Linear Programming Models

(many of these examples come from Introduction to Mathematical Programming by Wayne L. Winston)

Getting Started

- Read pp. 321-343 as a general introduction to optimization and decision modeling, and to master ***basic concepts***:

  * decisions and decision variables
  * constraints and constraint functions
  * objectives and criterion functions
  * using mathematical abstraction to describe real scenarios
  * approximation via simplifying assumptions
  * tractability versus validity and their trade-offs
  * descriptive versus prescriptive models
  * numerical search
  * exact versus heuristic/approximate solutions

Optimization: A Model Archetype

- "Deciding" can be abstracted as a mathematical statement
- The mathematical statement can be analyzed using algorithms and software
- The results of the analysis can help in making actual decisions

Today, focus on abstracting

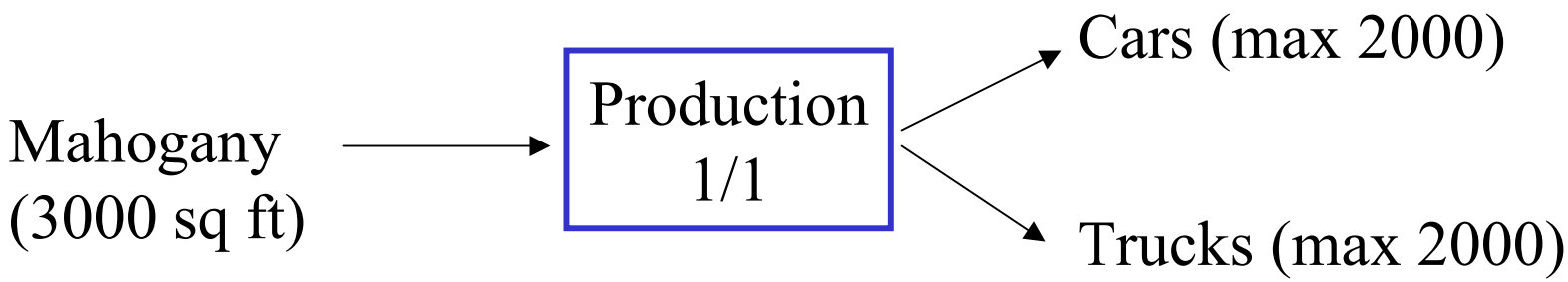
Later, we'll address analysis and software

Smallco, Inc.

Smallco, Inc. manufactures two products, wooden toy cars and wooden toy trucks, made from mahogany. The profit margin for the two toys is $1.10 and $0.70, respectively. Based on careful market analysis, it appears that Smallco can sell about 2000 of each toy each week. However, Smallco can only obtain a limited amount of mahogany, roughly 3000 board-feet per week. Producing either toy requires 1 board foot of wood.

What is the best mix of toys for Smallco to produce if the goal is to maximize total profit margin?

Smallco, Inc.


Mahogany
(3000 sq ft)
Production
1/1
Cars (max 2000)
Trucks (max 2000)

What is the objective?

- Maximize profit margin

What can be controlled?

- Amount produced for each toy, i.e.,
  - number of toy cars
  - number of toy trucks
- These are the ***decisions***
- Let C=number of cars produced,
- T=number of trucks produced

How do the decisions affect the objective?

- Each toy car contributes $1.10 and each toy truck contributes $0.70

profit margin = 1.1C + 0.7T

What limits the decisions?

- Sales potential is limited to 1000 units per week for each toy type

$$C \leq 2000,\ T \leq 2000$$

- Production is limited by availability of raw material--mahogany

$$C + T \leq 3000$$

Formal Model

$$\max z = 1.1 * C + 0.7 * T$$

$$subject\ to: C \leq 2$$

$$T \leq 2$$

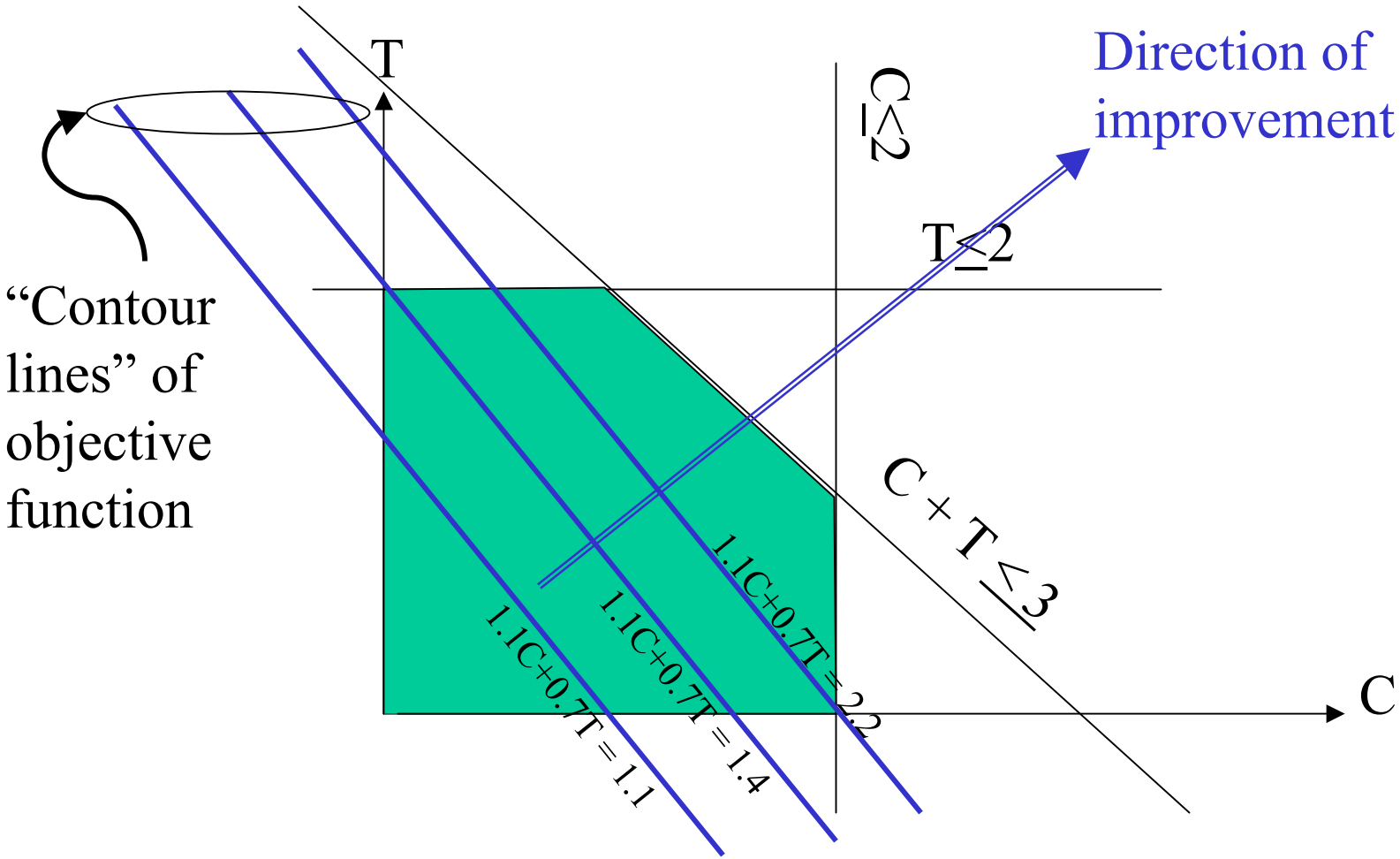
$$C + T \leq 3$$

$$C, T \geq 0$$

Solve using a spreadsheet

small.xls

Solve graphically



Any LP with 2 variables

- Can be solved graphically
- Must be careful with "sense" of inequality constraints
- Must be careful with equality constraints
- Must be able to plot objective function contours (choose a value!)
- Careful with max vs min problems

You WILL be required to do this on quiz 3 and the final!

Try this same problem, but

- $C \geq 2$
- $C + T \geq 3$
- minimize

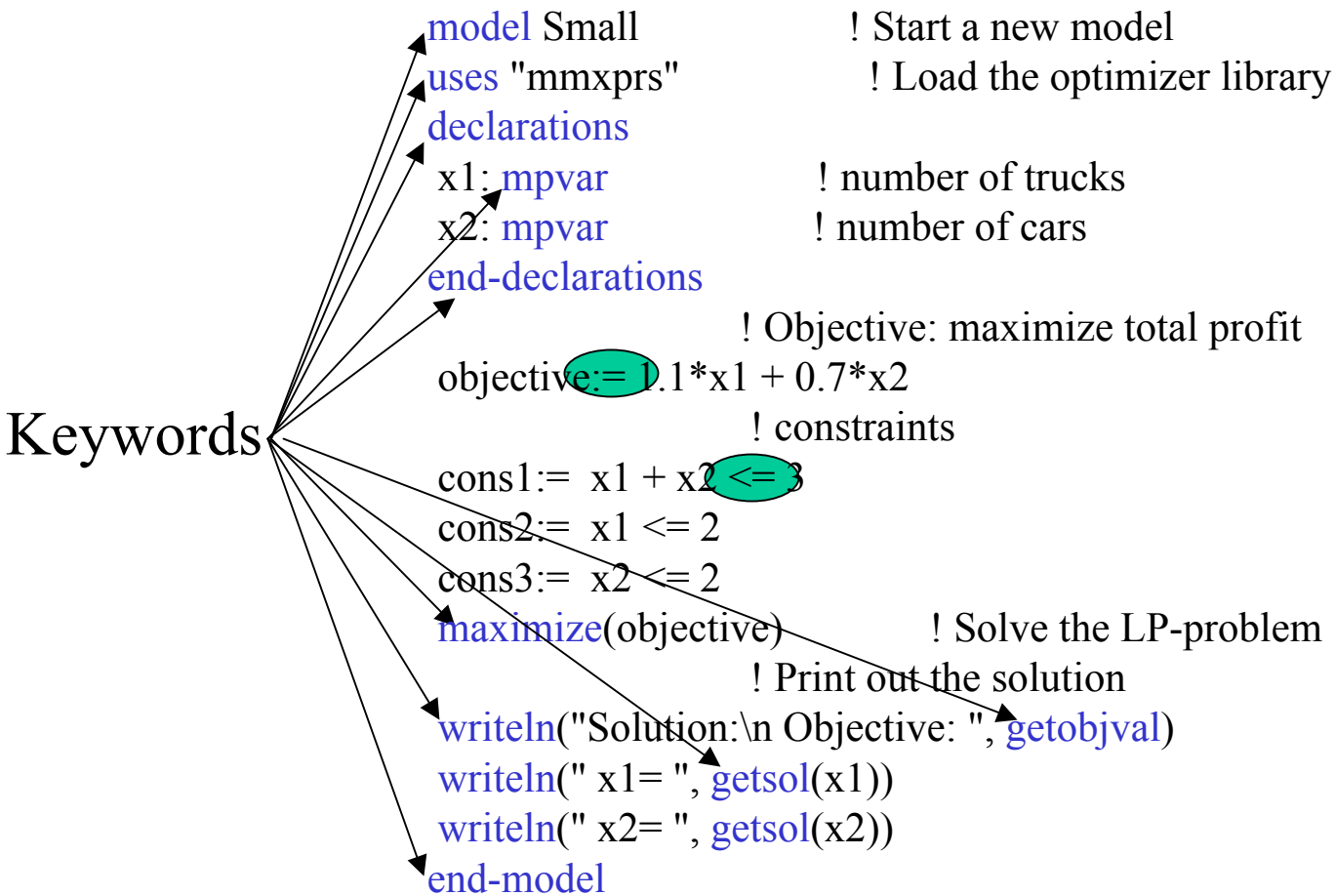
- What answer do you get?

Solve using Xpress-IVE

small.mod

download the “getting started” document!

Xpress-IVE Model



Xpress-IVE Solution

Solution:
Objective: 2.9
x1= 2
x2= 1

Modeling involves five distinct processes, which may be repeated a number of times in any application:

- ***cataloging*** what you know about the problem situation
- recalling relevant ***domain knowledge*** and facts pertinent to the situation
- recalling appropriate ***modeling archetypes*** that might be applicable to the situation
- ***creating*** a particular model
- using and ***evaluating*** the model, testing it against the problem situation and iterating the modeling process if necessary

Farmer Jones must determine how many acres of corn and wheat to plant this year in order to maximize his profit. An acre of wheat yields 25 bushels and requires 10 hours of labor per week. An acre of corn yields 10 bushels and requires 4 hours of labor per week. Wheat sells for $4 per bushel and corn sells for $3 per bushel. Seven acres are available for planting and 40 hours per week of labor are available. What should Farmer Jones do? Suppose he has a government subsidy that requires him to produce at least 30 bushels of corn. Does that change what he should do?

Cataloging

- What is the objective?
- What are the decisions (controllable)?
- How do the decisions affect the objective?
- What limits the decisions?

- Develop a formal model of Farmer Jones' problem.

Farmer Jones' Decision Problem

$$\max 30x_c + 100x_w$$

$$s.t. 4x_c + 10x_w \leq 40$$

$$x_c + x_w \leq 7$$

$$x_c, x_w \geq 0$$

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$$x_c + x_w \le 7$$
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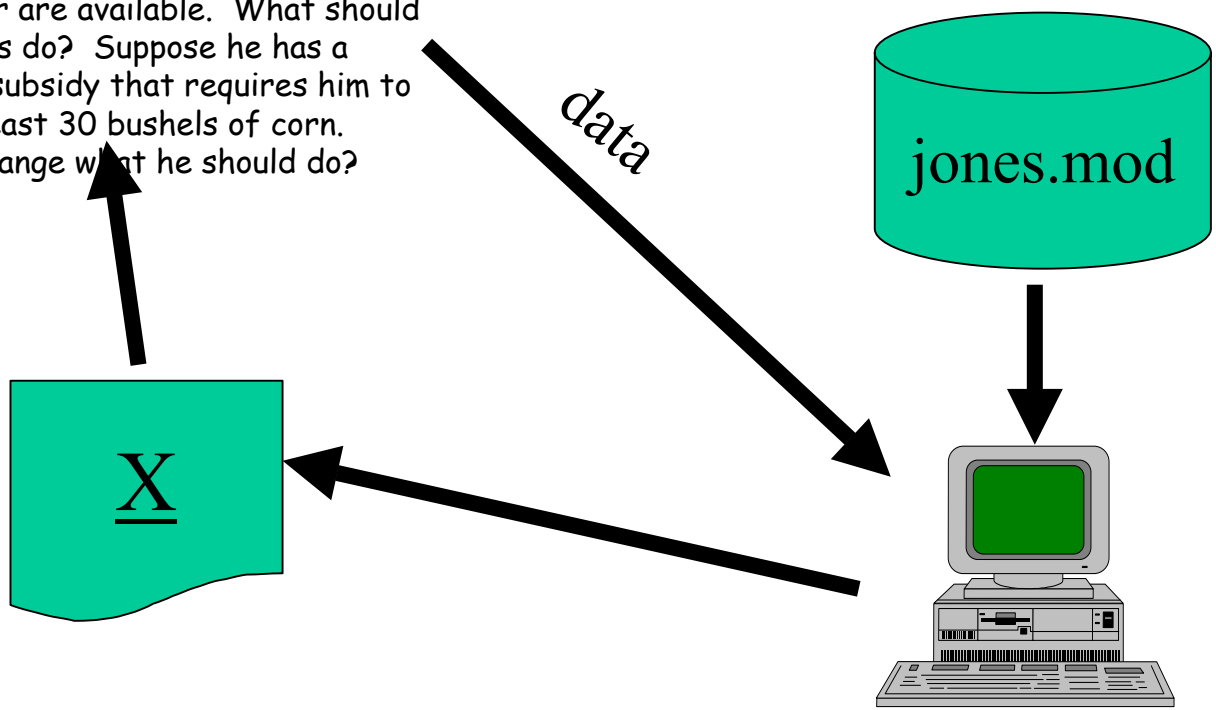

Assignment

- Modify small.xls to solve Farmer Jones' decision problem
- Modify small.mod and use Xpress-IVE to solve Farmer Jones' problem
- Solve Farmer Jones' problem graphically

DO IT TODAY!

See if you can develop the formal models for the following five problems.

We'll look at these in more detail during the next lecture, but you'll get more out of it if you try the problems before the lecture.

Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign, and has decided to purchase 1-minute ad spots on two types of programs; comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy spot costs $50,000 and a 1-minute football spot costs $100,000. Dorian wants to reach at least 28 million high-income women and 24 million high-income men. How should Dorian buy commercial time to reach their targets at the lowest possible cost?

Leary Chemical manufactures three chemicals: A, B, and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs $4 and produces 3 units of A, 1 of B, and 1 of C. Running process 2 for an hour costs $1, and produces 1 unit of A and 1 unit of B. Leary has contracts to provide 10 units of A, 5 units of B, and 3 units of C. How should Leary plan its production to minimize the cost of satisfying its contracts?

My diet requires that all the food I eat come from one of the four basic food groups: chocolate cake, ice cream, soda, and apple pie. The costs for the four items are, respectively, $1.50, $.60, $.90, and $2.40. Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional content of the four foods are shown below. What is the minimum cost for me to satisfy my dietary requirements?

	Calories	Chocolate	Sugar	Fat
Choc cake	400	3	2	2
Choc ice cream	200	2	2	4
soda	150	0	4	1
cheesecake	500	0	4	5

Star Oil Company is considering five different investment opportunities. The cash outflows and net present values (in millions) are given below. $40 million is available for investment now, and $20 million will be available next year. Star Oil can purchase a fraction of an investment; e.g., 1/2 of investment one will cost $5.5 million now, $1.5 million in a year, and have a net present value (NPV) of $6.5 million. What should Star Oil do to realize the maximum NPV, if money not spent now is used for something else?

	Inv. 1	Inv. 2	Inv. 3	Inv. 4	Inv. 5
Time 0 cost	$11	$53	$5	$5	$29
Time 1 cost	$3	$6	$5	$1	$34
NPV	$13	$16	$16	$14	$39

Chandler Oil Company has 5000 barrels of oil 1 and 10,000 barrels of oil 2. The company sells two products, gasoline and heating oil. The quality level of oil one is 10, and for oil 2 is 5. Gasoline must have an average quality level of at least 8, while heating oil must have an average quality level of 6. Demand is created through advertising, and each dollar spent advertising gasoline yields 5 barrels of sales at a price of $25, while each dollar spent advertising heating oil yields 10 barrels of sales at a price of $20. What should Chandler spend on advertising, and what should Chandler produce in order to maximize revenues?

What are the common attributes of the linear programming (LP) models for these decision problems?

LP's have these attributes

- Decision variables are continuous (divisibility)
- Impact of decision variable is proportional to its value (proportionality)
- Impact of one decision variable is independent of the impact of the others (additivity)
- Parameter values are known and constant (certainty)