NAME →

ISyE 3770 — Test 3x Solutions — Spring 2007

1. ______ 2. ______ 3. ______

4. ______ 5. ______ 6. ______

7. ______ 8. ______ 9. ______

10. ______ 11. ______ 12. ______

13. ______ 14. ______ 15. ______

16. ______ 17. ______ 18. ______

19. ______ 20. ______ 21. ______

22. ______ 23. ______ 24. ______

25. ______ 26. ______ 27. ______

28. ______ 29. ______ 30. ______

31. ______ 32. ______ 33. ______

34. State the precise reason that you had to take the test today. If you had a conflict, list the courses and class times that caused the conflict. Also, write your signature here affirming that you received no prior knowledge of this test.

1. Let X be the outcome of a die toss. Find $\mathsf{E}(X^3)$.

   Solution: 73.5. ♢

2. Suppose that the lifetime of a light bulb is exponential with a mean of 1000 hours. Further suppose that the bulb has already survived 2000 hours. Find the expected value of its total lifetime.

   Solution: 3000. ♢

3. If $X \sim \text{Unif}(0, 1)$, what's the distribution of $Y = -3\ell\text{n}(U)$?

   Solution: Exp(1/3). ♢

4. Suppose X and Y are both $\text{Nor}(0, 1)$ with $\mathsf{Cov}(X, Y) = 0.5$. What's the distribution of $3X + 2Y - 7$?

   Solution: $\text{Nor}(-7, 19)$. ♢

5. If $X \sim \text{Pois}(4)$, what's the m.g.f. of $3X - 2$?

   Solution: $e^{-2t} \exp\{4(e^{3t} - 1)\}$. ♢

6. If $X_1, \ldots, X_{100}$ are i.i.d. from the exponential distribution with mean 10, find the probability that the sample mean $\bar{X}$ is at least 12. You may use an asymptotic approximation, if necessary.

   Solution: 0.0227. ♢

7. Suppose that a random man's weight is Nor(165,200) and a random woman's weight is Nor(145,200). Find the probability that a random man weighs more than a random woman.

Solution: 0.8413. ♢

8. What theorem says that the sample mean (usually) approaches normality as you increase the number of observations?

Solution: CLT. ♢

9. Find the normal quantile value $z_{0.05}$.

Solution: 1.645. ♢

10. Find the normal c.d.f. value $\Phi(-1.5)$.

Solution: 0.0668. ♢

11. If $X \sim \chi^2(4)$, find $\mathsf{P}(X < 9.49)$.

Solution: 0.95. ♢

12. If $X \sim t(8)$, then X has a t distribution with 8 ______.. (Fill in the blank.)

Solution: degrees of freedom. ♢

13. Suppose $T \sim t(300)$. What's $\mathsf{P}(T < -1)$?

Solution: 0.1587. ♢

14. Find $F_{0.975,3,5}$.

Solution: 0.0672. ♢

15. Suppose X and Y are i.i.d. $\chi^2(3)$. What's the distribution of X/Y?

    Solution: $F(3,3)$. ♢

16. Find the sample variance of 7, 7, 7, and 3.

    Solution: 16. ♢

17. Suppose $X_1, \ldots, X_6$ are i.i.d. Pois(λ), and we observe $X_1 = 0$, $X_2 = 0$, $X_3 = 4$, $X_4 = 3$, $X_5 = 0$, and $X_6 = 3$. What is the maximum likelihood estimate of λ?

    Solution: $\bar{X} = 1.667$. ♢

18. Suppose $X_1, \ldots, X_6$ are i.i.d. Pois(λ), and we observe $X_1 = 0$, $X_2 = 0$, $X_3 = 4$, $X_4 = 3$, $X_5 = 0$, and $X_6 = 3$. What is the maximum likelihood estimate of $\ell n(\lambda)$?

    Solution: $\ell n(\bar{X}) = 0.5108$. ♢

19. If $X_1, \ldots, X_3$ are i.i.d. Exp(λ), with $X_1 = 11$, $X_2 = 3$, and $X_3 = 1$, find the MLE for λ.

    Solution: $1/\bar{X} = 0.2$. ♢

20. If $X_1, \ldots, X_3$ are i.i.d. Exp(λ), with $X_1 = 11$, $X_2 = 3$, and $X_3 = 1$, find the obvious unbiased estimate for $1/\lambda^2$.

    Solution: $S^2 = 28$. ♢

21. TRUE or FALSE? If an estimator T is unbiased for a parameter θ, then $5T$ is unbiased for 5θ.

    Solution: True. ♢

22. TRUE or FALSE? If $X_1, \ldots, X_n$ are i.i.d. $U(0, \theta)$, then the MLE for θ is unbiased.

    Solution: False. $\diamondsuit$

23. If $X_1, \ldots, X_n$ are i.i.d. $\text{Nor}(-3, 15)$, find $\mathsf{E}[S^2]$.

    Solution: 15. $\diamondsuit$

24. TRUE or FALSE? If $X_1, \ldots, X_n$ are i.i.d. $U(\theta, 0)$, then $\min_{1 \le i \le n}\{X_i\}$ is the MLE for θ.

    Solution: True. $\diamondsuit$

25. Suppose we have 2 estimators, T_1 and T_2, for some parameter θ. Further suppose that $\mathsf{Bias}(T_1) = 15$, $\mathsf{Var}(T_1) = 100$, $\mathsf{Bias}(T_2) = 10$, and $\mathsf{Var}(T_2) = 220$. Which estimator has the larger mean squared error?

    Solution: T_1. $\diamondsuit$

26. Suppose $X_1, \ldots, X_n$ are i.i.d. with p.d.f. $f(x) = \theta x^{-(\theta+1)}$, for $x \ge 0$ and $\theta > 1$. (This is called the Pareto distribution.) Find the MLE for θ.

    Solution: $n/\ell\text{n}(\prod_{i=1}^n x_i)$. $\diamondsuit$

27. When we calculate MLE's, what is the function $L(\theta) = \prod_{i=1}^n f(x_i)$ typically called?

    Solution: likelihood function. $\diamondsuit$

28. Suppose that $X_1, \ldots, X_{10}$ are i.i.d. normal with unknown mean and variance. Further suppose that $\bar{X} = 100$ and $S^2 = 900$. Find a 90% two-sided confidence interval for μ.

    Solution: 100 ± 17.39. $\diamondsuit$

29. Suppose that $X_1, \ldots, X_n$ are i.i.d. normal with unknown mean μ, but *known* variance of 60. How big would n have to be in order for a two-sided 95% confidence interval to have a half-length of 2? (Give the smallest such number.)

Solution: 58. $\diamondsuit$

30. After collecting your set of observations, what happens to the length of a confidence interval for the mean as you increase α?

 (a) It increases.
 (b) It decreases.
 (c) It stays the same.
 (d) You can't tell.

Solution: (b). $\diamondsuit$

31. Suppose my 95% confidence interval for the mean is $[-3, 10]$. Which of the following statements is true? (If there is more than one correct choice, write them all down.)

 (a) $\bar{X} = 3.5$.
 (b) The true mean is somewhere in $[-3, 10]$
 (c) We are 95% sure that μ is in the interval.
 (d) The half-length of the interval is 6.5.

Solution: (a), (c), (d) . $\diamondsuit$

32. We didn't get around to doing this in class, but it can be shown that if $X_1, \ldots, X_n$ are i.i.d. normal with unknown μ and σ^2, then a $100(1-\alpha)\%$ one-sided lower confidence interval for the variance is given by

$$\frac{(n-1)S^2}{\chi^2_{\alpha,n-1}} \le \sigma^2.$$

Suppose that $n = 6$ and $S^2 = 4$. Find such a 90% lower CI for σ^2.

Solution: $2.165 \le \sigma^2$. $\diamondsuit$

33. What do I mean when I write the symbol "//" in this class?

    Solution: I am done. ♢

Table 1: Standard normal values

z	$\mathsf{P}(Z \leq z)$
1	0.8413
1.28	0.9000
1.5	0.9332
1.645	0.9500
1.96	0.9750
2	0.9773

Table 2: $\chi^2_{\alpha,\nu}$ values

$\nu \setminus \alpha$	0.50	0.10	0.05	0.025
4	3.36	7.78	9.49	11.14
5	4.35	9.24	11.07	12.83
6	5.35	10.65	12.59	14.45

Table 3: $t_{\alpha,\nu}$ values

$\nu \setminus \alpha$	0.10	0.05	0.025
8	1.397	1.860	2.306
9	1.383	1.833	2.262
10	1.372	1.812	2.228

Table 4: $F_{0.025,m,n}$ values

$n \setminus m$	3	4	5
3	15.44	15.10	14.88
4	9.98	9.60	9.36
5	7.76	7.39	7.15