1. Let $X$ be the outcome of a 4-sided die toss. Find $\text{Var}(\ln(X) + 1)$.

2. Suppose that the lifetime of a transistor is exponential with a mean of 10 years. Further suppose that the transistor has already survived 20 years. Find the probability that it will not fail in the next 10 years.

3. Suppose that $X \sim c + \text{Exp}(\lambda)$, where the time units are in years. $X$ could represent the lifetime of a lightbulb that is guaranteed to last at least $c$ years. Find the median of $X$, that is, the point $m$ such that $P(X \leq m) = P(X > m) = 0.5$.

4. Suppose $X_1, X_2,$ and $X_3$ are i.i.d. Bernoulli($p$) random variables, which represent the functionality of three network components. Think of a signal passing through a network, where $X_i = 1$ if the signal can successfully get through component $i$, for $i = 1, 2, 3$ (and $X_i = 0$ if the signal is unsuccessful). Let’s consider two set-ups:

   (*) $X_1, X_2, X_3$ have $p = 0.95$ and are hooked up in series so that a signal getting through the network has to pass through all components 1 AND 2 AND 3.

   (**) $X_1, X_2, X_3$ have $p = 0.8$ and are hooked up in parallel so that a signal getting through the network only has to pass through components 1 OR 2 OR 3.

Which series is more reliable, i.e., more likely to permit a signal to pass through — (*) or (**)?

5. TRUE or FALSE? If $X$ is any normal distribution, then about 95% of all observations from $X$ will fall within two standard deviations of the mean.

6. TRUE or FALSE? The normal quantile value $\Phi^{-1}(0.975) = 1.96$. 
7. Suppose $X \sim \text{Nor}(1, 1)$. Find $c$ such that $P(-c \leq X \leq c) = 0.95$. Careful — this may require a little trial-and-error.

8. Suppose that $X$ and $Y$ are the scores that a Georgia Tech student and his twin who goes to the Univ. of Georgia will receive, respectively, on the same math test. Further suppose that $X$ is $\text{Nor}(90, 100)$, $Y$ is $\text{Nor}(60, 100)$ and $\text{Cov}(X, Y) = 50$. Find the probability that the GT kid will beat the UGA kid by at least 40 points. (You can assume that $X - Y$ is normal.)

9. If $X_1, \ldots, X_{400}$ are i.i.d. from some distribution with mean 0 and variance 1600, find the approximate probability that the sample mean $\bar{X}$ is between $-2$ and 2.

10. TRUE or FALSE? Consider any i.i.d. sequence of random variables having finite variance. Then the Central Limit Theorem says that a properly standardized sample mean can be approximated by a standard normal random variable as the sample size becomes large.

11. Short Research Question: Go to The Internets and write up a couple of paragraphs on who “invented” the Central Limit Theorem (there are several reasonably correct answers).

12. If $X \sim \chi^2(5)$, find $P(X < 11.07)$.

13. TRUE or FALSE? $t_{0.025,8} > z_{0.025}$.

14. Suppose $T \sim t(343)$. What’s $P(T < 1)$?

15. TRUE or FALSE? $P(F(5, 3) < F_{0.95,5,3}) = P(F(5, 3) < 1/F_{0.05,5,3})$.

16. Suppose $X_1, \ldots, X_6 \sim \text{Nor}(3, 9)$, $Y_1, \ldots, Y_7 \sim \text{Nor}(-3, 2)$, and everything is independent. Let $S_X^2$ and $S_Y^2$ denote the sample variances of the $X_i$‘s and $Y_j$‘s, respectively. Name the distribution (with parameter(s)) of $S_Y^2 / S_X^2$. 
17. Suppose $X_1, \ldots, X_n$ are i.i.d. Exp($\lambda$).

(a) TRUE or FALSE? The sample mean $\bar{X}$ is unbiased for the mean $1/\lambda$.

(b) TRUE or FALSE? $1/\bar{X}$ is unbiased for $\lambda$.

(c) Find $\mathbb{E}[1/\bar{X}]$. (This might take a little work if you do it from scratch.)

(d) TRUE or FALSE? $1/\bar{X}$ is the MLE for $\lambda$.

18. Suppose $X_1, X_2, X_3$ are i.i.d. Nor($\mu, \sigma^2$), and we observe $X_1 = 7$, $X_2 = 1$, and $X_3 = 4$.

(a) What is the sample variance of the $X_i$’s?

(b) What is the maximum likelihood estimate of $\sigma^2$?

(c) What is the maximum likelihood estimate of $P(X_i > 5)$?

19. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. Geom($p$). Thus, for all $i$, we have $P(X_i = k) = (1 - p)^{k-1}p$, for $k = 1, 2, \ldots, n$. What is the maximum likelihood estimate of $p$?

20. Suppose $X_1, X_2, X_3$ are i.i.d. Unif($\theta, 0$), and we observe $X_1 = -7$, $X_2 = -1$, and $X_3 = -4$. What is the maximum likelihood estimate of $\theta$?

21. Suppose $X_1, X_2, X_3$ are i.i.d. Nor($\mu, 16$). Define two estimators for $\mu$: $T_1 \equiv X_1 - X_2 + X_3$ and $T_2 \equiv (4X_1 + 3X_2 + X_3)/8$. Which of $T_1$ or $T_2$ has the smaller MSE?

22. Which family member is actually an estimation method?

(a) CAT
(b) DAD
(c) MOM
23. Suppose $X_1, \ldots, X_{10}$ are i.i.d. normal with unknown mean and known variance $\sigma^2 = 49$. Further suppose that $\bar{X} = -50$ and $S^2 = 60$.

(a) Find a 99% two-sided confidence interval for $\mu$.

(b) Find a 99% two-sided confidence interval for $2\mu - 4$.

24. Suppose $X_1, \ldots, X_{10}$ are i.i.d. normal with unknown mean and unknown variance $\sigma^2 = 49$. Further suppose that $\bar{X} = -50$ and $S^2 = 60$.

(a) Find a 99% two-sided confidence interval for $\mu$.

(b) Is your answer to Question 24a wider or narrower than your answer to Question 23a? Why?

(c) Find a 99% two-sided confidence interval for $\sigma^2$.

25. Suppose that $X_1, \ldots, X_n$ are i.i.d. Bernoulli with unknown mean $p$, and that we have carried out a preliminary investigation suggesting $p \approx 0.9$. How big would $n$ have to be in order for a two-sided 99% confidence interval to have a half-length of 0.01? (Give the smallest such number.)

26. After collecting your set of observations, what happens to the length of a confidence interval for the mean as the confidence level moves from 90% to 95%?

(a) It increases.

(b) It decreases.

(c) It stays the same.

(d) You can’t tell.
27. Consider i.i.d. normal observations $X_1, \ldots, X_{10}$ with unknown mean $\mu$ and unknown variance $\sigma^2$. What is the expected width of the usual 95% two-sided confidence interval for $\sigma^2$? You can keep your answer in terms of $\sigma$.

28. Suppose we conduct an experiment to test to see if people can throw farther right- or left-handed. We get 20 people to do the experiment. Each throws a ball right-handed once and throws a ball left-handed once, and we measure the distances. If we are interested in determining a confidence interval for the mean difference in left- and right-handed throws, which type of c.i. would we likely use?

(a) $z$ (normal) confidence interval for differences
(b) pooled $t$ confidence interval for differences
(c) paired $t$ confidence interval for differences
(d) $\chi^2$ confidence interval for differences
(e) $F$ confidence interval for differences

29. TRUE or FALSE? We reject the null hypothesis if we are given statistically significant evidence that it is false.

30. The quantity $\alpha$ is known as

(a) $P$(Type I error)
(b) $P$(Type II error)
(c) level of significance
(d) $P$(Reject $H_0$ | $H_0$ is true)

31. Suppose that we examine the IQs of 50 Justin Bieber concert attendees. We assume that the IQs are normally distributed with a standard deviation of 10. Suppose that the sample mean turns out to be 82. Test the null hypothesis that the mean IQ of the attendees is at least 90. Use $\alpha = 0.05$.

32. Referring to Question 31, how many observations should we take if we want the probability of a Type II error to be 0.10 when $\mu$ happens to equal 87?
33. Suppose we want to compare the means of two normal populations, both of which have unknown but approximately equal variances. We take \( n = 6 \) observations from the first population and find that the sample mean and sample variance are \( \bar{x} = 50 \) and \( s_x^2 = 120 \). We take \( m = 5 \) observations from the second population and find that the sample mean and sample variance are \( \bar{y} = 75 \) and \( s_y^2 = 100 \). Test the hypothesis that \( \mu_x = \mu_y \) with \( \alpha = 0.05 \), i.e., either accept or reject.

34. Short Research Question: Go to The Internets and write up a couple of paragraphs on what a “goodness-of-fit” hypothesis test is.

35. Let’s see if weight depends on height. We consider 6 i.i.d. people.

<table>
<thead>
<tr>
<th>Height (in.)</th>
<th>70</th>
<th>74</th>
<th>62</th>
<th>66</th>
<th>71</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (lbs.)</td>
<td>180</td>
<td>210</td>
<td>146</td>
<td>175</td>
<td>165</td>
<td>150</td>
</tr>
</tbody>
</table>

(a) Fit a regression line to this data and report your estimates for \( \beta_0 \) and \( \beta_1 \).

(b) What is the expected weight of a person who is 72 inches tall?

(c) Give a 95% confidence interval for \( \beta_1 \).