ISyE 6739 — Summer 2009

Homework #7 Solutions (Covers Modules 27–33) (revised 7/13/09)

All of the following problems are from Hines, et al.

8–1. Elementary data analysis. The shelf life of a high-speed photographic film is being investigated by the manufacturer. The following data are available (in days).

126	129	134	141
131	132	136	145
116	128	130	162
125	126	134	129
134	127	120	127
120	122	129	133
125	111	147	129
150	148	126	140
130	120	117	131
149	117	143	133

Construct a histogram and comment on the properties of the data.

Solution: $\bar{x} = 131.30$, $s^2 = 113.85$, $s = 10.67$. □

8–25. Interesting algebra question. Consider the quantity $\sum_{i=1}^{n}(x_i - a)^2$. For what value of a is this quantity minimized?

Solution: Differentiate and you eventually get $a = \bar{x}$. □

9–5. Normal distribution. A population of power supplies for a personal computer has an output voltage that is normally distributed with a mean of 5.00 V and a standard deviation of 0.10 V. A random sample of eight power supplies is selected. Specify the sampling distribution of $\bar{X}$.

Solution: $N(\mu, \sigma^2/n) = N(5, 0.00125)$. □

9–23(a). χ^2 quantile. Find $\chi^2_{0.95,8}$.

Solution: 2.73. □

9–24(a). t quantile. Find $t_{0.25,10}$.

Solution: 0.700. $\Box$

9–25(a). F quantile. Find $F_{0.25,4,9}$.

Solution: 1.63. $\Box$

10–1. MSE. Suppose we have a random sample of size $2n$ from a population denoted X, and $\mathsf{E}[X] = \mu$ and $\mathsf{Var}(X) = \sigma^2$. Let

$$\bar{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n} X_i \quad \text{and} \quad \bar{X}_2 = \frac{1}{n}\sum_{i=1}^{n} X_i$$

be two estimators of μ. Which is the better estimator of μ? Explain your choice.

Solution: Both estimators are unbiased. Now, $\mathsf{Var}(\bar{X}_1) = \sigma^2/2n$ while $\mathsf{Var}(\bar{X}_2) = \sigma^2/n$. Since $\mathsf{Var}(\bar{X}_1) < \mathsf{Var}(\bar{X}_2)$, $\bar{X}_1$ is a more efficient estimator than $\bar{X}_2$. $\Box$

10–13. Geometric MLE. Let X be a geometric random variable with parameter p. Find the maximum likelihood estimator of p, based on a sample of size n.

Solution: $L(p) = \displaystyle\prod_{i=1}^{n}(1-p)^{x_i-1}p = p^n(1-p)^{\Sigma x_i - n}$.

$\ell\text{n}\, L(p) = n\,\ell\text{n}\, p + (\sum_{i=1}^n x_i - n)\ell\text{n}\,(1-p)$. From $d\,\ell\text{n}\, L(p)/dp = 0$, we obtain

$$\frac{n}{p} - \frac{\sum_{i=1}^n x_i - n}{1-p} = 0,$$

so that

$$\hat{p} = \frac{n}{\sum_{i=1}^n x_i} = 1/\bar{x}. \quad \Box$$

10–41(a). Confidence interval (known variance). A civil engineer is analyzing the compressive strength of concrete. Compressive strength is approximately normally distributed with a variance of $\sigma^2 = 1000$ (psi)^2. A random sample of 12 specimens has a mean compressive strength of $\bar{x} = 3250$ psi. Construct a 95% two-sided confidence interval on mean compressive strength.

Solution: $3232.11 \le \mu \le 3267.89$. $\Box$