ISyE 6739 — Summer 2009

Homework #4 (Covers Module 15) — Solutions

1. Suppose $X \sim \text{Unif}(1,3)$. Find the p.d.f. of $Z = e^X$.

   Hint: The c.d.f. of Z is

$$\begin{aligned} G(z) &= \mathsf{Pr}(Z \le z) \\ &= \mathsf{Pr}(e^X \le z) \\ &= \mathsf{Pr}(X \le \ell\text{n}(z)) \\ &= \int_1^{\ell\text{n}(z)} f(x)\,dx \quad (\text{if } 1 \le \ell\text{n}(z) \le 3) \\ &= (\ell\text{n}(z) - 1)/2. \end{aligned}$$

   Now you can get the p.d.f.

$$g(z) = \frac{d}{dz} G(z) = \begin{cases} 0 & \text{if } z < e \text{ or } z > e^3 \\ \frac{1}{2z} & \text{if } e \le z \le e^3 \end{cases} \qquad \diamondsuit$$

2. Suppose X has p.d.f. $f(x) = 2xe^{-x^2}$, $x \ge 0$. Find the distribution of $Z = X^2$.

   Hint: The c.d.f. of Z is

$$\begin{aligned} G(z) &= \mathsf{Pr}(Z \le z) \\ &= \mathsf{Pr}(X^2 \le z) \\ &= \mathsf{Pr}(-\sqrt{z} \le X \le \sqrt{z}) \\ &= \mathsf{Pr}(0 \le X \le \sqrt{z}) \quad (\text{since } X \ge 0) \\ &= \int_0^{\sqrt{z}} 2xe^{-x^2}\,dx \\ &= 1 - e^{-z}. \end{aligned}$$

   Thus, Z is Exp(1). $\diamondsuit$

3. Computer Exercises — Random Variate Generation (see original assignment)