1–4:  
(a) $\bar{E}F\bar{G}$  
(b) $EFG$  
(c) $E \cup F \cup G$  
(d) $EF \cup EG \cup FG$ (Remark: $\cup EFG$ is redundant.)  
(e) $EFG$  
(f) $\bar{E} \cup F \cup \bar{G} = \bar{E}F\bar{G}$  
(g) “at most 1” = “none” $\cup$ “exactly 1” = $\bar{E}F\bar{G} \cup E\bar{F}\bar{G} \cup \bar{E}FG \cup \bar{E}\bar{F}G$  
(h) “at most 2” = “all 3” = $EFG$. □

1–8: (I’ll just do the general case.) Since  
\[ 1 \geq P(E \cup F) = P(E) + P(F) - P(EF), \]  
we have  
\[ P(EF) \geq P(E) + P(F) - 1. \] □

1–14: Let $S$ and $F$ denote “success” and “failure”, respectively.  
\[ P(A \text{ wins}) = P(S) + P(FFS) + P(FFFFS) + \cdots \]  
\[ = p + (1-p)(1-p)p + (1-p)^4p \]  
\[ = p \sum_{i=0}^{\infty} (1-p)^{2i} = \frac{p}{1-(1-p)^2} = \frac{1}{2-p}. \] □

1–20: The number of ways to roll 3 dice is $6^3 = 216$. We want exactly 2 of the dice to have the same value (say, $X$); let $Y$ be the value on the remaining die.  

There are 6 ways to pick $X$.  

5 ways to pick $Y(\neq X)$.  

3 ways to scramble two $X$’s and one $Y$.  

Hence, there are $6 \times 5 \times 3 = 90$ ways to pick two $X$’s and one $Y$. So the desired probability is $\frac{90}{216} = \frac{5}{12}$. □
Let $A_i$ be the event that man $i$ gets his own hat, for $i = 1, 2, \ldots, n$. Then

\[
P(\text{None of the men gets his own hat}) = P(A_1 \cup A_2 \cup \cdots \cup A_n) = 1 - P(A_1 \cup \cdots \cup A_n)
\]

\[
= 1 - \left[ \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_iA_j) + \sum_{i<j<k} P(A_iA_jA_k) + \cdots + (-1)^{n+1} P(A_1A_2\cdots A_n) \right]
\]

\[
= 1 - \left[ n \cdot \frac{1}{n} - \frac{n}{2} \cdot \frac{1}{n(n-1)} + \frac{n}{3} \cdot \frac{1}{n(n-1)(n-2)} + \cdots + (-1)^{n+1} \frac{1}{n!} \right]
\]

\[
= \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}.
\]

Note that this quantity goes to $1/e$ as $n \to \infty$. \hfill \Box

Let $W$ be the event that a white ball is selected. Then by Bayes Theorem, we have

\[
P(T|W) = \frac{P(W|T)P(T)}{P(W|T)P(T) + P(W|H)P(H)}
\]

\[
= \frac{1}{5} \cdot \frac{1}{2} = \frac{22}{12} \cdot \frac{1}{2} = 12 \cdot \frac{1}{37}.
\]

Thus, A is correct! \hfill \Box

Label $A$, $B$, and $C$ as the events that prisoners $A$, $B$, $C$ die, respectively. Whatever happens, the jailer will tell prisoner $A$ that either $B$ or $C$ won’t die. Without loss of generality, let $J_B$ be the event that the jailer says “$B$ won’t die”. We are therefore looking for the conditional probability that $A$ dies. By Bayes Theorem,

\[
P(A|J_B) = \frac{P(J_B|A)P(A)}{P(J_B|A)P(A) + P(J_B|B)P(B) + P(J_B|C)P(C)}
\]

\[
= \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}}
\]

\[
= \frac{1}{3}.
\]

Thus, A is correct! \hfill \Box

$X \sim \text{Bin}(3, 0.7)$. Then $P(X = k) = \binom{3}{k} (0.7)^k (1 - 0.7)^{3-k}$, $k = 0, 1, 2, 3$. \hfill \Box
2–8: We have
\[ P(X = b) = \begin{cases} 1/2, & \text{if } b = 0 \text{ or } 1 \\ 0, & \text{otherwise} \end{cases}. \]

\[ \square \]

2–14: \( X \sim \text{Bin}(3, 1/2) \). Then \( P(X = k) = \binom{6}{k} (1/2)^k \), which is maximized by \( k = 3 \).
\[ \square \]

2–22: Let \( X \) be the number of trials until \( H \) first appears. Then \( X \sim \text{Geom}(0.5) \), and so
\[ P(X = 5) = (1 - 0.5)^4(0.5) = 1/32. \]
\[ \square \]

2–23: In order for the \( r \)th \( H \) to appear exactly on the \( n \)th trial,
(i) Exactly \( r - 1 \) \( H \)'s must appear during the first \( n - 1 \) trials. This has probability \( \binom{n - 1}{r - 1} p^{r-1} q^{n-r} \) (think Binomial), AND
(ii) The \( n \)th trial must be \( H \). This has probability \( p \).
Since (i) and (ii) are independent, we get the desired result by multiplying the probabilities.
\[ \square \]

2–33: (a) \( 1 = \int_{-1}^{1} f(x) \, dx = c \int_{-1}^{1} (1 - x^2) \, dx \). Hence, \( c = 3/4 \).
\[ \square \]

(b) For \(-1 \leq x \leq 1\),
\[ F(x) = \int_{-\infty}^{x} f(t) \, dt = \int_{-1}^{x} \frac{3}{4} (1 - t^2) \, dt = \frac{3}{4} \left( \frac{2}{3} + x - \frac{x^3}{3} \right). \]
Further, \( F(x) = 1 \) for \( x > 1 \) and \( F(x) = 0 \) for \( x < 0 \).
\[ \square \]

2–39:
\[ \mathbb{E}[X] = \sum_{x} x P(X = x) = 1(1/2) + 2(1/3) + 24(1/6) = 31/6. \]
\[ \square \]

2–53: Let \( X \sim U(0, 1) \). Then \( f(x) = 1, \ 0 \leq x \leq 1 \). So,
\[ \mathbb{E}[X^n] = \int_{0}^{1} x^n \cdot 1 \, dx = \frac{1}{n + 1} \]
\[ \text{Var}(X^n) = \mathbb{E}[X^{2n}] - (\mathbb{E}[X^n])^2 = \frac{1}{2n + 1} - \frac{1}{(n + 1)^2}. \]
\[ \square \]
2–60: Let \( X \sim U(0, 1) \). Then
\[
M_X(t) = \mathbb{E}[e^{tX}] = \int_{\mathbb{R}} e^{tx} f(x) \, dx = \int_0^1 e^{tx} \, dx = \frac{1}{t}(e^t - 1).
\]
So
\[
\mathbb{E}[X] = \left. \frac{d}{dt} M_X(t) \right|_{t=0} = \left. \frac{te^t - e^t - 1}{t^2} \right|_{t=0} = \frac{te^t}{2t} \bigg|_{t=0} = \frac{1}{2}.
\]
where the third equality holds by the L’Hospital’s rule. □

Similarly, \( \text{Var}(X) = 1/12 \). □

3: A: test positive; B: test negative; E: contains oil; F: no oil

\[
P(E) = 0.6; \ P(F) = 0.4; \ P(A|E) = 0.9; \ P(B|E) = 0.1; \ P(B|F) = 0.8; \ P(A|F) = 0.2.
\]

So
\[
P(E|A) = \frac{P(A|E)P(E)}{P(A|E)P(E) + P(A|F)P(F)} = \frac{0.9 \times 0.6}{0.9 \times 0.6 + 0.2 \times 0.4} = 0.871. \quad \square
\]

4: (Think Geometric.)

\[
P(k\text{th shock kills the machine})
\]
\[
= P((k\text{th shock causes damage}) \cap (\text{first } k-1 \text{ shocks cause exactly 4 damages}))
\]
\[
= 0.1 \times \binom{k-1}{4} (0.1)^4 (0.9)^{k-5}, \quad k \geq 5. \quad \square
\]

5: (Conditional probability.) Let \( X \) be the time a typical item lasts, and let \( A \) (resp., \( B \)) be the events that the item came from the first (resp., second) vendor. Then
\[
P(X \geq 12) = P(X \geq 12|A)P(A) + P(X \geq 12|B)P(B)
\]
\[
= 0.5(e^{-12 \times 0.10} + e^{-12 \times 0.08}) = 0.342. \quad \square
\]

6: Let \( X_1 \) and \( X_2 \) be lifetimes of products from the first and second vendors, respectively. Since \( X_1 \sim \text{Exp}(\lambda) \) and \( X_2 \sim \text{Erlang}_2(\mu) \), we have \( \mathbb{E}[X_1] = 1/\lambda = 10 \) and \( \mathbb{E}[X_2] = 2/\mu = 10 \). Thus, \( \lambda = 0.1 \) and \( \mu = 0.2 \). This immediately implies that
\[
P(X_1 \geq 8) = e^{-0.1 \times 8} = 0.449
\]
\[
P(X_2 \geq 8) = \sum_{i=0}^1 \frac{e^{-0.2 \times 8} (0.2 \times 8)^i}{i!} = 2.6e^{-1.6} = 0.525.
\]
Hence the second vendor should be chosen. □

7: 
(a) \[ \sum_{i=k}^{n} \binom{n}{i} p^i (1 - p)^{n-i}. \] □

(b) Set \( p = e^{-t} \). Then \[ \sum_{i=2}^{3} \binom{n}{i} p^i (1 - p)^{n-i} = 3e^{-2t} - 2e^{-3t}. \] □

(c) Set \( p = e^{-3} \). Then the total cost is
\[ 0p^3 + 75 \left( \frac{3}{1} \right) p^2(1-p) + (75 \times 2 + 1000) \left( \frac{3}{2} \right) p^1(1-p)^2 + (75 \times 3 + 1000)(1-p)^3 = 1206.6. \] □

8: Let \( X \) be a lifetime of the machine, and \( X_i \) be that of the \( i \)th component, where \( X_i \sim \text{Exp}(0.1) \), \( i = 1, 2, 3 \). Then
\[
P(X > x) = P(\min(X_1, X_2, X_3) > x) = P(X_1 > x, X_2 > x, X_3 > X) = \prod_{i=1}^{3} P(X_i > x) = e^{-0.3x}.
\]
This implies that \( X \sim \text{Exp}(0.3) \), and so the mean lifetime of the machine is \( 1/0.3 \).

9: Let \( X \) be the number of operating machines after 10 hours. The probability that a typical machine is alive after 10 hours is \( e^{-10 \times 0.125} = 0.2865 \). Then clearly, \( X \sim \text{Bin}(10, 0.2865) \). Hence, \( \mathbb{E}[X] = np = 2.865 \), and \( \text{Var}(X) = npq = 2.044 \). □

10:
\[
P(Z > x) = \sum_{k=1}^{\infty} P(Z > x | N = k) P(N = k) = \sum_{k=1}^{\infty} \sum_{i=0}^{k-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!} (1 - p)^{k-i} p = \sum_{i=0}^{\infty} e^{-\lambda x} \frac{(\lambda x)^i}{i!} p \sum_{k=i+1}^{\infty} (1 - p)^{k-1} = \sum_{i=0}^{\infty} e^{-\lambda x} \frac{(\lambda x)^i}{i!} (1 - p)^{i} = e^{-\lambda x} e^{(1-p)\lambda x} \sum_{i=0}^{\infty} e^{-(1-p)\lambda x} \frac{(1 - p)^{i}}{i!} = e^{-\lambda x}.\]
Hence, $Z$ has an exponential distribution with parameter $p\lambda$. \hfill \blacklozenge

11: Let $A$ (resp., $B$) be lifetime of battery $A$ (resp., $B$). Then

$$
P(B > A) = \int_0^\infty P(B > A|A = x)f_A(x) \, dx
= \int_0^\infty e^{-\lambda_2 x} \lambda_1 e^{-\lambda_1 x} \, dx
= \frac{\lambda_1}{\lambda_1 + \lambda_2} = 0.455. \hfill \blacklozenge
$$