Name: SOLUTIONS
Answer the following questions (8 points each):

1. Explain the typical trade-off between the two strategic objectives of (i) providing a highly responsive service to the arising demand and (ii) quoting competitive prices. How has this trade-off been broken in the recent years?

In general, in order to maintain a high level of responsiveness to the customer demand, in the face of the uncertainty that exists in this demand, companies must maintain considerable safety stock and/or excessive processing capacity. Both of these elements inflate the operational costs and usually these extra costs are eventually reflected in the quoted prices. Similarly, if one tries to suppress these costs, the responsiveness will suffer.

However, the above trade-off has been broken in the recent years through the effective use of the available information technology, which helps reducing the demand uncertainty mentioned above.
2. Briefly discuss the meaning of the requirement for high levels of responsiveness that is experienced by modern corporations.

In modern corporations, "responsiveness" means primarily two things:

(i) The ability to meet the experienced demand w.r.t. the provided product portfolio in a "timely manner" (where the last concept is properly quantified in various operational contexts).

(ii) The ability to "sense" major market trends and other important developments in the overall "business" environment, and adapt accordingly. Such trends and developments might imply the design and offering of new product concepts, the deployment of new production technologies for the existing products, the penetrating to new markets, etc.
3. Consider a two-period uncapacitated dynamic lot sizing problem where
(a) the ratio of the ordering cost $A$ to the holding cost $h$ is equal to 40; (b) the demand $D_2$, for period 2 in the considered planning horizon, is equal to 30 units; and (c) the initial inventory $I_0$ is not sufficient to cover the entire demand for period 1, $D_1$. From this information we can infer that we should order for demand $D_2$ together with the net demand for period 1.

(a) YES  
(b) NO

Please, explain your answer.

Since $I_0 < D_2$, the company has to order in period 1 some quantity in order to meet the demand of that period, for an ordering cost $A$.

If $D_2$ is ordered in period 1 as well, the extra cost will be $h \cdot D_2 = \frac{A}{40} \cdot 30 = \frac{3}{4} A \leq A$, which would be the extra cost if $D_2$ is ordered separately in period 2.
4. Consider a basestock inventory system with basestock level \( R \) and suppose that at a certain time point \( t \) the number of the outstanding replenishment orders \( O(t) \) (i.e., the replenishment orders that have been placed but not received yet) is equal to \( R \). Use this information in order to compute the amount of the backorders at time \( t \), \( BO(t) \). Please, provide a clear explanation for your answer.

From the above information \( O(t) = R \). (1)

Next, from the theory of the basestock model we know that:

\[
IP(t) = OHIT(t) + O(t) - BO(t) = R \quad (2)
\]

From (1) and (2):

\[
OHIT(t) - BO(t) = \emptyset \quad (3)
\]

But from their definition:

\[
OHIT(t) \cdot BO(t) = \emptyset \quad (4)
\]

From (3) and (4):

\[
OHIT(t) = BO(t) = \emptyset.
\]
5. Consider an inventory control system with its operational conditions similar to those assumed by the EOQ model, except for the fact that the ordering cost consists of two parts: (i) a fixed cost of \( A_1 \) $ per order (consisting of a fixed transportation cost and possibly other clerical costs of a fixed nature) and (ii) a variable cost of \( A_2 \) $ per ordered unit (this cost can involve, for instance, handling costs for the ordered quantity). Item and holding costs remain the same as in the EOQ model. Develop a formula for the total annual cost for this new case, and provide the corresponding EOQ formula.

In this case:

\[
TA(Q) = C \cdot O + A_2 O + A_1 \frac{O}{Q} + h \frac{Q}{2}
\]

Then, setting \( \frac{\partial TA(Q)}{\partial Q} = 0 \), we get:

\[
Q^* = \sqrt{\frac{2A_1 O}{h}}
\]

Finally, in the particular case that \( h \) is computed on the basis of an annual interest rate, \( i \):

\[
Q^* = \sqrt{\frac{2A_1 O}{i (C + A_2)}}
\]
Problem 1 (20 points): A local store with an annual demand of 10,000 units for one of its main products is trying to select a supplier for this product by choosing among two different options. The first supplier (supplier A) quotes a purchasing price of $15 per unit, and a contract of 50 "free" deliveries per year at the price of $1,000 per year. The second supplier (supplier B) also quotes a purchasing price of $15 per unit but she will utilize a 3PL service provider for the deliveries, who charges a fixed cost of $15 per delivery and a variable cost of $0.5 per delivered unit. The considered store computes its holding cost on the basis of an annual interest rate of 5%. Assuming that both suppliers can provide comparable quality of product and service, use the above information in order to determine the right supplier for this product.

For Supplier A:

\[
TAC_A = 15 \times 10,000 + 1,000 + \frac{0.05 \times 15 \times 19000}{2} + \frac{19000}{50} = 151,075.
\]

In the above expression, we have taken into consideration that:

(i) the transportation cost reduces to the cost of the offered contract of $1,000 per year; and

(ii) under the proposed operational scheme, the holding cost is minimized by minimizing the order size \(a\) while respecting the imposed limit of "free" deliveries, i.e., \(a = \frac{19000}{50}\).

The model for Supplier B is similar to the model discussed in Question 5 in the previous page. Hence,

\[
a^* = \sqrt{\frac{2 \times 15 \times 10,000}{0.05 (15+0.5)}} \approx 622
\]

and

\[
TAC_B = (15+0.5) \times 19000 + 15 \times \frac{19000}{622} + 0.05 (15+0.5) \frac{622}{2} = 155,432.05
\]

Since \(TAC_A < TAC_B\), we should choose supplier A.
Problem 2 (40 points): Consider a news vendor who experiences a daily demand uniformly distributed over a certain interval \([a, b]\), buys his newspapers at a price of \(x\) per paper and sells them at the price of $1.5. At the end of the day, any remaining papers are returned to a recycling center for a salvage value of $0.2 per paper. The news vendor has a good estimate of his average daily demand, and being unfamiliar with the relevant theory, he has decided to buy a number of papers every morning that is equal to this estimate.

i. (10 pts) What must be the purchasing price \(x\) for his newspapers in order for him to luck out and maximize his expected daily profit under his current practice?

ii. (10 pts) Under the current purchasing policy, what is the expected number of the “good days” per year that the news vendor will sell all his newspapers?

iii. (10 pts) Assuming the purchasing price that you computed in part (i) above, what should be the average daily demand \(\mu\) so that, at anyone of the “good days” mentioned in part (ii), the news vendor makes a profit of $50.00?

iv. (10 pts) Consider that the minimum daily demand is 50 papers and the maximum 100. Compute the expected daily profit for this news vendor under the current purchasing policy and the purchasing price computed in part (i).

(i) According to the problem, the order quantity \(Q\) used by the newvendor is the mean of the corresponding distribution, i.e., \(Q = \frac{a+b}{2}\). For this choice to be optimal, we must have:

\[
G(Q) = G\left(\frac{a+b}{2}\right) = \frac{c_s}{c_s + c_o} \quad (1)
\]

Furthermore, in the uniform distribution:

\[
G\left(\frac{a+b}{2}\right) = \frac{1}{2} \quad (2)
\]

and from the provided data:

\[
c_s = 1.5 - x; \quad c_o = x - 0.2 \quad (3)
\]

From (1), (2) and (3):

\[
\frac{1.5 - x}{1.5 - x + x - 0.2} = \frac{1}{2} \quad \Rightarrow \quad x = 0.35
\]
(ii) Since \( q = \frac{e^{th^b}}{2} \), the probability that

the demand at any given day is greater than or equal to \( q \) or equal to \( \frac{q}{2} \). Hence, the expected number of days per year are \( \frac{1}{2} \times 365 = 182.5 \).

(iii) On a "good" day, the newsvendor pays \( 0.85p \) dollars to buy his stock for that day, and he sells all of that stock, for a revenue of \( 1.5p \). Hence, his profit is

\[ 1.5p - 0.85p = 0.65p \]. Setting this quantity equal to \( 4500 \), we get \( 0.65p = 50 \) \( \Rightarrow \) \( p = \frac{50}{0.65} = 76.923 \).

(iv) By its basic definition, expected profit = expected revenue - expected cost. \( \text{(1)} \)

Also,

\[
\text{Expected revenue} = \text{Expected sales} + \text{Expected salvage value} = 1.5 \int_{50}^{100} \min \left\{ \frac{x}{2}, \frac{50+100}{2} \right\} \frac{1}{100-50} \, dx + 0.2 \int_{50}^{100} \max \left\{ \frac{50+100}{2} - x, 0 \right\} \frac{1}{100-50} \, dx \quad \text{(2)}
\]

and

\[
\text{Expected cost} = \frac{50+100}{2} \times 0.85 \quad \text{(3)}
\]

From (1), (2) and (3), after some calculation we find that

\[
\text{Expected daily profit} = 40.625
\]