4.24 Other Continuous Distributions

Triangular

Beta

Weibull

Cauchy

Alphabet Soup

Normal

Triangular(a, b, c) **Distribution** — good for modeling RV's on the basis of limited data (min, mode, max).

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a < x \le b \\ \frac{2(c-x)}{(c-b)(c-a)} & b < x < c \\ 0 & \text{otherwise} \end{cases}$$

$$\mathsf{E}[X] = \frac{a+b+c}{3}, \qquad \mathsf{Var}(X) = \text{mess}$$

Beta(a, b) **Distribution** — good for modeling RV's that are restricted to an interval.

$$f(x) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathsf{E}[X] = \frac{a}{a+b}, \qquad \mathsf{Var}(X) = \frac{ab}{(a+b)^2(a+b+1)}$$

Weibull(a, b) **Distribution** — good for modeling reliability models. a is the "scale" parameter, and b is the "shape" parameter.

$$f(x) = \begin{cases} ab(ax)^{b-1}e^{-(ax)^b} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = 1 - \exp[-(ax)^b], \quad x > 0.$$

$\mathsf{E}[X] = (1/a)\Gamma(1+(1/b))$, $\quad$ $\mathsf{Var}(X) =$ slight mess

Remark: The exponential is a special case of the Weibull.

Example: Time-to-failure T for a transmitter has a Weibull distrn with rate $a = 1/(200 \text{ hrs})$ and parameter $b = 1/3$. Then

$$\mathsf{E}[T] \;=\; 200\Gamma(1+3) \;=\; 1200 \text{ hrs.}$$

The prob that it fails before 2000 hrs is

$$F(2000) \;=\; 1 - \exp[-(2000/200)^{(1/3)}] \;=\; 0.884.$$

Cauchy distribution — good for disproving things!

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad x \in \Re.$$

$$F(x) = \frac{1}{2} + \frac{\arctan(x)}{\pi}.$$

Theorem: The Cauchy distribution has an undefined mean and infinite variance!

Weird Fact: $X_1, \ldots, X_n \overset{\text{iid}}{\sim}$ Cauchy $\Rightarrow \sum_{i=1}^{n} X_i/n \sim$ Cauchy. Even you take the average of a bunch of Cauchys, you're right back where you started!

Alphabet Soup of Other Distrns

χ^2 distribution — coming up in the statistics portion

t distribution — coming up

F distribution — coming up

Pareto, LaPlace, Rayleigh, Gumbel distributions

Etc. . .

Normal Distribution

So important that we'll give it an entire chapter. Here are some quick tidbits.

X is normal with parameters μ and σ^2 if

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right].$$

$f(x)$ has a "bell-shaped" look. Also,

$$\mathsf{E}[X] = \mu, \qquad \mathsf{Var}(X) = \sigma^2.$$