

# SEMICONDUCTOR TOOL PLANNING VIA MULTI-STAGE STOCHASTIC PROGRAMMING\*

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## KEYWORDS

Tool planning, Capacity expansion, Stochastic programming, Value of multi-stage stochastic programming.

## ABSTRACT

This paper presents an optimization model for planning tool purchases for a semiconductor manufacturing facility under uncertain operating conditions. By modeling the uncertain parameters using a scenario tree, we develop a stochastic programming formulation for the problem. In contrast to earlier two-stage approaches for this problem, our model allows for revision of the tool purchase plan as more information regarding the uncertain problem parameters becomes available. The proposed model is a multi-stage stochastic integer program and, in general, is extremely difficult to solve to optimality. We propose a LP-relaxation based approximation scheme for the problem. Our preliminary numerical results indicate that even an approximate solution to the multi-stage model is far superior to any optimal solution to the two-stage model. These results confirm that the *value of multi-stage stochastic programming* for this class of problems is extremely high.

## INTRODUCTION

A crucial planning decision in semiconductor fabrication facilities is the planning of machine tool purchases to satisfy growing demand. Together with the high capital cost of semiconductor machines, the uncertainty surrounding demand growth, technology obsolescence and procurement lead times, makes tool purchasing decisions extremely complex. Consequently, various quantitative approaches for optimal machine tool capacity planning under uncertainty have been proposed.

Angelus et al. (1997, 2000) consider the problem of expanding the cumulative capacity of a single fabrication process — they do not distinguish between the individual tool capacities within the process. The authors (Angelus et al. 1997, 2000) apply inventory analysis to derive an optimal expansion policy for this simplified system. Çakanyilidirim and Roundy (1999) consider the capacity expansion of a bottleneck machine tool for processing a single product family. For the case when product demand is stochastically non-decreasing, the authors develop a polynomial time algorithm for computing the optimum expansion schedule for the bottleneck machine tool. Unfortunately, the above approaches are inapplicable for detailed capacity planning in systems involving multiple machine tools, products and processing steps.

Swaminathan (2000) suggests a stochastic programming approach (cf. Birge and Louveaux 1997) for single-period tool

planning in a wafer fab involving multiple products and machine tools. In this approach, the uncertainty in product demand is modeled using a set of scenarios, and the problem is formulated as a stochastic integer program. The model simultaneously optimizes the tool capacity plan as well as the production plan (i.e. the allocation of tool capacity to the products). The author suggests several heuristic strategies for solving this difficult optimization model. Extensions of the model involving multiple processing steps and multiple time periods are also discussed. Barahona et al. (2001) presents a comprehensive multi-period stochastic integer programming model for semiconductor tool planning involving multiple machine tools, product families, and processing steps. The authors describe several integer programming based techniques for solving the model to near-optimality.

Although existing stochastic programming models for semiconductor tool planning capture many important details regarding the interaction between machine tools, products, and processing steps, these models are based on a *two-stage* paradigm. That is, it is assumed that the tool purchase schedule for the entire (multi-period) planning horizon will be decided “here-and-now,” and the production planning decisions will be made on a period by period basis based upon realized demand and available tool capacity in that period. This assumption does not allow revising or updating of the tool purchase plan at a later period based upon the information available then. In this paper, we propose a multi-stage planning approach, where the tool purchase plan as well as the production plan is revised as time progresses and more information regarding the demand scenarios becomes available. The resulting model is an extremely difficult multi-stage stochastic integer program. We suggest an efficient scheme for obtaining good quality solutions to this multi-stage model. Our numerical results suggest that even an approximate solution to the multi-stage model is superior to an exact solution to the two-stage model, thereby emphasizing the value of multi-stage modeling.

## MODEL DEVELOPMENT

In this section, we develop a multi-stage stochastic programming model for semiconductor tool planning under uncertainty. We begin by examining a deterministic optimization model for tool capacity planning, and subsequently extend it to two-stage and multi-stage stochastic models.

### A Deterministic Model

The model described in this section is a multi-period extension of the model described in Section 6.4 of Swaminathan (2000). Consider a wafer fab consisting of  $M$  tool types, that can process  $N$  types of wafers. Each product (wafer type) goes through a subset of  $K$  processing steps, each of which can be performed on one or more tool types. Let  $a_{ijk}$  denote the time (in hrs) required by processing step  $k$  ( $1, \dots, K$ ) on wafer type

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$j (1, \dots, N)$  on tool type  $i (1, \dots, M)$ . We set  $a_{ijk} = 0$  if step  $k$  is not needed for wafer type  $j$ , and  $a_{ijk} = \infty$  if step  $k$  is required for wafer type  $j$  but cannot be performed on tool type  $i$ . Consider now a planning horizon of  $T$  periods. Let us use variables  $x_{it}$ ,  $u_{jt}$ ,  $v_{ijkt}$ , and  $w_{jt}$ , to denote the number of tool type  $i$  purchased in period  $t (1, \dots, T)$ , the shortage (in units of wafer starts) of wafer type  $j$  in period  $t$ , the allocation of processing step  $k$  of wafer type  $j$  to tool type  $i$  in period  $t$ , and the production of wafer type  $j$  in period  $t$ , respectively. In addition to  $a_{ijk}$ , let us also consider the problem parameters  $\alpha_{it}$ ,  $\beta_{jt}$ ,  $c_i$ , and  $d_{jt}$  corresponding to the (discounted) cost of tool type  $i$  in period  $t$ , the penalty cost of unit shortage in wafer type  $j$  in period  $t$ , the per-period capacity (in hours) of one tool of type  $i$ , and the per-period demand (in wafer starts) of wafer type  $j$  in period  $t$ , respectively.

With the above notation, an optimization model for multi-period (deterministic) scheduling of tool purchases and the allocation of tool capacity to production so as to minimize total tool purchase costs and shortage penalties can be stated as follows:

$$\begin{aligned}
\min \quad & \sum_{t=1}^T \left[ \sum_{i=1}^M \alpha_{it} x_{it} + \sum_{j=1}^N \beta_{jt} u_{jt} \right] \\
\text{s.t.} \quad & \sum_{j=1}^N \sum_{k=1}^K a_{ijk} v_{ijkt} \leq c_i \left( \sum_{\tau=1}^t x_{i\tau} \right) \quad \forall i, t \\
& \sum_{i=1}^M v_{ijkt} \geq w_{jt} \quad \forall j, k, t \\
& w_{jt} + u_{jt} \geq d_{jt} \quad \forall j, t \\
& u_{jt}, v_{ijkt}, w_{jt} \in \mathbb{R}_+ \quad \forall i, j, k, t \\
& x_{it} \in \mathbb{Z}_+ \quad \forall i, t.
\end{aligned}$$

For any period  $t$ , the first constraint in the above model assures that the total processing requirement (in hrs) allocated to tool  $i$  cannot exceed the installed capacity; the second constraint enforces that the actual production of wafer type  $j$  is equal to the number of wafers that has completed all of the required  $K$  processing steps; the third constraint enforces that the production and shortage together should exceed the demand; the fourth constraint enforces non-negativity of the production-allocation-shortage variables; and the fifth constraint enforces the integrality of the tool purchase decisions.

The above formulation can be compactly represented as

$$\begin{aligned}
\min \quad & \sum_{t=1}^T (\alpha_t x_t + \beta_t y_t) \\
\text{s.t.} \quad & Ay_t \leq c \odot \left( \sum_{\tau=1}^t x_\tau \right) \quad \forall t \\
& By_t \geq 0 \quad \forall t \\
& Dy_t \geq d_t \quad \forall t \\
& y_t \in \mathbb{R}_+^{2N+MNK} \quad \forall t \\
& x_t \in \mathbb{Z}_+^M \quad \forall t,
\end{aligned}$$

where  $x_t$  and  $y_t$  are the tool purchase and production-allocation-shortage decision vectors for period  $t$ , respectively;  $\alpha_t$ ,  $\beta_t$ , and  $d_t$  are the cost, shortage penalty, and demand vectors for period  $t$ , respectively;  $c$  is the vector of unit capacities of the various tools;  $A$ ,  $B$  and  $D$  are appropriate matrices; and for any two vectors  $a \in \mathbb{R}^M$  and  $b \in \mathbb{R}^M$ , the operation  $a \odot b$  denotes the component-wise product  $(a_1 b_1, \dots, a_M b_M)$ . Note that we have eliminated transposes for notational brevity.

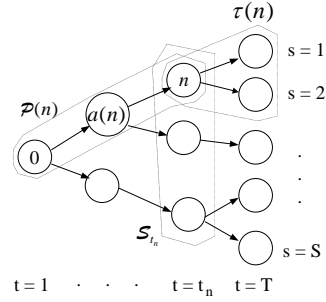


Figure 1: The Scenario Tree Notation

## Two-Stage and Multi-Stage Stochastic Models

Let us now extend the above deterministic tool planning model to a setting where the tool costs, shortage penalties, and product demands are stochastic. We assume that the uncertain problem parameters  $(\alpha_t, \beta_t, d_t)$  evolve as discrete time stochastic processes with a finite probability space. This information structure can be interpreted as a scenario tree where the nodes in stage (or level)  $t$  of the tree constitute the states of the world that can be distinguished by information available up to time stage  $t$ . Each node  $n$  of the scenario tree, except the root ( $n = 0$ ), has a unique parent  $a(n)$ , and each non-terminal node  $n$  is the root of a sub-tree  $\mathcal{T}(n)$ . Thus,  $\mathcal{T}(0)$  denotes the entire tree. The probability associated with the state of the world in node  $n$  is  $p_n$ . The set  $\mathcal{S}_t$  denotes the nodes corresponding to time stage  $t$ , and  $t_n$  is the time stage corresponding to node  $n$ . The path from the root node to a node  $n$  will be denoted by  $\mathcal{P}(n)$ . If  $n$  is a terminal (leaf) node then  $\mathcal{P}(n)$  corresponds to a *scenario*, and represents a joint realization of the problems parameters over all periods  $1, \dots, T$ . There are  $S$  leaf nodes corresponding to  $S$  scenarios. The notation just described is illustrated in Figure 1.

Let us first consider a two-stage model where the first-stage involves deciding the tool purchase plan for all subsequent periods, regardless of the state of the world, and the second-stage consists of deciding on the production plan subject to available capacity and the realized state. Thus the tool purchase variables are only indexed by the time periods (since these do not change with the realized state) while the production-allocation-shortage decisions are indexed by the nodes of the scenario tree. With an objective of minimizing the *expected* total costs, a two-stage stochastic programming formulation for semiconductor tool planning can be formulated as follows:

$$\begin{aligned}
(TS) : \quad & \min \sum_{t=1}^T \bar{\alpha}_t x_t + \sum_{n \in \mathcal{T}(0)} p_n \beta_n y_n \\
\text{s.t.} \quad & Ay_n \leq c \odot \left( \sum_{\tau=1}^{t_n} x_\tau \right) \quad \forall n \in \mathcal{T}(0) \\
& By_n \geq 0 \quad \forall n \in \mathcal{T}(0) \\
& Dy_n \geq d_n \quad \forall n \in \mathcal{T}(0) \\
& y_n \in \mathbb{R}_+^{2N+MNK} \quad \forall n \in \mathcal{T}(0) \\
& x_t \in \mathbb{Z}_+^M \quad \forall t,
\end{aligned}$$

where  $\bar{\alpha}_t = \sum_{n \in \mathcal{S}_t} p_n \alpha_n$ , i.e., the average tool purchase cost in period  $t$ . The stochastic programming model considered by Swaminathan (2000) is a special case of the model (TS)

when the number of periods  $T = 2$ . The model presented by Barahona et al. (2001) is identical to  $(TS)$ , however, there the uncertain parameters are defined over scenarios (paths in the scenario tree) rather than nodes of the scenario tree.

As mentioned earlier, the two-stage model does not allow any flexibility in the tool purchase plan with respect to the realized state of the world. To formulate a multi-stage stochastic programming model, we need to have the tool purchase decisions to be dependent on the realized state, and hence the resulting model can be stated as follows:

$$\begin{aligned}
(MS) : \quad & \min \sum_{n \in \mathcal{T}(0)} p_n \left( \alpha_n x_n + \beta_n y_n \right) \\
\text{s.t.} \quad & Ay_n \leq c \odot \left( \sum_{m \in \mathcal{P}(n)} x_m \right) \quad \forall n \in \mathcal{T}(0) \\
& By_n \geq 0 \quad \forall n \in \mathcal{T}(0) \\
& Dy_n \geq d_n \quad \forall n \in \mathcal{T}(0) \\
& y_n \in \mathbb{R}_+^{2N+MNK} \quad \forall n \in \mathcal{T}(0) \\
& x_n \in \mathbb{Z}_+^M \quad \forall n \in \mathcal{T}(0).
\end{aligned}$$

In the next section, we compare the two-stage and multi-stage stochastic programming formulations.

## THE VALUE OF MULTI-STAGE STOCHASTIC PROGRAMMING

Let us denote by  $v_{TS}^*$  and  $v_{MS}^*$  the optimal objective values of the two-stage  $(TS)$  and multi-stage  $(MS)$  formulations, respectively. For a given set of problem parameters, it can be easily verified that any solution to  $(TS)$  is feasible to  $(MS)$ , and the objective function values corresponding to this solution are equal in both problems. We thus have the following result:

**Proposition 1:** *For a given set of problem parameters*

$$v_{MS}^* \leq v_{TS}^*.$$

That is, the overall cost of the multi-stage solution is smaller than that of the two-stage solution. This should come as no surprise, since, the multi-stage solution offers more flexibility in the tool purchase decisions with respect to the uncertain states of the world. We refer to the difference between the optimal objective values of the two-stage and multi-stage formulations as the *value of multi-stage stochastic programming* (VMS):

$$\text{VMS} = v_{TS}^* - v_{MS}^*.$$

Unfortunately, the value of multi-stage stochastic programming comes at the expense of solving a much larger and difficult optimization model. Both  $(TS)$  and  $(MS)$  are stochastic integer programs, and in general, can be extremely difficult to solve (cf. van der Vlerk and Haneveld 1999). For our particular case, both models have the property that by fixing the tool purchase decisions (the  $x$  variables), we can break the problem down to independent production planning problems (in the  $y$  variables) corresponding to each node of the scenario tree. Owing to this structure, Benders decomposition algorithm (Benders 1962) is particularly attractive for these problems. Benders decomposition proceeds by first solving a reduced *master* problem in the  $x$ -variables. The master problem provides a lower bound to the optimal objective value. With the  $x$ -variables fixed, a sequence of *subproblems* in the  $y$ -variables are then solved. The cumulative objective values of the subproblems provide an upper bound to the optimal value. If the bounds have not converged, the current  $x$  solution is not optimal, and a set of constraints (cuts) are added

to the master problem to cut off this solution. The master problem is then re-solved to generate a better  $x$  solution, and the method iterates. The Benders decomposition algorithm for  $(TS)$  and  $(MS)$  would require us to solve integer master problems. While  $(TS)$  involves  $MT$  integer variables,  $(MS)$  involves  $M|\mathcal{T}(0)|$  integer variables, and for any non-trivial scenario tree  $|\mathcal{T}(0)| \gg T$ . Thus, for most practical problems, even if we are able to solve the two-stage stochastic integer program  $(TS)$  to optimality, there is little hope of solving the multi-stage stochastic integer program  $(MS)$  exactly. Consequently, we have to be content with an approximate solution. The question then remains whether this approximate solution to the multi-stage model is preferable to an exact optimal solution to the two-stage model. Next, we outline a particular approximation strategy for  $(MS)$ , and demonstrate that the scheme produces solutions that are superior to optimal solutions to the two-stage model.

## AN APPROXIMATION SCHEME

Our approximation scheme is based upon constructing an integer feasible solution to  $(MS)$  from a solution to its LP-relaxation. The scheme can be formally stated as follows:

1. Solve the LP relaxation of  $(MS)$ , and let  $(x_n^{LP}, y_n^{LP})$  be its optimal solution and  $v_{MS}^{LP}$  be the optimal objective value. If  $x_n^{LP}$  is integral stop.
2. For each tool  $i = 1, \dots, M$ , solve the following tool planning subproblem:

$$\begin{aligned}
(TP_i) : \quad & \min \sum_{n \in \mathcal{T}(0)} p_n \alpha_{in} x_{in} \\
\text{s.t.} \quad & A_i y_n^{LP} \leq c_i \left( \sum_{m \in \mathcal{P}(n)} x_{im} \right) \quad \forall n \in \mathcal{T}(0) \\
& x_{in} \in \mathbb{Z}_+ \quad \forall n \in \mathcal{T}(0),
\end{aligned}$$

and let  $x_{in}^H$  be the corresponding optimal solution.

3. For each  $n \in \mathcal{T}(0)$ , solve the following production planning subproblem:

$$\begin{aligned}
(PP_n) : \quad & \min p_n \beta_n y_n \\
\text{s.t.} \quad & Ay_n \leq c \odot \left( \sum_{m \in \mathcal{P}(n)} x_m^H \right) \\
& By_n \geq 0 \\
& Dy_n \geq d_n \\
& y_n \in \mathbb{R}_+^{2N+MNK}
\end{aligned}$$

and let  $y_n^H$  be the optimal solution.

The above procedure returns a solution  $(x_n^H, y_n^H)$  to  $(MS)$ . By construction, it is immediate that

**Proposition 2:** *The solution  $(x_n^H, y_n^H)$  is feasible to  $(MS)$ .*

Note that the objective value corresponding to the above approximate solution is

$$v_{MS}^H = \sum_{n \in \mathcal{T}(0)} p_n \left( \alpha_n x_n^H + \beta_n y_n^H \right).$$

The first step in our approximation scheme requires the solution of the LP relaxation of  $(MS)$ . This problem is multi-stage stochastic linear program and can be solved by the Nested L-Shaped Decomposition algorithm (Birge 1985). However, as mentioned earlier, the structure of  $(MS)$  allows for the much simpler Benders decomposition scheme. Furthermore, the Benders decomposition scheme will not affect the structure of the

production planning subproblems in each node of the scenario tree, while Nested L-shaped decomposition, by adding cuts to each stage of the tree, can affect this structure.

Step 2 of the proposed scheme requires the solution of the tool planning subproblem ( $TP_i$ ) for each tool of type  $i = 1, \dots, M$ . Since the variables are required to be integers, this problem can be restated as

$$\begin{aligned} (\overline{TP}_i): \quad & \min \sum_{n \in \mathcal{T}(0)} p_n \alpha_{in} x_{in} \\ \text{s.t.} \quad & \sum_{m \in \mathcal{P}(n)} x_{im} \geq \left\lceil \frac{A_i y_n^{LP}}{c_i} \right\rceil \quad \forall n \in \mathcal{T}(0) \\ & x_{in} \in \mathbb{Z}_+ \quad \forall n \in \mathcal{T}(0). \end{aligned}$$

The above problem is a multi-stage stochastic integer program and may appear difficult to solve. Fortunately, we have the following results (Ahmed and Takriti 2002):

**Proposition 3:** *The linear programming relaxation of  $(\overline{TP}_i)$  has an integer optimal solution.*

**Proposition 4:** *The linear programming relaxation of  $(\overline{TP}_i)$  can be solved in  $\mathcal{O}(|\mathcal{T}(0)|^2)$  operations.*

By virtue of the above results, for a given production plan  $y_n^{LP}$ , the subproblems ( $TP_i$ ) can be solved extremely efficiently to generate integer tool purchase decisions  $x_n^H$ .

The final step of the proposed scheme is to solve the production planning subproblems corresponding the tool purchase plan  $x_n^H$ . This involves solving  $|\mathcal{T}(0)|$  independent linear programs which are very similar to each other. By taking advantage of warm-start strategies, these problems can be solved quite efficiently.

## NUMERICAL RESULTS

In this section, we report on some preliminary numerical experiments with the proposed multi-stage approach for semiconductor tool planning under uncertainty.

As our test problem, we consider the small wafer fab example described in Stafford (1997). This problem involves 27 machine tools, 10 products, and 7 processing steps. We use the same process routing, tool cost parameters, and shortage penalty parameters as in Stafford (1997). The product demands are assumed to be stochastic with independent log-normal distributions. To generate a stochastic problem instance, we construct a scenario tree with a specified number of stages ( $T$ ) and number of branches per stage ( $K$ ) using conditional sampling of the demand parameters (Shapiro 2002). In our implementation of the proposed solution strategy, we used CPLEX 7.0 to solve the linear programming subproblems. All computations were carried out on a Sun Ultrasparc Workstation with 4 GB of RAM and a 480 MHz processor.

The proposed approximate multi-stage approach can be compared to the standard two-stage paradigm by examining the difference between the objective values  $v_{TS}^* - v_{MS}^H$ . However, computing the above quantity requires the solution of the two-stage stochastic integer program ( $TS$ ) to optimality. We avoid this expensive computation by estimating a lower bound on the objective value difference using  $v_{TS}^{LP} - v_{MS}^H$ . Figures 2 and 3 plot the value of the above lower bound with respect to increasing number of branches per stage ( $K$ ), and with increasing number of stages or time periods ( $T$ ). Except for the case  $T = 2$  and  $K = 2$ , the lower bound is strictly positive and quite large. It can also be observed, that the lower bound  $v_{TS}^{LP} - v_{MS}^H$  increases sharply as the number of branches per stage or the number of stages is increased. These results imply

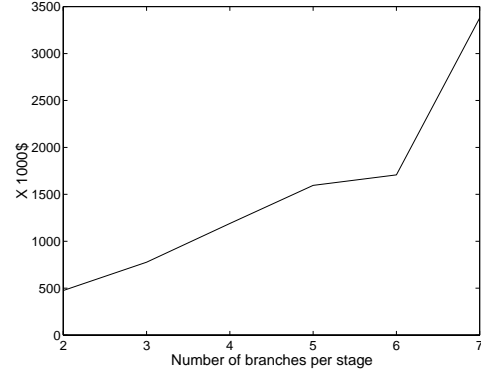


Figure 2:  $v_{TS}^{LP} - v_{MS}^H$  vs.  $K$  ( $T = 3$ ).

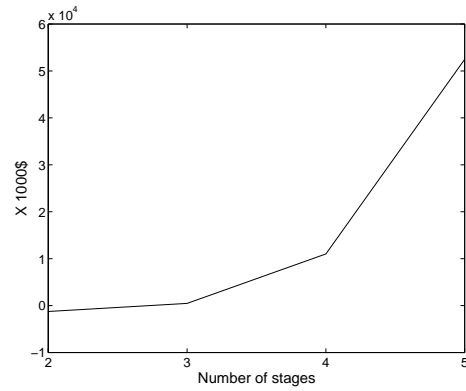


Figure 3:  $v_{TS}^{LP} - v_{MS}^H$  vs.  $T$  ( $K = 2$ ).

that an approximate solution to the multi-stage model is far superior to any optimal solution to the two-stage model. It is easily seen that  $VMS \geq v_{TS}^{LP} - v_{MS}^H$ , thus Figures 2 and 3 imply that the value in multistage modeling is quite significant. Furthermore, the VMS is greatly accentuated as the number of stages grow.

The relative optimality gap of an approximate solution to the multi-stage problem is given by  $\frac{v_{MS}^H - v_{MS}^*}{v_{MSH}^*} \times 100\%$ . An upper bound on this optimality gap can be computed by

$$\text{gap} \leq \frac{v_{MS}^H - v_{MS}^{LP}}{v_{MSH}^{LP}} \times 100\%.$$

In Figures 4 and 5, we plot the above upper bound on the optimality gap of the heuristic with respect to the growth in number of branches per stage, and the growth in the number of stages. For a fixed number of stages, the optimality gap is almost constant as the number of branches per stage is increased. However, for a fixed number of branches per stage, the optimality gap decreases sharply as the number of stages is increased. This observation conforms to the results in Ahmed and Sahinidis (2001) who proposed a LP-relaxation based heuristic for a different class of stochastic capacity expansion problems, and proved that the optimality gap of the heuristic solution asymptotically vanishes as the number of planning periods is increased.

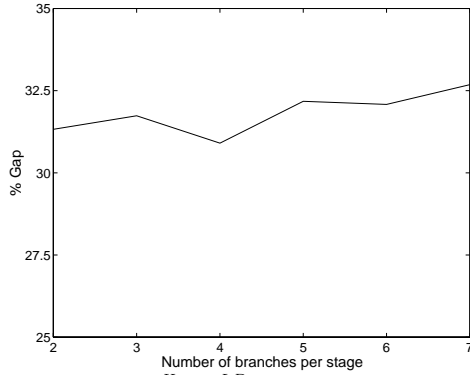


Figure 4:  $\frac{v_{MS}^H - v_{MS}^{LP}}{v_{MS}^{LP}}$  % vs.  $K$  ( $T = 3$ ).

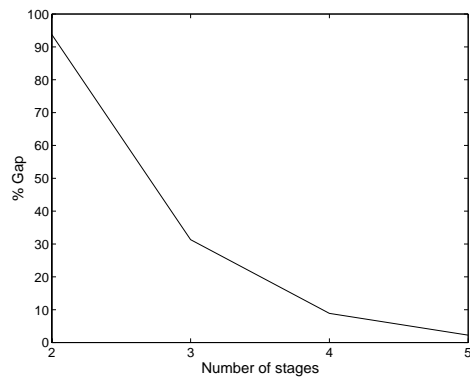


Figure 5:  $\frac{v_{MS}^H - v_{MS}^{LP}}{v_{MS}^{LP}}$  % vs.  $T$  ( $K = 2$ ).

## CONCLUDING REMARKS

In this paper, we have developed a multi-stage stochastic programming model for semiconductor tool planning under uncertainty. In contrast to earlier two-stage stochastic programming models for this problem, the proposed model allows for revision of the tool purchase plan as more information regarding the uncertain problem parameters becomes available. The multi-stage model is guaranteed to produce a solution with smaller overall cost than the two-stage model. However, the proposed model is a multi-stage stochastic integer program, and is extremely difficult to solve. We propose a LP-relaxation based approximation scheme for this problem. Our preliminary numerical results indicate even an approximate solution to the multi-stage model is far superior to any optimal solution to the two-stage model. These results confirm that the *value of multi-stage stochastic programming* for this class of problems is extremely high.

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