1 Storage Mode Alternatives

The 2 basic methods of storing pallets are floor storage and rack storage.

1.1 Floor Storage

As the name suggests, in floor storage pallets are stored on the floor arranged in lanes. The depth of a lane is the number of pallets stored back-to-back away from the pick aisle. The height of a lane is normally measured as the number of pallets that can be “stacked” one on top of each other. The characteristics of an SKU (e.g. weight, crushability, cases per pallet, etc.) will determine the maximum stack level. It is important to point out that the entire footprint of a lane is being used by an SKU if any part of a lane is currently storing a pallet. This rule is almost always applied, since if more than 1 SKU were stored in a lane, then a considerable amount of double-handling would be required to execute the receiving and shipping operations, which would offset any space savings. This loss of space is called honeycombing.

1.2 Rack Storage

The advantage of rack storage is that each level of the rack is independently supported, thus providing much greater access to the loads, and permitting possibly greater stack height. (Of course, one has to pay for the racks.) To access the loads in rack storage (other than an AS/RS) some type of lift truck is required. The type of lift truck determines, among other things, the amount of aisle space required. So, when space is tight, one can buy more expensive equipment (and it can get expensive). We shall first describe the most commonly used rack systems, and then describe the most commonly used lift trucks.

The most common types of rack storage are:

- **Selective rack or Single-Deep rack.** Pallets are stored one deep. Due to rack supports each pallet is independently accessible, and so any SKU can be stored in any pallet location at any level of the rack.

- **Double Deep rack.** Double deep rack essentially consists of 2 single deep racks placed one behind the other, and so pallets are stored 2 deep. Due to rack supports each 2 deep lane is independently accessible, and so any SKU can be stored in any lane at any level of the rack. With 2 deep storage a pallet located in the rear position is blocked by a pallet stored in the front position. To avoid double-handling, only 1 SKU occupies each lane, so some honeycombing will take place. This disadvantage is partially offset by the fewer aisles required to access the loads.

- **Drive-In or Drive-Through rack.** In this rack, the lift truck must drive within the rack frame to access the interior loads. With Drive-In rack the putaway and retrieval functions are performed from the same aisle. With Drive-Through rack the putaway
and retrieval functions are performed from different aisles. The depth of storage is usually limited to 8 pallets. Essentially, one can view this type of rack as functionally serving to support each layer of floor storage that is more than 2 deep.

- **Pallet Flow rack.** As with case flow rack, pallets are putaway from the back end and retrieved from the front end. As each load is removed, pallets move towards the front via gravity roller tracks. Storage depth is usually limited to 8 pallets. This type of rack is appropriate for high throughput facilities.

The most common type of lift trucks are:

- **Counterbalance Lift Truck.** The most versatile type of lift truck with respect to use within a facility. The sit-down version requires an aisle width of 12-15 feet, its lift height is limited to 20-22 feet, and it travels at about 70 ft/min. The stand-up version requires an aisle width of 10-12 feet, its lift height is limited to 20 feet, and it travels at about 65 ft/min.

- **Reach and Double-Reach Lift Truck.** This truck is equipped with a reach mechanism that allows its forks to extend to store and retrieve a pallet. The double-reach truck is required to access the rear positions in double deep rack storage. Each truck requires an aisle width of 7-9 feet, their lift height is limited to 30 feet, and they travel at about 50 ft/min.

- **Turret Truck.** This truck uses a turret that turns 90 degrees, left or right, to putaway and retrieve loads. Since the truck itself does not turn within the aisle, an aisle width of only 5-7 feet is required, its lift height is limited to 40-45 feet, and it travels at about 75 ft/min. Since the aisles are so narrow some kind of guidance device (e.g. rails, wire, tape) is usually required. It only operates within single deep rack and super flat floors are required. This type of truck is also not easily maneuverable outside the rack.

- **Stacker Crane within an AS/RS.** The stacker crane is the handling component of a unit load AS/RS, and so it is designed to handle loads up to 100 feet high. Roof or floor-mounted tracks are used to guide the crane. The aisle width is about 6-8 inches wider than the unit load. Often, the crane is “aisle-captive”, though transfer car mechanisms exist to move the crane from one aisle to another.

### 1.3 Costs

Floor storage and drive-in or drive-through racks require a specification of the depth of each lane. The aisle space required will depend on the type of truck used. Each truck will require a different up-front capital expense and operations and maintenance cost. Thus, there are conceivably many different storage mode alternatives, each with a different labor, space, maintenance, and up-front capital costs.

As a general guide, the counterbalance and reach trucks range in purchase cost from $20,000- $40,000, whereas the turret truck ranges in purchase cost from $60,000-$120,000. With respect to rack cost, single and double deep rack range in purchase cost from $35-$45 per slot, drive-in, drive-through rack range in purchase cost from $60-$80 cost per slot, and an AS/RS slot ranges in purchase cost from $175-$250 per slot.
2 Block Stacking Analysis

A warehouse receives 60 pallet loads of a product per shipment. Loads can be stacked 3 high. The load dimensions of the pallet are 48 x 40 inches. The first dimension records the pallet depth; the second dimension records the pallet width, as measured along the pick aisle. With block stacking, a 12 in. clearance between lanes is required, and a 12 ft. aisle is required to accommodate the counterbalance lift trucks used to store and retrieve loads.

2.1 Estimating Space Requirements

The billable sq. ft. is defined to be the product of the footprint sq. ft. and the average number of footprints in use per unit time. The footprint sq. ft. is the product of 2 terms: the width along the pick aisle and the depth of storage. The depth of storage is a function of the number of pallets deep, x, which is the decision variable.

Let's calculate the footprint sq. ft. Since the width of the pallet is 40 in. and the required clearance between lanes of storage is 12 in. the width along the pick aisle per storage row is \((40 + 12) = 52\) in. Since the depth of a pallet is 48 in., if x pallets are to be stored in an storage lane, the depth due to pallet storage is 48x. To assess the true cost of space, we add one-half of the required aisle space, since this space must be in front of each storage lane. The calculated storage depth is therefore \((48x + 72)\) in. (Note the dependency of the storage depth on the decision variable x.) Therefore, the footprint sq. ft. is \(52(48x+72)\).

The average number of footprints is dependent on how deep we configure the storage lane, namely, the variable x, and the rate of pallet withdrawal. With regard to the latter, as a first approximation, we shall assume pallets are withdrawn at a uniform rate. Since there are 60 loads to store, and the loads will be stored 3 high, there are \(60/3 = 20\) “stacks” to be stored. The maximum number of storage rows, M, equals \((Q/3x)\) rounded up to the nearest integer. For example, suppose we choose a 5-deep lane configuration, i.e., we choose x to be 5. There will be \(5 \times 3 = 15\) loads per lane, and thus \(60/15 = 4\) lanes required initially. Suppose, however, we choose x to be 6. Initially, there will still be 4 lanes required, since the first 3 lanes will only hold \(3 \times 18 = 54\) pallets. The “final” 6 pallets or 2 stacks will be stored in the 4th lane. Thus, given x, the number of lanes M initially required is determined (so \(M = M(x)\)) and the overall storage configuration is determined.

How do we estimate the average number of storage rows used? Consider the case when x = 5. It will take 60 time units to empty the load. Given the assumption of uniform withdrawal, for the first 15 time units there will be 4 lanes; for the next 15 time units there will be 3 lanes; for the next 15 time units there will be 2 lanes; and for the final 15 time units there will be 1 lane of storage being used. Another way to look at it is to say that for 25% of the time there will be either 4, 3, 2 or 1 storage rows. Thus, on average, there will be 0.25(4 + 3 + 2 + 1) = 2.5 storage rows. What about the case when x = 6? For the first 6 time units there will be 4 lanes; for the next 18 time units there will be 3 lanes; for the next 18 time units there will be 2 lanes; and for the final 18 time units there will be 1 lane of storage being used. Measured in percentages of time, 18/60 = 30% of the time there will be either 1, 2 or 3 lanes, and 6/60 = 10% of the time there will be 4 lanes. Thus, on average, there will be 0.30(1 + 2 + 3) + 0.10(4) = 2.2 lanes of storage being used. Note that 0.30(1 + 2 + 3) + 0.10(4) can be equivalently expressed as 0.90[(1 + 2 + 3)/3] + 0.10[4].

More generally, the average number of storage lanes used equals
\[
(1 - \pi)[(1 + 2 + \ldots + M - 1)/(M - 1)] + \pi[M] = M(1 + \pi)/2,
\]
(1)

where \(\pi\) denotes the percentage of time the maximum \(M\) lanes are being used. Note that when there is no “honeycombing” (i.e. when each lane is initially full), then \(\pi = 1/M\) and the average storage lanes used equals \((M + 1)/2\), which is simply the expectation of an \(M\)-sided die when each side is equally likely to come up. Remember that the average number of lanes used goes down as the depth of a lane goes up, but the footprint sq. ft. goes up.

One objective would be to choose the value of \(x\) to minimize the average billable sq. ft. by simply calculating for each value of \(x\) the product of the average number of lanes and the footprint sq. ft. and then selecting the minimum.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(M(x))</th>
<th>Footprint (sq. ft.)</th>
<th>Avg. No. of Rows</th>
<th>Avg. Sq. Ft. (E[SQ(x)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>43.3</td>
<td>10.5</td>
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<td>3.85</td>
<td>300</td>
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<td>5</td>
<td>95.3</td>
<td>3.00</td>
<td>286</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>112.6</td>
<td>2.50</td>
<td>282</td>
</tr>
<tr>
<td>6</td>
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<td>3</td>
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<td>1.95</td>
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<td>3</td>
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<td>3</td>
<td>182.0</td>
<td>1.65</td>
<td>300</td>
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<td>10</td>
<td>2</td>
<td>199.3</td>
<td>1.50</td>
<td>300</td>
</tr>
</tbody>
</table>

For the specific problem at hand the reader the best choice is \(x = 5\) and \(E[SQ(5)] = 282\).¹

3 A Closed-Form Approximation

In our example the function \(E[SQ(x)]\) is very flat around the optimal choice; that is, \(E[SQ(4)] = 286\) and \(E[SQ(6)] = 287\). We can take advantage of this general property to enable one to calculate an excellent choice for \(x\) by evaluating a simple formula.

Let \(A\) denote the aisle width, let \(d, w, g\), and \(z\) denote the pallet depth, width, lane gap, and stack level, respectively, and let \(E[SQ(x)]\) denote the average billable sq. ft. as a function of the decision variable \(x\). The maximum \(M(x)\) is the smallest integer at least as large as \(Q/(x\times z)\). Ignoring the “round-up” operation we set \(M(x) = Q/(x\times z)\) and treat it as a continuous variable. With regard to estimating the average number of lanes, it is technically true that the average value of a random variable that is uniformly distributed on the interval \([0, M(x)]\) is \(M(x)/2\); however, it will be more accurate to estimate the average number of lanes used by \((M(x)+1)/2\), which will be exact when there is no honeycombing.

Now given these approximations we have that

\[
E[SQ(x)] = \frac{1}{144}\left[\frac{M(x) + 1}{2}\right][w + g](A/2 + dx) \]

(2)

¹Filling out the table for \(x\) values of 1, 2, 3, 4, 5 and 10 is easy since the average number of storage rows will be \((M(x)+1)/2\). Notice also that the increment in footprint sq. ft. will be 17.33 as \(x\) in increased by one.
\[
\frac{1}{144} \left( \frac{w + g}{2} \right) \left( \frac{AQ}{2z} + dx + \left( \frac{A}{2} + \frac{dQ}{z} \right) \right). \tag{3}
\]

**Technical note.** Let \(a\) and \(b\) be positive numbers. The (convex) function \(f(x) = \frac{a}{x} + bx\) arises in many modeling situations, most notably in trading off replenishment costs with inventory costs when determining the optimum order size. It achieves its minimum when the derivative vanishes, i.e., when \(\frac{df}{dx} = -\frac{a}{x^2} + b = 0\). Thus, \(x^* = \sqrt{\frac{a}{b}}\) and \(f(x^*) = 2\sqrt{ab}\). One way to remember this is to note that the 2 functions \(g(x) = \frac{a}{x}\) and \(h(x) = bx\) intersect at the optimum.

Since \(E[SQ(x)]\) is an EOQ form, its minimum is achieved at
\[
x^* = \sqrt{\frac{AQ}{2zd}}, \tag{4}
\]
and the optimal (continuous) value is
\[
E[SQ(x^*)] = \frac{1}{144} \left( \frac{w + g}{2} \right) \left[ \sqrt{\frac{2AdQ}{z}} + \left( \frac{A}{2} + \frac{dQ}{z} \right) \right]. \tag{5}
\]
For our example,
\[
x^* = \sqrt{144 \times 20/2 \times 48} = 5.48, \tag{6}
\]
and
\[
E[SQ(x^*)] = \frac{1}{144} \left( \frac{40 + 12}{2} \right) \left[ \sqrt{2 \times 144 \times 48 + 20} + (72 + 48 \times 20) \right] = 281.3, \tag{7}
\]
which is extremely close to the integer optimal solution.

### 4 Single-deep and Double-deep pallet rack analysis

We continue with our example. Rack alternatives are single-deep and double-deep pallet rack and deep lane (drive-in, drive-through) storage. The horizontal clearance is 4 in., the upright rack member dimension is 3 in., and the rack flue space is 12 in.

The width along the pick aisle to store 2 pallet loads in single-deep or double-deep pallet rack is 80 in. for the 2 pallets plus 1 rack member clearance of 3 in. and 3 horizontal clearances of 4 in. totaling 95 in. Thus, the billable width for one pallet position is 95/2 = 47.5 in. The depth of single-deep pallet rack is the pallet depth of 48 in. plus one-half of the sum of the aisle width (144 in.) and the flue space (12 in.) totaling 126 in. For double-deep pallet rack it will be 126 + 48 = 174 in. since 2 pallets are stored in a lane. Thus, the footprint in sq. ft. on the floor for single-deep pallet rack is \((47.5)(126)/144 = 41.58\) and \((47.5)(174)/144 = 57.42\) for double-deep pallet rack, according to our previous approach.

However, the billable sq. ft. for racking should be much less, since each *layer* of the storage lane is *independently supported*. That is, each layer is sharing the footprint on the floor. To arrive at the *correct* billable sq. ft. number, we simply divide the floor footprint
by the number of rack positions per storage column or storage levels $z$, which in this case is 6. Thus, the billable sq. ft. for single-deep pallet rack is $41.58/6 = 6.93$ sq. ft. and for double-deep pallet rack is $57.42/6 = 9.57$ sq. ft. Note that in these footprint calculations there is no need to decide $x$, since $x = 1$ or 2 depending on the racking alternative.

As before, we now need to calculate the average number of footprints. For single-deep pallet rack, we initially need 60 pallet rack positions. Assuming once again a uniform withdrawal rate, the average number of single-deep pallet rack positions is simply $(60 + 1)/2 = 30.5$. For double-deep rack, we initially need 30 pallet rack positions; thus, the average number of double-deep pallet rack positions is $(30 + 1)/2 = 15.5$.

The billable sq. ft. for single-deep pallet rack is therefore $6.93 \times 30.5 = 211$, and the billable sq. ft. for double-deep pallet rack is therefore $9.57 \times 15.5 = 148$.

### 5 Deep-lane analysis

Deep-lane analysis is essentially identical to single- and double-deep analysis, except in 2 important respects: the depth $x$ of the storage position (lane) can be more than 2, and so is a decision variable, as in block stacking, and the width along the pick aisle is slightly different due to the racking configuration.

For deep-lane storage, the width along the pick aisle to store 1 pallet position in deep-lane storage is the width of the pallet (= 40 in.) plus the rack member clearance (= 3 in.) plus 2 horizontal clearances (= 8 in.) for a total of 51 in. The depth calculation is identical to the other racking alternatives and is $(48x + 78)$. Consequently, the footprint in sq. ft. on the floor is $51(48x + 78)/144$. Once again, we must divide this number by the number of rack positions per storage column $z$ (= 6) in order to correctly bill for the sq. ft. for each pallet position. Thus, the footprint sq. ft. if we store $x$ pallets deep is $(408x + 663)/144$. Given a choice for $x$, the calculation for the average number of storage lanes is identical to the calculation for single- and double-deep pallet rack, with the exception that the stack level variable $z$ should be set to one.

Again, one objective would be to choose the value of $x$ to minimize the average billable sq. ft. by simply calculating for each value of $x$ the product of the average number of lanes and the footprint sq. ft. and then selecting the minimum.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$M(x)$</th>
<th>Footprint (sq. ft.)</th>
<th>Avg. No. of Rows</th>
<th>Avg. Sq. Ft. ($E[SQ(x)]$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
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<td>116</td>
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</table>
For the specific problem at hand the reader the best choice is \( x^* = 10 \) and \( E[SQ(10)] = 115 \). Once again note that the average billable sq. ft. is 122 when \( x = 5 \) and is 116 when \( x = 12 \), and so, as before, this function is extremely flat around the optimum.

As in the block stacking case we can formulate a continuous approximation so that a closed-form expression for the optimal choice \( x^* \) can be found and \( E[SQ(x^*)] \). All one has to do is to modify the previous formulas (4) and (5), as follows:

- Replace \( A \) with \( A + f \), where \( f \) denotes the flue space.
- Replace \( (w+g) \) with \( (w+r+2c) \), where \( r \) and \( c \) denote the rack member and horizontal clearances, respectively.
- Set \( z = 1 \).
- Divide the expression for the expected square feet by the number of rack positions per storage column.

For our example, we would calculate

\[
x^* = \sqrt{\frac{(144 + 12) \cdot 60}{2 \cdot 48}} = 9.87
\]

and

\[
E[SQ(x^*)] = \frac{1}{144} \frac{40 + 3 + 2 \cdot 4}{2} \sqrt{2 \cdot (144 + 12) \cdot 48 \cdot 60 + (72 + 6) + 48 \cdot 60} = 115.28
\]

which is, as before, extremely close to the integer optimal solution.

6 Selecting the best alternative

To arrive at the “best” choice of storage alternative, we must estimate the various costs. The major cost categories are: space, labor, and the storage and handling equipment. We discuss each below.

- **Space cost.** If space is being leased then the lease cost per sq. ft. per period can be used. If the space is owned and there is a capacity, then an appropriate rental cost per sq. ft. per period should be determined.

- **Labor cost.** Here, one has to estimate for each storage alternative the time (in hours) it would take to execute all putaway and pick operations for the period, which is then multiplied by the labor cost per hour to arrive at an overall labor cost per period. One can estimate the (pallet or case) picking cost by estimating the average lines picked per hour for each type of operation.

- **Equipment cost.** The acquisition cost of a piece of equipment can be amortized to a per hour cost, as follows. Suppose a turret truck costs $70,000. For simplicity, suppose accounting has classified the equipment as 7-year property with no salvage value, and simple straight-line depreciation is used. The company’s marginal tax rate is 40%, and its average cost of capital is 12%. The after-tax equivalent annual cost
of the equipment purchase cost would then be $11,338. (The present value of $15,338 over 7 years discounted at 12% is $70,000, but there is a $4,000 = 0.40(1/7)(\$70,000) savings per year due to the depreciation write-off.) Now add to this equivalent per year acquisition cost the annual after-tax cost of maintenance and energy (fuel, battery, etc.) to obtain the annual cost of operating this piece of equipment. For sake of discussion suppose the combined cost is now $15,000 per year. Assuming a utilization rate of 75% on 2,000 hours per year would equate to a use 1,500 hours per year, and this would translate into approximately $10/hr. This $10/hr figure must then be multiplied by the usage rate over the period. A similar type of calculation must be performed for the cost of the different type of rack.

These 3 cost components are highly intertwined, of course. There is always a tradeoff between the cost of space, labor and equipment. A more expensive lift truck may lessen the overall space required (due to less aisle space) and may lower the associated labor cost, but may cost more per year to maintain itself and may have a higher up-front cost.

For simplicity assume that all costs have been lumped into a dollar cost per sq. ft. per year. For our example, suppose these costs are $6.00 for block stacking, $7.00 for single-deep pallet rack, $7.50 for double-deep pallet rack, and $12.00 for deep lane storage. We then have:

<table>
<thead>
<tr>
<th>Method</th>
<th>Estimated sq. ft</th>
<th>Cost per sq. ft</th>
<th>Annual cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block stacking</td>
<td>282</td>
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<td>1692</td>
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<tr>
<td>Single-deep</td>
<td>211</td>
<td>7.00</td>
<td>1477</td>
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<tr>
<td>Double-deep</td>
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</tr>
<tr>
<td>Deep-lane</td>
<td>115</td>
<td>12.00</td>
<td>1380</td>
</tr>
</tbody>
</table>

Without any other considerations the recommended storage alternative is double-deep pallet rack.

7 The Distribution of Withdrawal

In lieu of the uniform distribution, different distributions could be used to model faster or slower withdrawal rates or simply to provide a cushion, since once a storage lane is removed it may not be immediately useful to another sku. One simple model is to imagine that the number of storage lanes $S$ in use at any one time follows the cumulative distribution

$$P\{S \leq s\} = (s/m)^a, \quad 0 \leq s \leq m. \quad (10)$$

Note that the uniform distribution coincides when $a = 1$. When $a > 1$ the shape of the distribution is convex, which means that the initial withdrawal of pallets loads occurs more slowly during the beginning of the cycle and faster at the end. Conversely, when $a < 1$ the shape of the distribution is concave, which means that the initial withdrawal of pallets loads occurs more quickly during the beginning of the cycle and slower at the end. Now for nonnegative random variables,

$$E[S] = \int_0^m \{1 - (s/m)^a\} ds = m * a/(a + 1). \quad (11)$$
The correct choice of $m$ here would be $M(x)+1$, as before. Different values of the parameter $a$ other than 1 do not affect the choice of $x^*$, but is tantamount to simply scaling the original estimate of optimal square footage by the factor $[a/(a + 1)]/(1/2) = 2a/(a + 1)$. This basically justifies the usual industry practice of adjusting space estimates upwards by a “safety factor”, say 20%, which happens to coincide with $a = 1.5$. 