Random Variables — Modules

Intro and Definitions

Definition of Random Variable

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Discrete vs. Continuous RV's

Intro and Definitions

Definition: A **random variable** (RV) is a function from the sample space to the real line. $X : S \to \Re$.

Example: Flip 2 coins. $S = \{HH, HT, TH, TT\}$.

Suppose X is the RV corresponding to the # of H's.

$X(TT) = 0$, $X(HT) = X(TH) = 1$, $X(HH) = 2$.

$\Pr(X = 0) = \frac{1}{4}$, $\Pr(X = 1) = \frac{1}{2}$, $\Pr(X = 2) = \frac{1}{4}$.

Notation: Capital letters like X, Y, Z, U, V, W usually represent RV's.

Small letters like x, y, z, u, v, w usually represent particular values of the RV's.

Thus, you can speak of $\Pr(X = x)$.

Example: Let X be the sum of two dice rolls. Then $X((4,6)) = 10$. In addition,

$$\Pr(X = x) = \begin{cases} 1/36 & \text{if } x = 2 \\ 2/36 & \text{if } x = 3 \\ \vdots & \\ 6/36 & \text{if } x = 7 \\ \vdots & \\ 1/36 & \text{if } x = 12 \\ 0 & \text{otherwise} \end{cases}$$

Example: Flip a coin.

$$X \equiv \begin{cases} 0 & \text{if } T \\ 1 & \text{if } H \end{cases}$$

Example: Roll a die.

$$Y \equiv \begin{cases} 0 & \text{if } \{1,2,3\} \\ 1 & \text{if } \{4,5,6\} \end{cases}$$

For our purposes, X and Y are the same, since $\Pr(X = 0) = \Pr(Y = 0) = \frac{1}{2}$ and $\Pr(X = 1) = \Pr(Y = 1) = \frac{1}{2}$.

Example: Select a real # at random betw 0 and 1.

Infinite number of "equally likely" outcomes.

Conclusion: Pr(each *individual point*) $=$ Pr($X = x$) $=$ 0, believe it or not!

But Pr($X \leq 0.5$) $= 0.5$ and Pr($X \in [0.3, 0.7]$) $= 0.4$.

If A is any *interval* in [0,1], then Pr(A) is the length of A.

Definition: If the number of possible values of a RV X is finite or countably infinite, then X is a **discrete** RV. Otherwise,. . .

A **continuous** RV is one with prob 0 at every point.

Example: Flip a coin — get H or T. Discrete.

Example: Pick a point at random in $[0, 1]$. Continuous.