p_i \left[ \frac{\Delta_0}{\lambda(s^2 + (i + 1)\sigma_0^2)} - b \right] + Kc'(b).
\]

\[
\tag{A5}
\]

**Proof of Proposition 1.2.** We must show that \( b_i^* \leq \max(\Delta_i/p_i, 1) \). Obviously, the result immediately follows if \( b_i^* \leq 1 \), so suppose that \( b_i^* \geq 1 \). It remains to show that \( b_i^* \leq \Delta_i/p_i \). By Equation (29), we can use backward induction to derive the equilibrium by first considering the last possible investment period \([T - 1, T] \).

**Proof of Proposition 1.3.** We write \( F_i(\cdot; \Delta_i) \) to make explicit the functional dependence of \( F_i \) on the parameter \( \Delta_i \), and write \( F_i'(\cdot; \Delta_i) \) to denote its derivative with respect to \( b \). Let \( b(\Delta_i) \) denote the optimal pay-performance sensitivity when the degree of agent optimism parameter is \( \Delta_i \).

To prove the first statement of part (i), pick two values \( \Delta_1 \) and \( \Delta_2 \) such that \( \Delta_1 < \Delta_2 \). We must show that \( b(\Delta_1) < b(\Delta_2) \). Since \( b(\Delta_1) \) maximizes \( F_i(\cdot; \Delta_1) \), its derivative (A5) vanishes, and since \( \Delta_1 < \Delta_2 \), it follows that

\[
0 = \Delta_1 - p_1 b(\Delta_1) + Kc'(b(\Delta_1)) < \Delta_2 - p_1 b(\Delta_1) + Kc'(b(\Delta_1)) = F_i'(b(\Delta_1); \Delta_2). \tag{A6}
\]

Since \( F_i(\cdot, \Delta_2) \) is strictly concave, its derivative can only be positive at values below the unique maximizer \( b(\Delta_2) \), which proves that \( b(\Delta_1) < b(\Delta_2) \), as required. As to the second statement of part (i), note that since \( c'(1) = 1 \) by Proposition 1.1(i), the derivative of \( F_i \) at \( b = 1 \) is \( F_i'(1, \Delta_i) = \Delta_i - p_i \). Consequently, \( \Delta_i \) is less than, equal to, or greater than \( p_i \) if and only if \( F_i'(1, \Delta_i) \) is less than, equal to, or greater than zero. Since \( F_i \) is strictly concave, \( F_i'(1, \Delta_i) \) is less than, equal to, or greater than zero if and only if \( b(\Delta_i) \) is less than, equal to, or greater than zero, which establishes the result.

**Proof of Theorem 3.1.** By Equation (29), we can use backward induction to derive the equilibrium by first considering the last possible investment period \([T - 1, T] \). Suppose that the project has not been terminated as of date \( T - 1 \). Recall that the agent and the principal’s priors on \( \Theta \) as of date \( T - 1 \) are \( N(\mu_{T-1}^j, \sigma_{T-1}^2) \) with \( \mu_{T-1}^j \) and \( \sigma_{T-1}^2 \) given by Equations (22) and (21), respectively, with the index \( i \) set to \( T - 1 \). For subsequent convenience, we use the index \( i \) to denote the date, which is set to \( T - 1 \) for now, but later denotes an arbitrary date when we establish the inductive step in our analysis.

**The Optimal Contractual Parameters in Period \([T - 1, T] \).** Suppose that in period \([i, i + 1] \) (recall that \( i = T - 1 \)), the principal’s investment is \( c_i \) and the agent’s contractual parameters
are \((a_i, b_i)\); see Equation (28). Following the arguments in Section 1.3, the contract is incentive compatible for the agent if and only if the effort \(\hat{h}\) specified by the contract minimizes the agent’s current utility ratio

\[
CUR_i = E_i^{Ag} \left[ \exp\left( -\lambda (a_i + b_i) (V_{i+1} - V_i) - k \hat{h}^2 \right) \right],
\]

which implies that the agent’s optimal effort is

\[
\eta_i = \eta(b_i, c_i) = \left( \frac{A b_i c_i}{\gamma k} \right)^{1/\gamma}.
\]  \hspace{1cm} (A8)

The promise-keeping constraint must be satisfied by the principal’s investment \(c_i\) and the agent’s contractual parameters \((a_i, b_i)\); that is, the agent’s continuation utility ratio must equal one:

\[
CUR_i \equiv 1.
\]  \hspace{1cm} (A9)

See the discussion following Equation (26). After some algebra, Equation (A9) implies that the relation between the parameters \(a_i, b_i,\) and \(c_i\) in period \([i, i+1]\) (here \(i = T - 1\)) satisfies

\[
a_i = a_i(b_i, c_i) := 0.5\lambda b_i^2 (\sigma^2_i + \delta^2) + \kappa \eta(b_i, c_i) - b_i (A c_i \eta(b_i, c_i) - l_i + \mu_i^{Ag}).
\]  \hspace{1cm} (A10)

Given the contractual parameters \((b_i, c_i)\), the principal’s additional value from continuing the project or her continuation value at date \(i, CV_i,\) is

\[
CV_i := E_i^{Pr}[(1 - b_i) (V_{i+1} - V_i) - a_i(b_i, c_i) - c_i].
\]  \hspace{1cm} (A11)

Substituting the agent’s effort (A8) into Equation (A11) and using the fact that

\[
E_i^{Pr} [V_{i+1} - V_i] = E_i^{Pr} \left[ \left( A c_i \eta^i b_i - l_i \right) + \xi_{i+1} \right] = \left( A c_i \eta^i b_i - l_i + \mu_i^{Pr} \right),
\]

the principal’s continuation value simplifies to

\[
\Lambda_i(b_i, c_i) := \Delta_i b_i - 0.5 p_i b_i^2 + \phi(b_i) c_i^{\alpha/\gamma} - c_i + \mu_i^{Pr} - l_i.
\]  \hspace{1cm} (A12)

In Equation (A12), \(\Delta_i\) and \(p_i\) are, of course, the degree of agent optimism and the price of risk in period \([i, i+1]\), and \(\phi()\) is defined as in Equation (14).

The principal’s capital investment and the agent’s pay-performance sensitivity maximize the principal’s continuation value (A12). By exactly the same arguments as in Section 1.3, the principal’s optimal investment as a function of the agent’s pay-performance sensitivity is given by Equation (15). The principal’s continuation value as a function of the pay-performance sensitivity is therefore

\[
\Lambda_i(b_i, c(b_i)) = \Delta_i b_i - 0.5 p_i b_i^2 + K c(b) + (\mu_i^{Pr} - l_i).
\]  \hspace{1cm} (A13)

(The constant \(K\) in Equation (A13) is identical to the constant \(K\) in the static model.) We conclude that the principal chooses the agent’s pay-performance sensitivity in period \([i, i+1]\) to solve

\[
b_i^* := \arg \max_{b_i \geq 0} F_i(b_i),
\]  \hspace{1cm} (A14)

where, of course, \(F_i()\) is the principal’s periodic flow function at date \(i\). The optimal termination time is \(i = T - 1\) if the principal’s maximum continuation value \(\Lambda_i(b_i^*, c(b_i^*))\) is negative.
The Inductive Step. We now set \( i = T - 2 \), and suppose the project has not been terminated as of date \( T - 2 \). If the investment is \( c_i \), the agent’s contractual parameters are \((a_i, b_i)\), and he exerts effort \( \tilde{h}_i \), his continuation utility ratio (26) is

\[
CV_i = E^{P_t}_i[(1 - b_i)(V_{i+1} - V_i) - a_i(b_i, c_i) - c_i + \max\{CV_{i+1}, 0\}],
\]

(A16)

The first line of Equation (A15) follows by the law of iterated expectations and the second line follows by Equation (A9). Since the expression (A15) is identical to Equation (A7), we may use our previous arguments to show that the agent’s effort is \( \eta (b, c) \) given in Equation (A8) and the “fixed” component of the agent’s compensation is \( a(b, c) \) given in Equation (A10).

It remains to determine the principal’s investment and the agent’s pay-performance sensitivity. The principal’s continuation value at the beginning of period \( [i, i + 1] \) is

\[
CV_i = E^{P_t}_i[(1 - b_i)(V_{i+1} - V_i) - a_i(b_i, c_i) - c_i + \max\{CV_{i+1}, 0\}] = \Lambda_i(b_i, c_i) + E^{P_t}_i[\max\{CV_{i+1}, 0\}].
\]

(A17)

The above follows from the fact that the expression

\[
E^{P_t}_i[(1 - b_i)(V_{i+1} - V_i) - a_i(b_i, c_i) - c_i]
\]

is identical to Equation (A11) and hence Equation (A12). As the second term of Equation (A17) is unaffected by the actions taken by the principal and agent during period \([i, i + 1]\), we may use our previous arguments to show that the principal’s continuation value at date \( i \) is maximized when the pay-performance sensitivity solves Equation (A14) and the investment is given by \( c(b^*_p) \).

The optimal termination time of the relationship is \( i \) if the principal’s continuation value \( CV_i \) is negative. \( \blacksquare \)

The incremental change in the termination payoff (19) depends on \( \eta \) only through the terms \( \eta^\beta, \eta^\gamma \). There is no loss of generality if the unit of effort is redefined as \( z := \eta^\beta \), the production function is taken as \( c^z \), and the disutility of effort is taken as \( z^{\gamma/\beta} \). The equilibrium, as characterized in Theorem 3.1, depends on the parameters \( \beta \) and \( \gamma \) only through their ratio \( \gamma/\beta \). Hence, we normalize \( \beta \) to 1 in the subsequent proofs to simplify the notation.

Proof of Theorem 4.1. We begin by proving the claims concerning the path of the pay-performance sensitivities \( b^*_i \)'s. From Equation (36) and the definition of \( b^*_p \) (recall, \( F'(b^*_p) = 0 \)),

\[
F'(b^*_p) = \left( \frac{\Delta_0}{\sigma_0^2} - \lambda b^*_p \right) \sigma^2.
\]

(A18)

Therefore, the sign of \( F'(b^*_p) \) is identical to the sign of \( \Delta_0 - \lambda \sigma_0^2 b^*_p \). The strict quasi-concavity of each \( F_i(\cdot) \) ensures that if this sign is negative (positive), then \( b^*_i \) is strictly less than (greater than) \( b^*_p \). If the sign is zero, \( b^*_i \) coincides with \( b^*_p \).

It remains to show that the convergence is monotonic in the first and third cases. To this end suppose \( \Delta_0 < \lambda \sigma_0^2 b^*_p \). Pick a period \( i \). The optimal solution \( b^*_i \) satisfies

\[
0 = F'(b^*_i) = \left( \frac{\Delta_0}{\sigma_0^2} - \lambda b^*_i \right) \sigma^2 + F'(b^*_p).
\]

(A19)
Since $F(\cdot)$ is strictly quasi-concave, it follows from $b_t^* < b_p^*$ and $F'(b_p^*) = 0$ that $F'(b_t^*) > 0$. We may conclude from Equation (A19) that $\Delta_0/\sigma_0^2 - \lambda b_t^* < 0$. Since $\sigma_t^2 > \sigma_{t+1}^2$,

$$F_{t+1}'(b_t^*) = \left(\frac{\Delta_0}{\sigma_0^2} - \lambda b_t^*\right)\sigma_{t+1}^2 + F'(b_t^*) > F_t'(b_t^*) = 0.$$ 

Hence, $b_{t+1}^* > b_t^*$ as $F_{t+1}(\cdot)$ is strictly quasi-concave. Thus, the $b_t^*$ increase monotonically toward $b_p^*$.

Suppose $\Delta_0 > \lambda \sigma_0^2 b_p^*$. Since $F(\cdot)$ is strictly quasi-concave, it follows from $b_t^* > b_p^*$ and $F'(b_p^*) = 0$ that $F'(b_t^*) < 0$. We may conclude from Equation (A19) that $\Delta_0/\sigma_0^2 - \lambda b_t^* > 0$. Since $\sigma_t^2 < \sigma_{t+1}^2$,

$$F_{t+1}'(b_t^*) = \left(\frac{\Delta_0}{\sigma_0^2} - \lambda b_t^*\right)\sigma_{t+1}^2 + F'(b_t^*) < F_t'(b_t^*) = 0.$$ 

Hence, $b_{t+1}^* < b_t^*$ since $F_{t+1}(\cdot)$ is strictly quasi-concave. Thus, the $b_t^*$ decrease monotonically toward $b_p^*$.

We now turn our attention to the principal’s investments. Suppose first that $\Delta_0 < \lambda \sigma_0^2 b_p^*$. In this case the $b_t^*$ increase monotonically toward $b_p^*$, which is less than one. Since $c(\cdot)$ is strictly quasi-concave with a maximum at one (see Proposition 1.1), the $c_t^*$ increase monotonically toward $c_p^*$. The second case is obvious. As for the third case, the ratio $\frac{\Delta_0}{\lambda \sigma_0^2 \eta^*(\pi)}$ in Equation (A5) is greater than, equal to, or less than one depending on whether $i$ is less than, equal to, or greater than $i^*$. Since $c^*(\cdot)$ is negative on $(1, \gamma)$ and positive on $(0, 1)$, it now follows easily from Equation (A5) that $b_t^* > 1$ when $i < i^*$; $b_t^* = 1$ when $i = i^*$; and $b_t^* < 1$ when $i > i^*$. Since the $b_t^*$ decrease monotonically toward $b_p^*$, initially the $c_t^*$ increase until $i = i^*$, and then decrease monotonically toward $c_p^*$, as claimed.

The results for the agent’s effort choices, $\eta_t^*$, is an immediate consequence of the optimal effort function (A8) and the results for the pay-performance sensitivity and investment paths. ■

For each parameter “$\Pi$” we let $b_i(\pi)$ and $c_i(\pi)$ denote, respectively, the value of $b$ and $c$ at date $i$ when the parameter $\Pi$’s value equals $\pi$, and we let $b(\pi)$ and $c(\pi)$ denote the entire time path of pay-performance sensitivity and optimal investment when the parameter $\Pi$’s value equals $\pi$. We will also write $F_i'(b_i, \pi)$ to make explicit the functional dependence of the derivative of $F_i$ on the parameter value $\pi$. To ease notational burdens in what follows, we will view the time index $i$ as a real variable.

The following lemma is used repeatedly in the proofs to follow.

**Lemma 6.1.** If $F_i'(b_i, \pi)$ is an increasing (decreasing) function of $\pi$, then $b_i(\pi)$ is an increasing (decreasing) function of $\pi$.

**Proof.** Let $\pi^1 < \pi^2$. Suppose first that $F_i'(b_i, \pi)$ is an increasing function of $\pi$. By definition,

$$0 = F_i'(b_i(\pi^2), \pi^2) = F_i'(b_i(\pi^1), \pi^1) < F_i'(b_i(\pi^1), \pi^2),$$

which immediately implies $b(\pi^1) < b(\pi^2)$ by the strict quasi-concavity of $F_i(\cdot)$. The proof in the decreasing case is analogous. ■
Proof of Theorem 4.2.

(a) Set the parameter \( \pi = \lambda, \sigma_0^2 \). By substituting Equation (21) in Equation (A5),

\[
F'(b, \pi) = \frac{\Delta_0 s^2}{s^2 + i\sigma_0^2} - \frac{\lambda b s^2}{s^2 + i\sigma_0^2} + (i + 1)\sigma_0^2 + K c'(b),
\]

(A20)

we see that \( F'(b, \pi) \) is clearly decreasing in \( \pi \). The result now follows from Lemma 6.1.

(b) Theorem 4.1 shows that the \( b \) trajectory increases toward \( b_p \), which is less than one. Since the trajectory of \( b \) is pointwise decreasing by part (i), and since \( c(\cdot) \) is an increasing function on \([0, 1]\), the result follows.

(c) Please refer to Figure 3(b). Suppose \( i_j^* = (\Delta_0 - p_0)/(i \sigma_0^2), j = 1, 2 \), denote the value of \( i^* \) defined in Theorem 4.1c(ii) corresponding to \( \pi^1 \). We will prove the case only when both \( i_1^* \) and \( i_2^* \) lie above one; since \( b(\pi^1) > b(\pi^2) \), it immediately follows that \( c(\pi^1) < c(\pi^2) \) in this interval. Analogously, the proof of Theorem 4.1c(i) also shows that in the interval \( (i_1^*, \infty) \) both \( b(\pi^1) \) and \( b(\pi^2) \) lie below one; since \( b(\pi^1) > b(\pi^2) \), it immediately follows that \( c(\pi^1) > c(\pi^2) \) in this interval. By Theorem 4.1c(ii), we know \( c(\pi^1) \) is increasing in the interval \([i_2^*, i_1^*]\), whereas \( c(\pi^2) \) is decreasing in this interval. Moreover, since \( c_1(\pi^1) = c(1) > c_1(\pi^2) \) and \( c_2(\pi^1) < c_2(\pi^2) = c(1) \), the trajectories \( c(\pi^1) \) and \( c(\pi^2) \) cross exactly once in this interval.

(d) By Lemma 6.1 it is sufficient to show that \( F'(b^*_1, s^2) \) is decreasing in \( s^2 \). By Equation (A20), \( F'_j(b^*_0, s^2) \) is clearly decreasing in \( s^2 \). Now suppose \( i \geq 1 \). The sign of the derivative of Equation (A20) with respect to \( s^2 \) coincides with the sign of

\[-(\lambda b s^2 + i\sigma_0^2(2s^2 + (i + 1)\sigma_0^2) - \Delta_0),\]

(A21)

and therefore the result will follow if we can establish that \( b^*_j \lambda(2s^2 + (i + 1)\sigma_0^2) \geq \Delta_0 \) and \( b^*_j > 0 \). To this end let

\[
\hat{b}_i := \frac{\Delta_0}{\lambda(2s^2 + (i + 1)\sigma_0^2)}.
\]

Since \( \Delta_0 \leq 2p_0 \) by assumption, \( \hat{b}_i \leq 1 \), which implies \( c'(\hat{b}_i) \) is nonnegative. Therefore,

\[
F'(\hat{b}_i, s^2) = K c'(\hat{b}_i) + \frac{\Delta_0 s^2}{s^2 + i\sigma_0^2} \left[ 1 - \frac{s^2 + (i + 1)\sigma_0^2}{2s^2 + (i + 1)\sigma_0^2} \right] \geq 0.
\]

(A22)

Hence, we may conclude that \( b^*_j \geq \hat{b}_i > 0 \) since \( F_i(\cdot) \) is strictly quasi-concave and \( b^*_j \in [0, b^M] \). Thus,

\[
b^*_j \lambda(2s^2 + (i + 1)\sigma_0^2) \geq \hat{b}_i \lambda(2s^2 + (i + 1)\sigma_0^2) = \Delta_0.
\]

Proof of Theorem 4.3.

(a) This follows by a straightforward application of Lemma 6.1.

(b) Theorem 4.1 shows that the \( b \) trajectory increases toward \( b_p \), which is less than one. Since \( c(\cdot) \) is an increasing function on \([0, 1]\), the result follows.

(c) Please refer to Figure 3(b). Suppose \( \Delta_0^1 < \Delta_0^2 \). Let \( i_j^*, j = 1, 2 \), denote the value of \( i^* \) corresponding to \( \Delta_0^j \). We will prove the case only when both \( i_j^*, j = 1, 2 \), are positive (which occurs when \( \Delta_0 > p_0 \)), as the other cases follow the same logic. Clearly, \( i_1^* < i_2^* \). The proof of Theorem 4.1c(ii) shows that in the interval \([0, i_1^*]\) both \( b(\Delta_0^1) \) and \( b(\Delta_0^2) \) lie above one;
since \( b(\Delta_1^j) < b(\Delta^2_j) \), it immediately follows that \( c(\Delta_1^j) > c(\Delta^2_j) \) in this interval. Analogously, the proof of Theorem 4.1c(i) also shows that in the interval \((i^*_2, \infty)\) both \( b(\Delta_1^j) \) and \( b(\Delta^2_j) \) lie below one; since \( b(\Delta_1^j) < b(\Delta^2_j) \), it immediately follows that \( c(\Delta_1^j) < c(\Delta^2_j) \) in this interval. By Theorem 4.1, we know \( c(\Delta_1^j) \) is decreasing in the interval \([i^*_1, i^*_2]\), whereas \( c(\Delta^2_j) \) is increasing in this interval. Moreover, since \( c_{i^*_1}(\Delta_1^j) = c(1) > c_{i^*_1}(\Delta^2_j) \) and \( c_{i^*_2}(\Delta_1^j) < c_{i^*_2}(\Delta^2_j) = c(1) \), the trajectories \( c(\Delta_1^j) \) and \( c(\Delta^2_j) \) cross exactly once in this interval.

We now make explicit the functional dependence of the principal continuation value (A17) on the current assessment of the project’s core output, and write it as \( CV_p(\mu_i) \). We drop the superscript on \( \mu_i \), since it will always refer to the principal’s assessment. Let \( Z \) denote the standard normal random variable. By Equations (20) and (22), the continuation value may be expressed as

\[
CV_i(\mu_i) = [P_i^* + \mu_i - l_i] + E[\max\{CV_{i+1}(\delta_{i+1}Z + \mu_i), 0\}],
\]

(A23)

where \( \delta^2_{i+1} \) is defined in Equation (42).

**Lemma 6.2.** The continuation value \( CV_j(\cdot) \) is a continuous, nondecreasing function of \( \mu_j \) for \( 0 \leq j \leq T - 1 \).

**Proof.** We use backward induction. To establish continuity, we show that there exist positive constants \( \kappa^1_j, \kappa^2_j \) such that

\[
CV_j(\mu_j) \leq \kappa^1_j + \kappa^2_j \max\{\mu_j, 0\}.
\]

(A24)

The assertions of continuity, monotonicity, and Equation (A24) immediately follow when \( j = T - 1 \), since \( CV_{T-1}(\mu_{T-1}) = F_{\mu_{T-1}}^* + \mu_{T-1} - l_{T-1} \). Suppose the assertions are true for \( j = i + 1, \ldots, T - 1 \). We will establish the assertions are true for \( j = i \).

The monotonicity of \( CV_i(\cdot) \) is a direct consequence of the fact that the expectation on the right-hand side of Equation (A23) is taken with respect to the standard normal density, which is independent of the problem parameters, and the monotonicity of \( CV_{i+1}(\cdot) \) (by the inductive assumption).

The proof of continuity will follow from Equation (A23) if the limit and expectation operators may be interchanged, since \( CV_{i}(\cdot) \) is continuous in \( \mu_i \), by the inductive assumption. By the inductive assumption (A24), the function \( CV_i(\cdot) \) is bounded above by a positive function whose expectation

\[
E[\kappa^1_{i+1} + \kappa^2_{i+1} \max\{\mu_{i+1}, 0\}] = \kappa^1_{i+1} + \kappa^2_{i+1} \left[ \frac{\delta_i}{\sqrt{2\pi}} e^{-1/2\left(\frac{\mu^2_i}{\delta_i}\right)} + \mu_i P\left( Z > -\frac{\mu_i}{\delta_i}\right) \right].
\]

(A25)

is finite, and thus the interchange is justified by the dominated convergence theorem. Furthermore, the inequality (A25) shows that the expectation on the right-hand side of Equation (A23) is bounded above by

\[
\left( \kappa^1_{i+1} + \kappa^2_{i+1} \frac{\delta_i}{\sqrt{2\pi}} \right) + \kappa^2_{i+1} \max\{\mu_j, 0\}.
\]

(A26)

Hence, it is possible to define \( \kappa^1_j, \kappa^2_j > 0 \) such that Equation (A24) holds for \( j = i \), as required.

**Proof of Proposition 4.4.** By Lemma 6.2 each function \( CV_i(\cdot) \) is continuous and nondecreasing. The proof of Lemma 6.2, in particular Equation (A26), shows that each \( CV_i(\cdot) \) is negative for
sufficiently small $\mu_i$. Since each $CV_i(\cdot)$ is obviously positive for sufficiently high $\mu_i$, there exists a unique value $\mu_i^*$ for which $CV_i(\mu_i^*) = 0$. By the dynamic programming principle of optimality, the contract is terminated at date $i$ only if $\mu_i < \mu_i^*$.

Proof of Proposition 4.5. The function $F_i(\cdot)$ is an increasing function of $\Delta_0$, which implies that $F_i^*$ is also an increasing function of $\Delta_0$. One may proceed exactly as in the proof of Lemma 6.2 to establish that each $CV_i(\cdot)$ is a pointwise nondecreasing function of $\Delta_0$, too, and it should be clear from the proof of Proposition 4.4 that the trigger values will decrease. Since a change in this parameter has no effect on the sample paths, the result (i) follows. The proof of (ii) is the same, except that each $F_i(\cdot)$ is now a decreasing function of either $\lambda$ or $k$, and thus the trigger values increase. □

Appendix B: Numerical Implementation and Calibration

We use the simulated method of moments to calibrate the thirteen parameters,

$$\pi = (V_0, A, \alpha, \mu_0^{true}, s, \ell_1, \ell_2, \mu_0^P, \Delta_0, \sigma, \lambda, k, \gamma / \beta),$$

of the model. For a given candidate parameter vector $\pi$, we simulate a sufficiently large number $N = 40,000$ of sample paths for the termination payoff process described in Equation (19). Each sample path represents a realization of the termination payoff, agent payoff, and capital investments associated with one possible pharmaceutical R&D project. The set of sample paths $\mathcal{P}$ is used to construct accurate empirical distributions for the project terminal value and R&D expenditures from which the vector $V(\pi)$ of simulated values of the fourteen statistics reported in Table 2 can be computed. Let $d_i(V_i(\pi), O_i) := V_i(\pi) - O_i$ denote the difference between the simulated and observed values of $i^{th}$ statistic in Table 2 and let $d(\pi)$ denote the vector of differences. Define

$$f(\pi) := d(\pi)^T \Sigma d(\pi). \quad (B1)$$

We use the Nelder-Meade optimization subroutine available in MATLAB to obtain the vector $\pi^*$ of parameter estimates that solves

$$\pi^* = \arg \min_{\pi} f(\pi). \quad (B2)$$

In Equation (B1), we take $\Sigma$ to be a diagonal matrix whose respective entries are the reciprocals of the corresponding observed statistics reported in Table 2. We now precisely describe how we determine the vector $V(\pi)$ of simulated values of the fourteen statistics.

Monte Carlo simulation of the termination payoff process. The incremental termination payoff described in Equation (19) is used to determine a sample path of project termination values and capital investments, as follows. The maximum project duration is set to $T = 12$ years and the period length is set to one year. In Equation (19), the value for the operating cost $l_i = l_i^{1/2}$ is directly obtained from the loss parameter components of $\pi$. As a result of Theorem 3.1, assuming the project has not been terminated, the equilibrium value for the discretionary output $\Phi(c_i^*, \eta_i^*)$ in Equation (19) is deterministic and, therefore, can be precomputed. The value of $\theta$ is drawn from a normal distribution with mean $\mu_0^{true}$ and standard deviation $\sigma_0$. The value of $S_{i+1}$ is drawn from a normal distribution with mean zero and standard deviation $s$. Let $\hat{S} := (\hat{S}_1, \hat{S}_2, \ldots, \hat{S}_T)$ denote a vector of independent random draws from a standard normal distribution, and let $\hat{\theta}$ denote a random draw from a standard normal distribution independent of the $\hat{S}_i$. If the project has not been terminated as of date $i$, then the terminal project value at date $i + 1$ is given by

$$V_{i+1} = V_i + (\mu_0^{true} + \sigma_0 \hat{\theta}) - l_i^{1/2} + s \hat{S}_{i+1} + \Phi(c_i^*, \eta_i^*). \quad (B3)$$
We identify a sample path with the vector \((\hat{\theta}, \hat{S})\). It remains to describe how the stopping time \(\tau\) is determined for each sample path.

By Proposition 4.4, the principal will terminate the relationship at date \(i\) if and only if \(\mu_{i}^{Pr}\) is less than a trigger \(\mu_{i}^{*}\). To determine the \(\mu_{i}^{*}\), we, therefore, directly model the evolution of \(\mu_{i}^{Pr}\) using a discrete lattice. At date 0 the principal assesses the core output as \(\mu_{0}^{Pr}\). Let \(n(i)\) denote the number of states at date \(i > 0\) and let \(\mu_{i,j}^{Pr}\) denote the core output at the \(j^{th}\) state at date \(i\), \(j = 1, \ldots, n(i)\). The lattice is designed so that the minimal and maximal states at date \(i\), \(\mu_{i,1}^{Pr}\), and \(\mu_{i,n(i)}^{Pr}\), respectively, are \(\kappa = 2.5\) standard deviations below and above the minimal and maximal states at date \(i - 1\). More precisely,

\[
\mu_{i,n(i)}^{Pr} = \mu_{i-1,n(i-1)}^{Pr} + \kappa \sigma_{i}^{Pr} \quad \text{and} \quad \mu_{i,1}^{Pr} = \mu_{i-1,1}^{Pr} - \kappa \sigma_{i-1}^{Pr},
\]

(B4)

where \(\sigma_{i}^{Pr}\) is the standard deviation of the evolution of the assessment of core output over the period \((i - 1, i)\) and is given by Equation (42). The values for the remaining \(n(i) - 2\) states are equally spaced between the minimum and maximum states. The number of states in the lattice increases linearly from period to period. That is, \(n(i) = Mi\) for \(i > 0\), where we set \(M = 20\). We found that an increase in \(M\) or \(\kappa\) both did not appreciably change the optimal trigger values.

Let \(CV_{i,j}\) denote the principal’s continuation value at state \(\mu_{i,j}^{Pr}\). At date \(T - 1\) the continuation value is independent of the future and is \(CV_{T-1,j} = \mu_{T-1,j}^{Pr} + F_{T-1}^{*} - I_{T-1}\) for each state \(\mu_{T-1,j}^{Pr}\), \(j = 1, 2, \ldots, n(T - 1)\). At earlier dates the continuation values are

\[
CV_{i,j} = \mu_{i,j}^{Pr} + F_{i}^{*} - I_{i} + e^{-r} \sum_{k=1}^{n(i+1)} p_{i,j,k} \max(CV_{i+1,k}, 0). \tag{42}
\]

In the above, \(p_{i,j,k}\) denotes the probability that the assessment of core output will transition from state \(\mu_{i,j}^{Pr}\) at date \(i\) to state \(\mu_{i+1,j}^{Pr}\) at date \(i + 1\). If \(\mu_{i+1,j}^{Pr}\) is within \(\pm \kappa \sigma_{i}^{Pr}\) from \(\mu_{i,j}^{Pr}\), we set

\[
p_{i,j,k} := \Phi \left( \frac{1}{2} (\mu_{i+1,j}^{Pr} + \mu_{i+1,k+1}^{Pr}) - \mu_{i,j}^{Pr} \right) \frac{1}{\sigma_{i}^{Pr}} - \Phi \left( \frac{1}{2} (\mu_{i+1,j}^{Pr} + \mu_{i+1,k-1}^{Pr}) - \mu_{i,j}^{Pr} \right),
\]

where \(\Phi(\cdot)\) denotes the cdf of the standard normal distribution. Otherwise, the transition probability is zero. Starting from the last investment period \([T - 1, T]\) and working backward through time, we use dynamic programming to compute the continuation values for all states and dates. Since the true continuation value function is continuous and increasing (see Lemma 6.2 in Appendix A), we complete the approximation to \(CV_{i}(\cdot)\) by linear interpolation. We then determine the optimal trigger \(\mu_{i}^{*}\) that solves \(CV_{i}(\mu_{i}^{*}) = 0\).

In our application of the model to firm R&D, recall that we assumed that shareholders earn competitive returns on their investments. Since the shareholders initially invest the seed capital \(V_{0}\), they will never accept any value for \(P_{0}\) for which the principal’s stake (at date 0),

\[
S_{\pi}(P_{0}) := E_{0}^{Pr} \left[ e^{-rT}(V_{T} - P_{T}) - \sum_{j=0}^{n-1} e^{-rj} I_{j}^{*} \right]. \tag{B5}
\]

is less than \(V_{0}\). Because shareholders competitively supply capital, the initial promised payoff, \(P_{0}\), for the agent is such that the principal’s stake at date zero is equal to the initial seed capital \(V_{0}\), that is, \(P_{0}\) is determined by the identity \(S_{\pi}(P_{0}) = V_{0}\). We construct a set \(P\) of 40,000 sample paths to estimate \(S_{\pi}(P_{0})\). To eliminate simulation error, we keep \(P\) fixed and iterate until the identity \(S_{\pi}(P_{0}) = V_{0}\) is satisfied. Keep in mind that for this calculation, the right-hand side of Equation (B5) uses the principal’s assessment of project value, so that along each sample path \(\mu_{0}^{true}\) in Equation (B3) is replaced with \(\mu_{0}^{Pr}\).
Determination of the vector of simulated statistics. Once \( P_0 \) has been determined, the project termination value and capital investments can now be determined for each sample path (B3) using the true distribution for core output. The same fixed set of sample paths \( P \) is used again to determine the vector of simulated statistics \( V(\pi) \) from which the vector of differences \( d(\pi) \) used in Equation (B1) is determined.

Appendix C: Parametric Bootstrapping

We use parametric bootstrapping to determine the confidence intervals for the estimated parameters and the standard deviations of the fourteen predicted statistics (see Davison and Hinkley 1997).

First, we use the parameter vector \( \pi = \pi^* \) and the value for \( P_0 \) obtained from the parameter estimation procedure described in Appendix B to generate \( M \) samples of 120 projects denoted as \((S_1, \ldots, S_M)\). We use 120 instead of 118, the number of NCE projects in the sample, so that there will be 12 NCEs in each decile of the terminal project value distribution. We use samples \( S_j \) to compute the \( j \)th vector of the fourteen statistics reported in Table 2. In this manner, we obtain a set of \( M \) vectors of “bootstrapped” statistics \( V_j, j = 1, 2, \ldots, M \). Next, we replace the vector \( O \) of actual values of the fourteen statistics of Phase I with the vector \( V_j \) of bootstrapped values. For each bootstrapped vector \( V_j \), we solve Equation (B2) to obtain a set of \( M \) “bootstrapped” estimates of the parameter vector \((\pi_1^*, \ldots, \pi_M^*)\). We use these vectors to obtain standard errors for the estimated parameters \( \pi^* \). Since this calculation is computationally intensive, we set \( M = 1,000 \) for this calculation.

To determine the standard deviations of the fourteen predicted statistics, we once again use the parameter vector \( \pi = \pi^* \) and the value for \( P_0 \) to generate \( M \) samples \((S_1, \ldots, S_M)\) of 120 projects. These vectors are used in the obvious way to determine the standard deviations associated with the fourteen predicted statistics that are reported in the fourth row of Table 2. Since this calculation does not involve optimization, it is not computationally intensive, and so we set \( M = 10,000 \).

References


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