Math in the Real World: A High School Course in Operations Research for Georgia

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National Challenges

• Performance gaps – Americans fare poorly
  – Multi-step problems
  – Interpretation of results
  – Word problems

• Lack of motivation and relevance of math with respect to many career choices
  – Students turned off to math do not take four years
  – Often end up in remedial courses in first year of college
What is Operations Research?

• Scientific approach to analyzing problems and making decisions.
• Study of certain physical and human systems in order to improve or optimize such systems.
• OR is the “theoretical sibling” of Industrial Engineering.
• Examples: Airlines, Service Systems, Manufacturing, Logistics, Telecomm
What Is OR (cont’d)?

Specific Topics of Interest:
- Linear Programming
- Probability and Statistics
- Queueing Theory
- Inventory and Scheduling
- Computer Simulation
- etc., etc.
Why Should We Care?

We Encounter IE/OR in Everyday Life.
• Queue at the bank
• Disneyland
• Travel Industry
• Financial Engineering
• Postal System
• etc., etc.
Why Should We Care (cont’d)?

• Interesting, Real-Life Topics
• Interesting Theory
• Nice Alternative to Other Math Courses
• Lots of Jobs Available at All Levels
  – Airlines
  – Consulting Companies
  – Wall Street
  – Manufacturing
  – Schools
IMAGINE!

• Can you imagine a more exciting high school math course than one driven by real-world OR and IE applications?
MINDSET NSF Grant 2007-2012

- $3 million – 5 year NC State, Wayne State, UNCC
- Currently Developing…
  - HS math course and text with Algebra II prerequisite
  - Graduate math education course for teachers including distance learning version
  - Extensive support services for teachers
  - 80-100 teachers piloting in 2009-2010
  - 50 high schools in NC and Mich. with 1500 kids
  - Deterministic portion of course is READY
What’s the Competition?

- Pre-calculus and Calculus
- Statistics AP
- Discrete Mathematics – hodgepodge
  - Routing and discrete probability
  - No unifying theme
  - Limited organizational support
- NC (and Colorado) – broad and specific objectives with syllabus but no text
- Mich. – every school district on its own
Course Outline – Probabilistic Models

- Probability Review and Applications
- Data Collection
- Statistics Review and Applications
- Queueing Theory — Waiting in Lines
- Forecasting — Planning for the Future
- Quality Control — Making Things Right
- Computer Simulation I — Modeling Complex Systems
- Computer Simulation II — Software
Course Outline – Deterministic Models

- Review of Linear Inequalities
- Linear Programming Techniques and Applications
- Integer Programming Techniques and Applications
- Graph and Network Theory — Getting From Here to There
- Scheduling — Who Will Work When
- Inventory Theory — How to Order Supplies
- Supply Chains and Logistics — Moving Material Efficiently
- Decision Theory — Making the Best Choice
Course Outline – Applications

• Humanitarian Logistics
• Operations Research in Sports
• OR in the Airline Industry
• Applications in Renewable Resources — OR Goes Green
• Financial Planning
• Healthcare
Resources on Hand

- Georgia Tech School of ISyE
- Georgia Tech CEISMC
- INFORMS / MINDSET / NSF?
  - www.informs.org
  - www.mindsetproject.org
  - www.hisor.org
- Rockwell Software
- State of Georgia??
Required Resources?

- Deterministic material is “ready”
- Probabilistic material “ready” by end of year
- Need to match up material with Georgia standards
- Need to vet material with Georgia teachers
- Train the teachers
- Pilot studies with live classes
- Solicit comments from teachers / students
- Eventually, formal adoption of course
Example Topic: Computer Simulation

- What is Simulation?
- Some Easy Examples
- Generating Randomness
- Some Bigger Examples
What is Simulation?

What I Used to Think...

\[
E(m) \equiv \sum_{i=m-2\delta_m+1}^{m} \sum_{j=m-\delta_m+1}^{m} \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} f_{ijk\ell}
\]

\[
\leq 4\delta_m^4 E[X_i^4] \leq 4\delta_m^4 C_8 = O(m^{4/3}) \quad \text{(by Lemma 1 applied twice).} \quad (7)
\]

We finally complete the proof by examining the second term of (2). Since \(|g_{ij}(m)| \leq C_q/m\), we have (using the obvious abuse of summation notation)

\[
\frac{m^2}{C_q^2} |\text{Cov}(Q_1(m), Q_2(m))| \leq \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} f_{ijk\ell}
\]

\[
= \sum_{i=1}^{m} \sum_{j=1}^{m} \left\{ \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} + 2 \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} \sum_{k=m+\delta_m+1}^{m+2\delta_m} \sum_{\ell=m+\delta_m+1}^{m+2\delta_m} \right\} f_{ijk\ell}
\]

\[
\leq A(m) + 2B(m) + 2 \left\{ \sum_{i=1}^{m} \sum_{j=1}^{m-\delta_m} \sum_{j=m-\delta_m+1}^{m+\delta_m} + 2 \sum_{i=1}^{m} \sum_{j=m-\delta_m+1}^{m+\delta_m} \right\} \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} f_{ijk\ell}
\]

\[
\leq A(m) + 2B(m) + 2C(m) + 4 \left\{ \sum_{i=1}^{m} \sum_{j=m-2\delta_m+1}^{m} \sum_{k=m+1}^{m+\delta_m} \sum_{\ell=m+1}^{m+2\delta_m} f_{ijk\ell} \right\}
\]

\[
\leq 4[A(m) + B(m) + C(m) + D(m) + E(m)] = O(m^{4-\gamma/6}) + O(m^{10/3-\gamma/6}) + O(m^{4/3}) = O(m^{4-\gamma/6}) + O(m^{4/3}).
\]
What is Simulation?

- Simulation is the imitation of a real-world process or system over time.
- Simulation involves the generation of an artificial history to draw inferences concerning the operating characteristics of the real system that is represented.
- One of the top three OR technologies
- It’s an indispensable problem-solving methodology
• We use simulation to:
  – Describe and analyze the behavior of a system
  – Ask “what if” questions about the system
  – Aid in the design of systems

• After about a week, a student will be able to model and evaluate simple stochastic systems
Reasons to Simulate

- Will the system accomplish its goals?
- Current system won’t accomplish its goals. Now what?
- Resolve disputes
- Solve a problem, like a bottleneck
- Sell an idea
- Create a specification or plan of action
Origins: Manufacturing Systems

• Simulation is the technique of choice
  – Calculates movement of parts and interaction of system components
  – Evaluates flow of parts through the system
  – Examines conflicting demand for resources
  – Examines contemplated changes before their introduction
  – Eliminates major design blunders
Typical Questions

- What will be the throughput?
- How can we change it?
- Where are the bottlenecks?
- Which is the best design?
- What is the reliability of the system?
- What is the impact of breakdowns?
Some Easy Examples

- Happy Birthday
- Let’s Make Some Pi
- Fun With Calculus
- Evil Random Numbers
- Queues ‘R Us
- Stock Market Follies
Happy Birthday

- How many people do you need in a room in order to have a 50% chance that at least two will have the same birthday?
  - 9
  - 23
  - 42
  - 183
# Birthday Paradox

1. **Enter Seed:**

   47475

2. **Click Start Button:**

   ![START]

**New Birthday Date:**

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</table>

**Total # Birthdays:**

| 24

**!!! MATCH !!!**

![BACK]
Let’s Make Some Pi

• Use Monte Carlo simulation to estimate π.
• Idea:
  – Area of a unit square is 1.
  – Area of an inscribed circle is π /4.
  – Probability that a dart thrown at the square will land in the circle is π /4.
  – Throw lots of darts. Proportion that will land in circle should approach π /4.
<table>
<thead>
<tr>
<th>Monte Carlo Simulation</th>
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<tbody>
<tr>
<td>Number of Points ?</td>
</tr>
<tr>
<td>Random Seed ?</td>
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<tr>
<td>Animation Delay (1-100)</td>
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<tr>
<td>Points Plotted:</td>
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<tr>
<td>Real Value of Pi:</td>
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<tr>
<td>Estimator for Pi:</td>
</tr>
</tbody>
</table>

![Monte Carlo Simulation Diagram]

![Monte Carlo Simulation Graph]
Fun With Calculus

• Use simulation to integrate the function $f(x) = \sin(\pi x)$ over $[0,1]$.

• Idea:
  – Sample $n$ rectangles.
  – Each is centered randomly on $[0,1]$ and has width $1/n$ and height $f(x)$.
  – Add up areas.
  – Make $n$ really, really big.
Monte Carlo Integration

1. Number of Points: 64
2. Enter Seed: 567893
3. Animation Delay: 1

ESTIMATE INTEGRAL

Actual Result: 0.6366
Estimator: 0.5886

Points Plotted: 64
Evil Random Numbers

• See what happens when you use a bad random number generator.

• Idea:
  – Simulate heights vs weights.
  – Should be a 2-D bell curve (normal distribution) with most observations in the middle and some on the outside.
  – Do observations “look” random?
Box Muller Method

1. **Number of Points:** 2000
   (Max 5000 points)

2. **Enter Seed:**
   (Any Positive Integer)
   585867

3. **Animation Delay:**
   (Value between 1 and 100)
   (Default is 1)

4. **Select Type of Generator:**
   - Random Number Generator
     - Good
     - Bad

5. **Click Start:**
   - START
   - BACK

Points Plotted: 2000
Box Muller Method

1. Number of Points: 2000 (Max 5000 points)
2. Enter Seed: 64 (Any Positive Integer)
3. Animation Delay: (Value between 1 and 100) (Default is 1)
4. Select Type of Generator:
   Random Number Generator
   - Good
   - Bad
5. Click Start:

START
BACK
Queues ‘R Us

• Single-server queue at McDonalds.
• Customers show up, wait in line, get served first-in-first-out.
• What happens as arrival rate approaches service rate?
  – Nothing much?
  – Line gets pretty long?
  – Hamburgers start to taste better?
Queues ‘R Us (cont’d)

• Can analyze queues via simulation.
• Can analyze via numerical or exact methods.

• Fun fact: Notice anything interesting about the word “queueing”? How about “queueoid”?
# MM1 Queue Simulation

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- **Interarrival Mean:** 4
- **Service Mean:** 3
- **Interarrival time:** 6
- **Service Time:** 1

![Graph showing Current Queue Length and Average Server Utilization]
# Output Analysis

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<th>SIMTIME: 083</th>
<th>THROUGHPUT: 021</th>
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<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
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<tr>
<td>Average # of Customers in the System</td>
<td>2.30</td>
</tr>
<tr>
<td>Average Time in the System</td>
<td>10.19</td>
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<tr>
<td>Server Utilization</td>
<td>93 %</td>
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<tr>
<td>Average Length of the Queue</td>
<td>1.37</td>
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<tr>
<td>Average Waiting Time in the Queue</td>
<td>6.05</td>
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Stock Market Follies

• Simulate a portfolio of various stocks
• Stock prices change randomly from year to year, with various volatilities
• Can consider different mixes for portfolio
• Simple spreadsheet application
<table>
<thead>
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<th>year0</th>
<th>mean</th>
<th>spread</th>
<th>year1</th>
<th>year2</th>
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</table>

![Graph](image1.png)

```
50000  60684.09  74235.77  81168.92  96092.43  106400.8  116847.1  132018.5  157688.1  188868  226685.8
```
Generating Randomness

• Need random variables to run the simulation, e.g., interarrival times, service times, etc.
• Generate $\text{Unif}(0,1)$ pseudo-random numbers
  – Use a deterministic algorithm
  – Not really random, but appear to be.
• Generate other random variables
  – Start with $\text{Unif}(0,1)$’s
  – Apply certain transformations to come up with just about any other type of random variable
Unif(0,1) PRN’s

• Deterministic algorithm
• Example: Linear Congruential Generator
  – Choose an integer “seed,” $X(0)$
  – Set $X(i) = a \times X(i-1) \mod (m)$, where $a$ and $m$
    are carefully chosen constants, and $\mod$ is
    the modulus function
  – Set the $i$th PRN as $U(i) = X(i)/m$
Unif(0,1) PRN’s

• Pretend Example:
  – Set $X(i) = 5 \times X(i-1) \mod(7)$, with $X(0) = 4$

  – Then $X(1) = 20 \mod 7 = 6$
  – $X(2) = 2$, $X(3) = 3$, $X(4) = 1$, $X(5) = 5$, etc.

  – So $U(1) = X(1)/m = 6/7$
  – $U(2) = 2/7$, $U(3) = 3/7$, etc.

  – Numbers don’t look particularly random.
Unif(0,1) PRN’s

• Real Example
• $X(i) = 16807 \times X(i-1) \mod (2^{31} - 1)$
• $U(i) = X(i) / m$
  – This generator is used in a number of simulation languages
  – Has nice properties, including long “cycle times”
  – Even better generators are out there
Generating Other R.V.’s

- Start with $U(i) \sim \text{Unif}(0,1)$
- Apply some appropriate transformation
- Example: $-(1/a) \ln(U(i)) \sim \text{Exp}(a)$
  - Inverse transform method – can use this for lots of important distributions
  - Many other more-sophisticated methods available, e.g., Box-Muller method for normals
Health-Related Examples

- The Medical Cold Chain Problem
- Childhood Immunization Problems
- Mitigating the Spread of Infectious Diseases
- Control of Asian Bird Flu
- Guinea Worm Eradication in The Sudan
- Hurricane Katrina Relief
- Earthquake Relief in Third World Countries
- A Maternity Ward in Kabul
- Military Humanitarian Logistics
A Simulation Model of Guinea Worm Disease in Sudan

Dave Goldsman, Paul Goldsman, Georgia Tech

James Zingeser — Carter Center / CDC / WHO
• Background on Guinea worm disease (dracunculiasis – sickness with little dragons)
• Simulation model
• Future work
Guinea Worm Disease
*(dracunculiasis)*
Larvae undergo two molts in the copepod and becomes a L3 larvae.

Human drinks unfiltered water containing copepods with L3 larvae.

Larvae are released when copepods die. Larvae penetrate the host's stomach and intestinal wall. They mature and reproduce.

Larvae are released when copepods die. Larvae penetrate the host's stomach and intestinal wall. They mature and reproduce.

Female worm begins to emerge from skin one year after infection.

L1 larvae released into water from the emerging female worm.

Fertilized female worm migrates to surface of skin, causes a blister, and discharges larvae.

<i> = Infective Stage

<i>d> = Diagnostic Stage
Filters
Community development
Niger
Distribution by Country of 49,886 Indigenous Cases of Dracunculiasis Reported during 2002

Number of cases

- **Sudan**: 37,023
- **Ghana**: 5,497
- **Nigeria**: 3,820
- **Togo**: 1,476
- **Mali**: 852
- **Burkina Faso**: 562
- **Niger**: 237
- **Cote d'Ivoire**: 192
- **Benin**: 154
- **Mauritania**: 42
- **Ethiopia**: 24
- **Uganda**: 7
- **Ethiopia**: 10
- **Chad**: 0 (1998^)
- **Cameroon**: 0 (1997^)
- **Yemen**: 0 (1997^)
- **Senegal**: 0 (1997^)
- **India**: 0 (1996^)
- **Kenya**: 0 (1994^)
- **Pakistan**: 0 (1993^)

* Provisional.
^ Year last indigenous case reported.

Pakistan was certified by WHO to be free of the disease in 1996 and India in 2000.
SUDAN GUINEA WORM ERADICATION PROGRAM
NUMBER OF REPORTED CASES OF DRACUNCULIASIS: 2006 - 2007*

2006 = 20,582 cases
2007* = 6,068 cases

From CDC Wrap-up #179
GHANA GUINEA WORM ERADICATION PROGRAM
NUMBER OF CASES OF DRACUNCULIASIS REPORTED DURING EPIDEMIOLOGIC YEAR JULY 2006 - JUNE 2007, AND JULY 2007* - JANUARY 2008*

From CDC Wrap-up #179
<table>
<thead>
<tr>
<th>Country</th>
<th>Indigenous Cases Reported</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2006</td>
</tr>
<tr>
<td>Togo</td>
<td>25</td>
</tr>
<tr>
<td>Cote d'Ivoire</td>
<td>5</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>1</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>3</td>
</tr>
<tr>
<td>Niger</td>
<td>108</td>
</tr>
<tr>
<td>Sudan</td>
<td>20582</td>
</tr>
<tr>
<td>Ghana</td>
<td>4134</td>
</tr>
<tr>
<td>Mali</td>
<td>323</td>
</tr>
<tr>
<td>Nigeria</td>
<td>16</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>25197</td>
</tr>
<tr>
<td><strong>All countries, excluding Sudan and Ghana</strong></td>
<td>481</td>
</tr>
</tbody>
</table>
Simulation Model

- Simulate movement of people from refugee camps back to home villages
- Allow possibility of picking up Guinea worm on the way.
- Allow for education and other intervention methods.
- **Goal of Simulation:** Where should we allocate resources?
Simulation Inputs & Outputs

• **Inputs** (used in spreadsheet to establish risk along routes)
  – Populations
  – # Health / Education Centers
  – Availability of other resources

• **Outputs** (from simulation)
  – # cases of G. worm disease
  – Travel times
  – Costs
Preliminary Example

• “Normal Operations”
  – Total cases from certain camps after 90 days = 1328
  – City 1 has 577 cases
  – City 2 has 8

• Reallocate resources to City 1; simulate again
  – Total cases = 952
  – City 1 = 265
  – City 2 = 29
Future Work

• Better data becoming available
• Simulate all camps
• Appropriate statistical analysis
• Optimize performance
Partnership of Immunization Providers

Project supported by funds received from the National Immunization Program, Centers for Disease Control and Prevention, Public Health Service, Department of Health and Human Services (#U66/CCU912985-01).
A DISCRETE EVENT SIMULATION
FOR CLINICS SERVING THE POOR

UCSD Partnership of Immunization Providers
“Collaboration for Improved Immunization Delivery”

John Fontanesi, PhD
Christos Alexopoulos, PhD*
Dave Goldsman, PhD*
Michelle De Guire, MPH, MSW
David Kopald
Kathy Holcomb, MA
Mark Sawyer, MD

University of California, San Diego School of Medicine

*Georgia Tech
BACKGROUND

• Medicine as a “high velocity” environment
  – Demand for *documented* quality
  – Demand for increased services
  – Demand for cheaper service
IMPACT ON IMMUNIZATIONS

• Increasingly complex schedule
• More and more expensive vaccines
• Larger cohort “eligible”
• Coverage rates tied to quality rating
Study Objectives

• Develop easy-to-use, low cost tools clinics can use to evaluate delivery practices.

• Development of the Observational Checklist of Patient Encounters (OCPE)
  – workflow observation protocol
  – community health worker “friendly”
  – pre-packaged reports

• Development of Outpatient Staging schema
OCPE RESULTS

• Inadequate routine assessment of patients’ immunization status for all visit types
• Patient flow through the clinics varied significantly across and within clinics
• Time analysis of clinic stages showed long waiting times
• Time analysis of the exam room process indicate patients spend more time waiting for the provider than interacting with provider
OCPE RESULTS

- Forms analysis showed that immunization information is captured on multiple forms.
- Families brought their immunization cards more often than providers asked.
- Because of overworked staff, billing was often not submitted.
- Because of overworked staff, immunization documentation often did not occur.
NEED FOR A VISUALIZATION TOOL

• Convert OCPE data into a visual representation
• Realistically represent “what is”
• Capable of performing “what if”
SIMULATING THE CLINICS

• The workflow analysis provided the basis for simulating a “generic” clinic.
• Simulation should provide feedback on
  – Resource utilization
  – Patient flow
  – Cost benefits of staff / facility changes
• Simulation should be easy to use.
What Does the Model Need?

• How do patients arrive?
• How do patients flow thru system?
• How many “servers”?
• How do “servers” flow thru system?
• How long do services take?
• Etc., etc.
MODELING ARRIVAL TIMES

• There are two types of arrivals:
  – Scheduled arrivals: Patients never arrive exactly on time!
  – Random arrivals: Arrival patterns are crazy... they vary by hour, day, season.
The Clinic Simulation

- Used the ARENA discrete-event simulation language
- Test cases: Various San Diego clinics
- Excel spreadsheet interface feeds ARENA model.
- Additional spreadsheet will populate Excel sheet with actual data.
- Demo...
It Works!

- Identified key areas of improvement for Mid-City clinic, at little or no cost.
- Anticipate lower patient renege rate (high correlation between renege rate and missed opportunities).
- Increased patient throughput.
- Lower patient delays.
A Maternity Ward in Kabul

• Joint work with Denise Giles and Brian McCarthy, (Centers for Disease Control and Prevention) and Sungjoo Lee (Georgia Tech)

• Reproductive health services discrete-event simulation model for a maternity hospital located in a low-resource setting.
  – Varying patient flow patterns
  – Tremendous patient risks
  – Extensive patient waiting times
  – Questionable workload distributions
  – Inefficient service delivery
Game Plan

- Followed 500 maternity patients entering Rabia Hospital (over two one-week periods)
- Identified various paths pts took thru hospital (including visits to outpatient care, labor & delivery, lab, operating room, post partum area, etc.)
- Identified doctor, nurse, mid-wife availability
- Simulated system using Arena
- Re-allocated room use as well as provider schedules
Snapshot of Patient Flow

Patient Arrival Rate
Disease Spread Modeling and Monitoring

Dave Goldsman
Kwok Tsui
Wenchi Chiu
Peter Gierlich
Paul Goldsman
Mi Lim Lee
Xuyuan Liu
Thomas Maschek

School of ISyE, Georgia Tech
Importance of Disease Spread

• Bioterrorism
• Epidemic Diseases
  – Anthrax
  – Severe Acute Respiratory Syndrome (SARS)
  – Avian influenza
  – Hepatitis
  – Smallpox
  – …
Simulation for Disease Spread Monitoring and Modeling

- Display dynamics of disease outbreaks
- Investigate impact of prevention activities
- Investigate impact of intervention activities
- Generate training and test data for monitoring methods
Challenges in Infectious Disease Simulation

• Uncertainty of assumptions
  – due to a lack of historical data (disease dynamics, behaviour of population...)
• Complex modeling of spatiotemporal settings
• Simulation models only in limited (geographical) settings
  → Integration into macroperspective
Geographical Extension: Multi-Layer Simulation Model

- Nationwide
- State level
- County level
- ZIP code level
Assign each person’s characteristics: Age, household, school, …

Opportunities of infection in different environments: Household, school, community

If the person is infected, Undergo latent & infectious period

Record number of deaths, hospitalizations, and recoveries
Latent Period and Incubation Period

- **Latent period**
- **Infectious period**
- **Symptomatic period**

- Exposure
- Time of infected
- Development of symptom
- Development of infectiousness
- End of infectiousness
- End of symptom

Time progression: Exposure → Latent period → Infectious period → Symptomatic period → End of symptom
Outcomes

- Total number of dead persons
- Total number of hospitalized persons
- Total number of recovered persons_symptomatic
- Total number of recovered persons_nonsymptomatic

Number of persons dead
Number of persons hospitalized
Number of persons recovered
Number of persons without symptoms recovered
Scenarios

• Scenario 1: do nothing
• Scenario 2: forbid to go outside after work/school
• Scenario 3: close all daycares, schools, workplaces
• Scenario 4: forbid to go outside, close all daycares, schools, workplaces
Comparison – 10 runs

Scenario 1

Scenario 2

Daily total With symptom

Daily total Without symptom

Daily total With symptom

Daily total Without symptom

15.90 (0.39)

15.61 (0.26)

7.51 (0.30)

8.03 (0.24)
Summary

• Computer simulation can be used for humanitarian logistics problems just as for “industrial” problems
• All elements of good simulation study are needed
• Many, many applications
Q & A?

Thanks very much!