ISyE 6201: Manufacturing Systems
Instructor: Spyros Reveliotis

Solutions for Homework #3
Chapter 8

Problem 1

a. The mean is 5 and the variance is 0. The coefficient of variation is also zero. These process times could be from a highly automated machine dedicated to one product type.

b. The mean is 5, the standard deviation is 0.115 and the CV is 0.023. These process times might be from a machine that has some slight variability in process times.

c. The mean of these is 11.7 and the standard deviation is 14.22 so the CV is 1.22. The times appear to be from a highly regular machine that is subject to random outages.

d. The mean is 2. If this pattern repeats itself over the long run, the standard deviation will be 4 (otherwise, for these 10 observations it is 4.2). The CV will be 2.0. The pattern suggests a machine that processes a batch of 5 items before moving any of the parts.

Problem 3

a. The natural CV is 1.5/2 = 0.75.

b. The mean would be (60)(2) = 120 min. The variance would be (60)(1.5)^2 = 135. The CV will be CV = √135/120 = 0.0968

c. The availability will be A = m_t/(m_t + m_r) = 60/(60+2) = 0.9677. The effective mean will be t_e = t_0/A = 120/0.9677 = 124.

The effective SCV will be

\[ c_e = c_0^2 + 2(1-A)Am_r/t_0 = 0.009375 + 2(1-0.9677)(0.9677)(120\text{min})/120 = 0.07181. \]

So, c_e = 0.268.

Problem 5

In the tables below TH is the throughput that is given, t_e, is the mean effective process time, c_e^2 is the effective SCV of the process times, u is the utilization given by (TH)(t_e), CT_q is the expected time in queue given by

\[ CT_q = \frac{1+c_e^2}{2} \frac{u}{1-u} t_e \] for the single machine case and

\[ CT_q = \frac{1+c_e^2}{2} \frac{u\sqrt{m(1-u)-1}}{m(1-u)} t_e \] for the case with m machines.

Finally, CT is the sum of CT_q and t_e

a. In the first case, even though B has greater capacity, A has shorter cycle time since its SCV is much smaller.
b. Doubling the arrival rate (TH) and the number of tools makes station A have a longer average cycle time than B.

<table>
<thead>
<tr>
<th>Machine</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH</td>
<td>1.84</td>
<td>1.84</td>
</tr>
<tr>
<td>te</td>
<td>1</td>
<td>0.85</td>
</tr>
<tr>
<td>c_e^2</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>m</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>u</td>
<td>0.92</td>
<td>0.782</td>
</tr>
<tr>
<td>CT_q</td>
<td>3.4616</td>
<td>3.4125</td>
</tr>
<tr>
<td>CT</td>
<td>4.4616</td>
<td>4.2625</td>
</tr>
</tbody>
</table>

c. Note the large increase in cycle time with the modest increase in throughput as compare to (a).

<table>
<thead>
<tr>
<th>Machine</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>te</td>
<td>1</td>
<td>0.85</td>
</tr>
<tr>
<td>c_e^2</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>m</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u</td>
<td>0.95</td>
<td>0.8075</td>
</tr>
<tr>
<td>CT_q</td>
<td>11.8750</td>
<td>8.9140</td>
</tr>
<tr>
<td>CT</td>
<td>12.8750</td>
<td>9.7640</td>
</tr>
</tbody>
</table>

d. We now consider Machine A only.
   i) First we increase TH by 1% from 0.5. The increase in cycle time is less than one percent.
   ii) Next we increase TH by 1% from 0.95. The increase in cycle time is almost 23%.

<table>
<thead>
<tr>
<th>Machine</th>
<th>A</th>
<th>A</th>
<th>A</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH</td>
<td>0.5</td>
<td>0.505</td>
<td>0.95</td>
<td>0.9595</td>
</tr>
<tr>
<td>te</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>c_e^2</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>m</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u</td>
<td>0.5</td>
<td>0.505</td>
<td>0.95</td>
<td>0.9595</td>
</tr>
<tr>
<td>CT_q</td>
<td>0.6250</td>
<td>0.6376</td>
<td>11.8750</td>
<td>14.8071</td>
</tr>
<tr>
<td>CT</td>
<td>1.6250</td>
<td>1.6376</td>
<td>12.8750</td>
<td>15.8071</td>
</tr>
<tr>
<td>% Increase</td>
<td>0.7770</td>
<td>22.7736</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 9

Problem 10

(a) If we reduce the buffer sizes, the number of jobs that balk will increase, thereby decreasing TH. The maximum WIP level also decreases as does cycle time. Cycle time goes down because it is a convex function of WIP.

(b) Reducing the variability should increase TH (slightly by reducing the amount of balking and decrease CT.)

(c) If we unbalance the line without changing \( r_b \) we must add capacity to the other stations (otherwise a different station would become the bottleneck with a capacity lower than the current value of \( r_b \)). If we add capacity, we increase TH (slightly) and decrease CT (more significantly).

(d) The opposite of b.

(e) Decreasing the arrival rate decreases TH (obviously) and also decreases utilization which thereby decreases CT.

(f) If we decrease the variability enough we might see an increase in TH and a reduction in CT.

G/G/1 station Problem:

\[
\begin{align*}
\lambda &= 0.1/\text{min} \\
\mu &= 0 \\
u &= \lambda \mu = 0.95 \\
\mu' &= 9.5/\text{min} \\
CT &= \frac{\sigma^2 + \mu^2}{2} \times \frac{1}{1-\mu} \times \mu' = 45 \text{ min} \\
\delta &= \frac{4\mu'}{\lambda(1-\mu')} = 0.4986 \\
\sigma^2 &= \mu^2 \delta^2 + (1-\mu^2) \mu^2 = 0.95^2 \times 0.4986 = 0.45
\end{align*}
\]

Since it is a stable system,

\[
\mu' = \lambda
\]

Therefore,

\[
\begin{align*}
\mu' &= \frac{1}{\lambda} = 10 \text{ min} \\
\sigma^2 &= \mu' \cdot \mu' = 45
\end{align*}
\]
Two-Station, Single-Machine Production Line Problem:

i. \[ A = \frac{m_f}{m_f + m_r}, \quad A_1 = \frac{7}{7 + 1.5} = 0.824, \quad A_2 = \frac{5}{5 + 0.5} = 0.909 \]

\[ t_e = \frac{t_0}{A}, \quad t_{e1} = \frac{11}{0.824} = 13.36 \text{ min} = 0.223 \text{ hr}, \quad t_{e2} = \frac{11}{0.909} = 12.1 \text{ min} = 0.202 \text{ hr} \]

Station 1 is the effective bottleneck of the line.

ii. \[ u = r_a t_e, \quad u_1 = \frac{35}{8} \cdot \frac{13.36}{60} = 0.974, \quad u_2 = \frac{35}{8} \cdot \frac{12.1}{60} = 0.882 \]

The utilization is less than 1 at both stations, so the production line can sustain the production rate of 35 parts per 8-hour shift.

iii. For station 1, the SCV of the effective processing time is

\[ c_e^2 = c_0^2 + (1 + c_r^2)(1 - A) \frac{m_r}{t_0} = 0.5^2 + (1 + 0.75^2)(1 - 0.824)(0.824) \frac{1.5}{11/60} = 2.108 \]

\[ CT = \left( \frac{c_a^2 + c_e^2}{2} \cdot \frac{u}{1 - u} + 1 \right) t_e = 2 \text{ hr} \text{ and } 7.815 \text{ min} = 2.13 \text{ hr} \]

\[ \Rightarrow \left( \frac{0 + 2.108}{2} \cdot \frac{r_a \cdot 0.223}{1 - r_a \cdot 0.223} + 1 \right) 0.223 = 2.13 \]

\[ \Rightarrow r_a = 4 \text{ parts per hour} \]

The inter-release interval is 15 minutes.

iv. \[ CT_q = CT - t_e = 1.908 \text{ hr}, \quad WIP_q = TH \cdot CT_q = 4 \times 1.908 = 7.63 \]

v. Mean = 15 minutes = \( \frac{1}{4} \) hrs

Variance = variance of inter-departure times

\[ = c_{d1} \cdot \left( \frac{1}{4} \right)^2 = \left( u_1^2 \cdot c_{a1}^2 + (1 - u_1^2) \cdot c_{a1}^2 \right) \left( \frac{1}{4} \right)^2 = \left[ (4 \cdot 0.223)^2 \cdot 2.108 + 0 \right] \left( \frac{1}{4} \right)^2 \]

\[ = 0.1045 \text{ (hrs)}^2 = 376.2 \text{ (min)}^2 \]
C. Extra Credit

1. Let \( T_0 \) be the natural processing time.  
   \( N \) be the number of failures that occur during the processing of a single part; since inter-failure times are exponentially distributed with mean equal to \( m_r \), \( N \) follows a Poisson distribution with \( \text{E}(N) = \text{var}(N) = T_0/m_r \).  
   \( Y_j \) be the repair time of the \( j \)-th failure, \( j = 1, 2, \ldots, N \), where \( \text{E}(Y_j) = m_r \), \( \text{var}(Y_j) = \sigma_r^2 \).  
   \( f(\cdot) \) be the probability density function of \( T_0 \).  
   \( T_e \) be the effective processing time.  

\[
E[T_e^2] = \int_0^\infty \sum_{n=0}^\infty E\left[T_e^2 \mid T_0 = x, N = n\right] e^{-x/m_r} \frac{(x/m_r)^n}{n!} f(x)dx
\]

\[
= \int_0^\infty \sum_{n=0}^\infty \left[ x^2 e^{-x/m_r} \frac{(x/m_r)^n}{n!} f(x)dx \right] + \left[ 2x \sum_{j=1}^n Y_j + \left( \sum_{j=1}^n Y_j \right)^2 \right] e^{-x/m_r} \frac{(x/m_r)^n}{n!} f(x)dx
\]

\[
= \int_0^\infty \left[ x^2 + 2x \sum_{j=1}^n Y_j + n(n-1)E^2(Y_j) + nE(Y_j^2) \right] e^{-x/m_r} \frac{(x/m_r)^n}{n!} f(x)dx
\]

\[
= \int_0^\infty \left[ x^2 + 2xE(N)E(Y_j) + E(N^2 - N)E^2(Y_j) + E(N)E(Y_j^2) \right] f(x)dx
\]

\[
= \int_0^\infty \left[ x^2 + 2xE(N)m_r + \left[ \text{var}(N) + E^2(N) - E(N) \right] m_r^2 + E(N)(m_r^2 + \sigma_r^2) \right] f(x)dx
\]

\[
= \int_0^\infty \left[ x^2 + 2x \frac{m_r^2}{m_f} + \left( \frac{x}{m_f} \right)^2 m_r^2 + \left( \frac{x}{m_f} \right) \left( m_r^2 + \sigma_r^2 \right) \right] f(x)dx
\]

\[
= E(T_e^2) \left( \frac{m_f + m_r}{m_f} \right)^2 + (m_r^2 + \sigma_r^2) \frac{t_0}{m_f}
\]

\[
= (\sigma_0^2 + t_0^2) \left( \frac{1}{A} \right)^2 + (m_r^2 + \sigma_r^2) \frac{(1 - A)t_0}{Am_r}
\]

Since \( A = \frac{m_f}{m_f + m_r} \),

So we have equation (8.5): \( \sigma_e^2 = E[T_e^2] - E^2[T_e] = \left( \frac{\sigma_0}{A} \right)^2 + (m_r^2 + \sigma_r^2) \frac{(1 - A)t_0}{Am_r} \).

Taking \( c_0^2 = t_0^2 \sigma_0^2 \) and \( c_r^2 = m_r^2 \sigma_r^2 \), we can derive Equation (8.6):
2. 

\[ \begin{align*} 
E(T_{\text{effective}}) &= E(T_0 + T_{\text{disruption}}) = E(T_0) + E(T_{\text{disruption}}) \\
&= t + E\left[E\left(T_{\text{disruption}} \mid \text{type 1 occurs (may simultaneously)}\right)\right] \\
&= t + p_1 \times (1 - p_2) \times \frac{1}{\lambda_a} + p_2 \times (1 - p_2) \times \frac{1}{\lambda_a} \\
&\quad + p_1 \times p_2 \times E[\max(T_{\text{DRE 1}}, T_{\text{DRE 2}})] \\
\end{align*} \]

\[ E[\max(T_{\text{DRE 1}}, T_{\text{DRE 2}})] = E(\max(T_{\text{DRE 1}}, T_{\text{DRE 2}}) \mid T_{\text{DRE 1}} < T_{\text{DRE 2}}) \times P(T_{\text{DRE 1}} < T_{\text{DRE 2}}) \\
+ E(\max(T_{\text{DRE 1}}, T_{\text{DRE 2}}) \mid T_{\text{DRE 1}} > T_{\text{DRE 2}}) \times P(T_{\text{DRE 1}} > T_{\text{DRE 2}}) \\
= E(T_{\text{DRE 1}} + (T_{\text{DRE 1}} - T_{\text{DRE 2}}) \mid T_{\text{DRE 1}} < T_{\text{DRE 2}}) \times P(T_{\text{DRE 1}} < T_{\text{DRE 2}}) \\
+ E(T_{\text{DRE 1}} + (T_{\text{DRE 1}} - T_{\text{DRE 2}}) \mid T_{\text{DRE 1}} > T_{\text{DRE 2}}) \times P(T_{\text{DRE 1}} > T_{\text{DRE 2}}) \\
= \left( \frac{1}{\lambda_a + \lambda_e} + \frac{1}{\lambda_e} \right) \times \frac{\lambda_a}{\lambda_a + \lambda_e} + \left( \frac{1}{\lambda_a + \lambda_e} + \frac{1}{\lambda_e} \right) \times \frac{\lambda_e}{\lambda_a + \lambda_e} \\
\end{align*} \]

Plug in all the data,

\[ E(T_{\text{effective}}) = 2 + 0.3 \times 0.3 \times 5 + 0.2 \times 0.7 \times 10 + 0.3 \times 0.2 \times \frac{25}{2} = 5.3 \text{ min} \]

Therefore, the effective processing capacity is \( 60/5.3 = 11.32 \) units per hour.