1. (Short answer questions — Just write your answer.)

(a) Suppose that $U_1 = 0.35$ and $U_2 = 0.65$ are realizations of two i.i.d. $U(0,1)$’s. Use the Box-Muller method to generate two Nor(0,1)’s.

ANSWER: ______________________

(b) Use your answer in Question 1a to generate a $\chi^2$ random variate with two degrees of freedom. [Hint: If $Z_1$ and $Z_2$ are i.i.d. normal(0,1), then $Z_1^2 + Z_2^2 \sim \chi^2(2).$]

ANSWER: ______________________

(c) Using the two $U_i$’s from Question 1a, generate a realization from an $\text{Erlang}_{k=2}(\lambda = 1)$ distribution.

ANSWER: ______________________

(d) Suppose $U_1 = 0.55$ is a $U(0,1)$ random number. Use this number to generate a Geometric($0.4$) random variate.

ANSWER: ______________________

(e) If $Z$ is a standard normal random variable, what is the distribution of $3Z - 2$?

ANSWER: ______________________
(f) If $U$ is Unif(0,1), what is the distribution of $-0.5\ln(1 - U)$?

ANSWER: ____________________

(g) Suppose $U_1, U_2, \ldots, U_{12}$ are i.i.d. Unif(0,1). What is the approximate distribution of $\sum_{i=1}^{12} U_i$?

ANSWER: ____________________

(h) Who gave you the lecture on input analysis?

ANSWER: ____________________

(i) BONUS: The Zombies’ lead singer, Colin Blunstone, recorded a solo album in 1978. Name the album and the record company’s owner.

ANSWER: ____________________
2. Suppose we observe 1000 pseudo-random numbers to obtain the following data.

<table>
<thead>
<tr>
<th>interval</th>
<th>[0.0, 0.2)</th>
<th>[0.2, 0.4)</th>
<th>[0.4, 0.6)</th>
<th>[0.6, 0.8)</th>
<th>[0.8, 1.0]</th>
</tr>
</thead>
<tbody>
<tr>
<td>number observed</td>
<td>195</td>
<td>210</td>
<td>195</td>
<td>189</td>
<td>211</td>
</tr>
</tbody>
</table>

Conduct a $\chi^2$ goodness-of-fit test to see if these numbers are approximately $\text{Unif}(0,1)$. Use level of significance $\alpha = 0.05$. Here are some table entries that you may need: $\chi^2_{0.05,3} = 7.81$, $\chi^2_{0.05,4} = 9.49$, and $\chi^2_{0.05,5} = 11.1$. 


3. Suppose the random variable $X$ has the following c.d.f.

$$F(x) = \begin{cases} \frac{1}{2}e^{5x} & \text{if } x \leq 0 \\ 1 - \frac{1}{2}e^{-2x} & \text{if } x > 0 \end{cases}.$$

(a) Give an inverse transform method for generating realizations of $X$.

(b) Use the $U(0,1)$ random number 0.65 to generate a realization of $X$. 

4. (Questions on Poisson random variables.)

(a) Use the acceptance-rejection technique to generate one Poisson(4) random variate. Use as many of the following $U(0, 1)$ numbers as is necessary.

$$0.89 \ 0.75 \ 0.83 \ 0.02 \ 0.11 \ 0.23$$

(b) How many $U(0, 1)$’s would you have expected to use in Question 4a?

(c) What would you have had to do if I had asked you to generate a Poisson(400) random variate?
5. Suppose that $Z_1 = 3$, $Z_2 = 5$, and $Z_3 = 4$ are three batch means resulting from a long simulation run. Find a 90% two-sided confidence interval for the mean.