Optimization of Automated Float Glass Lines

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Statement of Scope and Purpose

Flat glass is approximately a $20 billion/year industry worldwide, with almost all flat glass products being manufactured on float glass lines. New technologies are allowing float glass manufacturers to increase the level of automation in their plants, but the question of how to effectively use the automation has given rise to a new and difficult class of optimization problems not yet studied in the literature. These optimization problems combine aspects of traditional cutting problems and traditional scheduling and sequencing problems, the hybrid of which has not previously been modeled, let alone studied or solved. In this paper, we define and model the problem, and provide heuristic solution methods that produce manufacturing yields greater than 99%. These high-yield solutions can save millions of dollars, so our methods are currently being implemented at a major float glass manufacturer.

Abstract

In automated float glass manufacturing, a continuous ribbon of glass is cut according to customer orders and then offloaded using robots. We consider the problem of laying out and sequencing the orders on the ribbon so as to minimize waste. The problem combines characteristics of cutting, packing, and sequencing problems. Given the size and the complexity of the problem, we propose a two-phase heuristic approach where the layout and sequencing aspects of the problem are addressed independently. On a set of real-world and random instances, our approach achieves schedules with over 99% yield (ratio of glass input and output). This is a significant improvement for the manufacturer where this work has been implemented.

Key words: Float Line, Glass, Automation, Cutting, Scheduling, Heuristics

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1 Introduction

Flat glass manufacturing is a continuous process whereby a ribbon of molten glass is produced in a furnace and then cooled on a bath of molten tin to ensure flatness. The continuous glass ribbon is then carried on rollers through an annealing lehr, machine-cut according to customer size requirements, and offloaded for distribution. The equipment in this process, beginning with the furnace and ending with the offloading equipment, constitutes a float line (see Figure 1).

Assuming that any glass cut could easily be offloaded, the problem of fitting all of the customers' orders into the smallest-possible piece of ribbon is very similar to traditional 2-dimensional cutting problems. In this case, the only way glass is wasted (other than breakage) is in the process of laying out orders on the ribbon. This wasted glass, or scrap, is known as *layout scrap*.

This paper considers a float line in a glass plant that has recently fully automated their previously-manual offloading process. There are clear safety and cost advantages to performing offloading with robots rather than humans, but the automation is more restricted in the amount of glass it can offload per unit time. As a result, some additional glass might be wasted if it is cut but cannot be picked up before it gets to the end of the conveyor. This additional scrap is known as *cycle time scrap* because it is caused by the cycle time (minimum time between pickups) of the picking robots. Of course, cycle time scrap could be eliminated by purchasing more automated equipment, but each robot costs millions of dollars. It is preferable to optimize the sequence and layout of the glass being cut so that the total amount lost to cycle time scrap and layout scrap is small.

Although the literature addresses individual aspects of the float glass problem mentioned above, we are unaware of any model in the literature that deals with the full complexity of the problem. The float glass problem resembles a guillotine version of the two-dimensional cutting-stock problem (2D CSP) studied by Gilmore and Gomory [2, 3]. However, the 2D CSP addresses only layout scrap. The float glass problem has the added complexity of cycle time scrap caused by the relation between the cutting and offloading operations. Another related problem is two-stage hybrid flow shop (HFS) scheduling with no intermediate storage and identical parallel machines in the second stage. See Linn and Zhang [6] for general HFS scheduling, and see Gupta and Tunc [4] for the HFS with parallel machines at the second stage. Also, see Pinedo [8] for the flow shop with limited intermediate storage. The cutting and offloading operations in the float glass problem make up the two stages of a flow shop, and minimizing cycle time scrap is equivalent to scheduling to minimize processing time. However HFS scheduling does not address layout scrap. Dash et al. [1] studied the problem of producing rectangular plates for a steel company to minimize scrap, but do not consider the sequencing issue. Thus, none of these earlier works adequately captures the full complexity of the float glass problem.

In [7], we show that the float glass problem is NP-hard in general, and study special cases that can be solved in polynomial time. We also introduce a mixed-integer programming (MIP) formulation for it. However, because of the MIP's size and difficulty, a state-of-the art commercial solver such as CPLEX 11 is unable to find good solutions within a reasonable amount of time. Therefore, we present a heuristic solution approach to solving the float glass problem. We will empirically show that the proposed heuristic solution approach produces nearly-optimal solutions for a collection of randomly generated and real-world test problems.

In Section 2, we introduce the relevant characteristics of a float line, and describe the sequencing and layout optimization problem. In Section 3, we present heuristic algorithms to solve the problem. Computational results are provided in Section 4.

2 Problem Description

2.1 Basic Terminology

Our problem concerns processing a given set of customer orders (usually about 60) over a 24 hour working shift. Each customer order consists of a specified number of identical pieces of glass (*plates*) of specified dimension (length $\times$ width $\times$ thickness). For example, a customer might order 200 plates with dimensions 20 inches by 40 inches by 1 inch. Because it is difficult and costly to change the thickness of the ribbon, each production shift is typically composed of orders of the same thickness.

Plates are created by a two-step cutting process in which the ribbon of glass is first scored (etched where plates will be divided), and then snapped along the scores. The *x-cuts* stretch across the width of the ribbon perpendicular to its direction of flow, and the *y-cuts* are at right angles to the x-cuts and stretch between two consecutive x-cuts.

The glass between two consecutive x-cuts is referred to a *snap*, and the *snap time* is the time between the two x-cuts. (Because glass of a given thickness and width is produced at a constant rate, snap time is proportional to the length of glass between the two x-cuts.) Between two consecutive x-cuts, the y-cuts divide a snap into two or more plates. Figure 2 illustrates the terminology of a float glass line. Layout scrap and cycle time scrap are also shown in the figure. Recall that layout scrap is caused by not using the ribbon width optimally when laying out the plates, and cycle time scrap is caused by improper sequencing of snaps on the ribbon.

After being cut, the plates are offloaded into storage containers by offloading machines. There are several types of offloading machines. This paper deals with two common types: *high-speed-stacker* (HSS) and *pick-on-the-fly* (POF) machines. HSS machines simultaneously pick up all the plates from one snap of the ribbon. The requirement for using an HSS machine is that all plates in the snap have the same size and that each plate's size must not be too large. On the other hand, each POF machine can pick up large plates, but can pick up only one plate at a time, so a snap with m plates requires m POF machines to be simultaneously available for picking. Figure 3 shows photos of HSS and POF machines.

2.2 Constraints, Objective, and Decisions

Because of the limitations of the glass-making process and the equipment involved, there are a number of operational restrictions imposed by the manufacturer.

The first set of restrictions deals with the width and usage of the ribbon.

Continuous Production Production of glass on the float line is continuous. Therefore, even if no offloading machine is available to pick glass, the glass will still be produced (and therefore wasted as scrap).

Thickness Grouping Because the thickness of the ribbon is the most difficult attribute to change, many orders of the same thickness are run in one shift before the thickness is changed. These shifts often take multiple days to complete, and the focus of this paper is on scheduling within these shifts.

Monotone Ribbon Width Every change in ribbon width incurs scrap while the line is being re-adjusted. Therefore, the manufacturer has specified that the ribbon width may only be varied in a monotone way (either non-increasing or non-decreasing) during a shift of a fixed thickness.

Single-Order Snaps Although in related cutting-stock problems a snap sometimes contains different-sized objects in order to make best use of the ribbon, in glass manufacturing all of the plates in a snap must come from the same order. Thus, a solution may not contain snaps that contain plates from multiple orders.

There are also some restrictions related to the offloading machines.

Machine Dedication A float line has a limited number of offloading machines, and each offloading machine can deal with only one container of glass at a time. Since each container stores only one order's glass, an offloading machine cannot pick any glass from the next order before all snaps of the previous order are completed.

Machine Cycle Time Each offloading machine has a required cycle time (the time it takes to pick a plate, put it into the container, and return to the ready position). The cycle time of offloading machines can vary slightly by machine type and according to the size of plates being picked up. However, the difference is negligible in practice.

Within a shift, a set of customer orders is given, with each order specifying the size of plates and the number of plates to be produced. The measure of a schedule is the amount of glass required to produce all of the given set of orders. We quantify this measure by defining the *yield ratio* :

$$\text{yield ratio} \quad = \quad 1 - \frac{(\text{cycle time scrap}) + (\text{layout scrap})}{(\text{total glass required for the shift})}.$$

The objective of the float glass problem is to maximize the yield ratio subject to the restrictions already mentioned. To produce a schedule that maximizes yield, we need to determine:
(1) the construction of snaps by laying out the plates in the given orders,
(2) the sequence in which the snaps are processed (cut and offloaded),
(3) the assignment of plates and/or snaps to offloading machines.

Given the size of real-world instances of the float glass problem, our approach is to decompose the solution procedure into two phases. In phase one, we determine the construction of snaps (or layout of plates), and in phase two we make the remaining decisions, sequencing and assignment, together to create a production and offloading schedule.

2.3 Schedule Structure

Two of the float glass line's operational restrictions, continuous production and machine dedication, provide additional structure to the set of feasible solutions. In this section, we explain how this structure can be used to represent the solution space in a way that motivates our sequencing and assignment algorithm.

First, we note that the cutting of snaps requires much less time than the offloading machines' cycle times. (Most snaps are cut in 2.5-5 seconds, compared to machine cycle time of about 13 seconds.) Because glass flows through the system continuously, a schedule that cuts one snap of an order after another without any intervening snaps will incur significant cycle time scrap. For example, if machine cycle times are 10 seconds and a snap requires 3 seconds of glass, then cutting this order's snaps consecutively would require 7 seconds of wasted glass between snaps, i.e. the sequence on the ribbon would be: snap (3 seconds), scrap (7 seconds, until offloading machine is ready), snap (3 seconds), scrap (7 seconds), etc. (see Figure 4 (a)). Therefore, good schedules will require snaps of other orders between two snaps of the same order, to either decrease or eliminate cycle time scrap (see Figures 4 (b) and (c)).

Because offloading machines must finish loading the container(s) of one order before starting the next, an order may not be preempted. Therefore, since snaps from different orders are interspersed with each other, rotating through the orders' snaps until one of the orders finishes (as shown in Figures 4 (b) and (c)) is necessary to minimize cycle time scrap. We refer to such a set of orders being simultaneously processed in rotation as a *covey.*

A schedule can be defined to be a sequence of coveys where between two consecutive coveys, either one or more orders have finished and/or one or more orders have started. The optimization problem thus consists of determining a sequence of coveys that produce all orders such that the yield ratio is maximized.

Table 1 introduces an example set of five orders. For this example, we assume that two HSS machines and two POF machines are available and that their cycle time is 10 seconds. In this example, we define three coveys. The first covey (Covey P) consists of Orders A, B, and C. (This set of orders uses all four offloading machines, since Order C has two plates per snap and thus requires both POF machines.) After 100 snaps of this covey, Order C is completed and is replaced by Order D. Thus, Orders A, B, and D define the second covey (Covey Q). After 50 snaps of Covey Q, Order A is completed (a total of 150 snaps) and is replaced by Order E. Orders E, B, and D thus define the third covey (Covey R). The sequence of snaps produced by the cutter and its relation to the coveys are described in Figure 5. The Gantt chart in Figure 6 illustrates this schedule for each machine. The top row in the figure shows the schedule for the cutters, and the other four rows show the schedule for each offloading machine.

Another useful representation of coveys is a snap-based view illustrated in Figure 7. This view specifies the sequence of orders, their assignment to offloading machines, and the number of snaps to be produced of each order and of each covey. In the example, Figure 7 shows that machine HSS1 first offloads 150 snaps of Order A and then offloads 80 snaps of Order E, HSS2 offloads 200 snaps of Order B and then remains idle, POF1 offloads 100 snaps of Order C and 100 snaps of Order D (and is then idle), POF2 offloads 100 snaps of Order C and is then idle for the rest of the time. In this view, it is easy to see the change of coveys; Covey P runs for 100 snaps, followed by 50 snaps of Covey Q and 50 snaps of Covey R (and then 30 snaps of a fourth covey consisting only of Order E).

For each covey, it is easy to calculate the required ribbon width. For ribbons with non-decreasing (non-increasing) width, the ribbon width for a covey is the maximum width of a snap in that covey or any preceding (subsequent) covey.

The volume of glass sent through the system per unit time is constant, as is the length and thickness of a snap. Therefore, for a snap of given length and thickness, the time required to cut a snap is a function of ribbon width. Consider a snap whose dimensions (length l, width w, and thickness t) is being cut on a line that produces V volume of glass per second. Such a snap requires $d = \frac{lwt}{V}$ seconds of glass on a ribbon of width w. However, if the snap is run on a wider ribbon of width $w' > w$, the snap time increases to $d' = \frac{lw't}{V} = d\frac{w'}{w}$. (Note that although this means wider ribbon widths might help reduce cycle time scrap, they simultaneously increase layout scrap, since $l(w - w')t$ volume of glass is wasted.)

Figure 8 gives a ribbon-based view of one cycle through each of the coveys P, Q, and R. It shows the amount of scrap incurred in each covey. In Covey P, all three orders require the same snap width as the ribbon width (132), so this covey has no layout scrap. In addition, the sum of snap times is $3.5 + 3.0 + 4.5 = 11$ seconds, which exceeds the cycle time of the offloading machines. Therefore, Covey P does not have any cycle time scrap. In Covey Q, the snap width of Order D is 130, but the ribbon width must remain 132 because Orders A and B require a ribbon width of at least. Thus, Covey Q incurs layout scrap corresponding to the area of difference between 132 and 130, multiplied by the ribbon's thickness. The sum of snap times is $3.5 + 3.0 + 4.0 = 10.5 > 10$ seconds, so this covey does not have any cycle time scrap. In Covey R, the snap width of Order E is 128 and that of Order D is 130, but the ribbon width is 132. Therefore, Covey R has layout scrap corresponding to these two areas times the ribbon's thickness. The sum of snap times is $2.0 + 3.0 + 4.0 = 9.0$ seconds, which is less than the offloading machines' cycle time of 10 seconds. Thus, Covey R has cycle time scrap corresponding to the volume of glass produced in one second of idle time.

3 Solution Approach

Our heuristic approach to the float glass problem is in two phases:

1. Define the *standard snap* for each order by selecting the number and orientation of plates in the order's snaps.
2. Construct a schedule for cutting and offloading the correct number of each order's standard snap in a way that satisfies the operational restrictions and machine cycle time limitations.

3.1 Snap Construction

The first step in our algorithm is to create a standard snap for each order, so that each order is filled by repeatedly cutting its standard snap. As mentioned earlier, all plates in a snap come from the same order. Thus, for each order we only need to determine the orientation and the number of the plates in each snap. Snap width is determined by these two factors.

There are only a few choices for each order. For example, to produce plates of size of 35×40 on a ribbon with minimum and maximum width 100 and 140, the manufacturer can produce three plates or four plates of dimensions 35×40, or three plates of dimensions 40×35 (see Figure 9).

The preference in snap construction is to have smaller variance in snap widths in order to reduce layout scrap. Thus, we simply choose the rotation of a plate and the number of plates in a snap so that snap width is closest to the average of the maximum and minimum ribbon widths in the shift. Since the snap width of the last alternative is closest to $\frac{100+140}{2}$, we would select it to be the order's standard snap.

3.2 Constructing Cutting and Offloading Schedules

The scheduling phase of our approach has three main steps: preprocessing, construction, and local improvements. The preprocessing steps help to balance the workload among machines. The construction step uses a greedy scheme to construct an initial schedule. Finally, we use a number of local improvement schemes to reduce scrap.

3.2.1 Preprocessing

We have two preprocessing steps.

(i) Splitting Long Orders:

At the end of a schedule, we found that because no other orders remain, a covey often consists of only one or two long orders. Thus, the sum of the snap times of orders in the last coveys is less than the cycle time of the offloading machines, which causes the schedule to have cycle time scrap. We refer to this phenomenon as the *end effect*. The amount of cycle time scrap caused by the end effect often comprises a large portion of a schedule's total scrap.

As a pre-processing step, in order to reduce the size of large orders and hence reduce end effect, we split orders with a large number of snaps into several sub-orders with fewer snaps. We can only apply this splitting rule as long as the order is large enough to require multiple containers so as not to violate the packaging constraint. For example, if an order of 2,000 snaps requires five containers, we can split it into one order of 1,200 snaps (three containers) and one order of 800 snaps (two containers). Figure 10 illustrates how splitting long orders contributes to decreasing the end effect.

(ii) Balancing between HSS and POF machines

If the assignment of orders to offloading machines is not balanced between the HSS and POF machines, one machine type may be busy while the other type is idle, thus increasing the chance of incurring cycle time scrap. Although some orders' characteristics require them to be picked by a certain type of machine, orders which satisfy the size requirements of both HSS and POF machines can be offloaded by either type. For these orders, we decide the assignment of offloading machine type in order to minimize the difference between the total number of snaps offloaded by HSS machines and the total number of snaps offloaded by POF machines.

3.2.2 Construction

We use greedy construction algorithms to obtain initial schedules. Orders are prioritized according to some characteristics (discussed below). By priority, they are then assigned to offloading machines in the following way. If the sum of snap times in the current covey is less than the machine cycle time (which would result in this covey having cycle time scrap), then it is myopically beneficial to add another order to the covey. Thus, the next order in the priority list is assigned to that covey if a machine is available to offload it. Otherwise, if the covey already has no cycle time scrap or if no machine is available, this covey is considered "full" and the algorithm advances to the next covey. The process iterates until all orders are assigned.

Assume that the ribbon width is monotonically non-increasing. Then, wider orders should be produced earlier; otherwise, narrow orders will incur layout scrap. Therefore, we first sort the orders in non-increasing order of width.

It is not uncommon to have many orders of the same width. We have two different ways of breaking ties between such orders, leading to two slightly-different construction heuristics. Both tie-breaking methods are designed to help reduce the end effect.

One tie-breaking method is to produce orders with shorter snap times first, to save longer-snap-time orders for coveys that might be subject to the end effect; in such a case, the cycle time scrap of the end effect would be smaller. We refer to the heuristic using this tie-breaking method as the Width-Then-Time (WTT) method.

A second tie-breaking method is to begin orders with more snaps first, so that these orders are less likely to have long end effects. We refer to the heuristic using this tie-breaking method as the Width-Then-Snaps (WTS) method.

3.2.3 Local Improvement Algorithms

Once an initial schedule has been constructed, we use three local improvement methods, *push-back*, *relocation*, and *exchange*. We first apply push-back to the initial schedule, followed by repeated application of relocation and exchange until there is no more improvement in the yield ratio. Since the push-back algorithm can be applied to improve any schedule, we apply it not only to the initial schedule, but also to all potential relocation and exchange schedules before evaluating their yield ratio.

(i) Push-back of Orders:

The purpose of the push-back step is to eliminate as much of the end effect as possible. We try to push backward (earlier in time) the orders located in the later coveys in order to combine them with orders of the earlier coveys. This can reduce the number of snaps with few orders (and consequently shorter total snap times) and decrease the total cycle time scrap at the end of a shift.

At each iteration, the algorithm pushes backward the orders in the last covey by the number of snaps in that last covey. The push-back algorithm is repeatedly applied to the last covey of a schedule as long as the covey currently incurs cycle time scrap and push-back is feasible.

Figure 11 shows an example of the push-back algorithm. In Figure 11 (a), the last covey of the current schedule consists only of Order 8, and the number of snaps in this covey is δ_1. So, the push-back algorithm moves Order 8 leftward by δ_1 snaps. As a consequence, Order 4 also gets pushed back δ_1 snaps to make room, creating the schedule in Figure 11 (b).

Assume now that sum of snap times of Order 7 and Order 8 is less than the machine cycle time. Then, the schedule in Figure 11 (b) still has an end effect. Thus, we want to push back Orders 7 and 8 by δ_2 snaps. Orders 6 and 4 are also pushed backward to make room, resulting in the schedule shown in Figure 11 (c).

Figure 11 (c) shows that no end effect now exists, since the sum of snap times of Orders 5, 7, and 8 is greater than the machine cycle time. However, even if there still was an end effect, no push-back could be done, since there is no room to push Orders 8 and 4 backward on the first POF machine.

(ii) Relocation of Orders

The relocation algorithm iteratively selects an order and evaluates the benefit of changing its position in the schedule. It tries to relocate the order to the starting point of each covey on each machine of the same type as the order is currently on. For each possible relocation, we run the push-back algorithm (if necessary) and then compute the yield ratio of the updated schedule. If the new yield ratio is higher than the yield ratio before the relocation, then the new schedule is retained; otherwise, we return to the schedule before the relocation. The algorithm cycles through all orders, attempting to relocate each one in turn, until a complete cycle is made with no changes.

Figures 12 (a-1) and (a-2) illustrate one step of the relocation algorithm.

It might happen that when an order is relocated, it has enough snaps that its end overlaps the scheduled start of the next order on the same machine. In such a case, we first apply a *push-forward* operation. This operation might produce an increased end effect. Therefore, we apply the push-back algorithm after each relocation of orders in order to remove as much of this end effect as possible. The push-forward and subsequent push-back steps are illustrated in Figure 13.

(iii) Exchange of Orders

The exchange algorithm iteratively selects two orders and evaluates the benefit of exchanging their positions in the schedule. Similar to relocation, orders are only exchanged between the same type of machine, and the push-back algorithm is run after each potential exchange before calculating its yield ratio and comparing with the yield ratio of the current schedule.

Figures 12 (b-1) and (b-2) illustrate one step of the exchange algorithm.

3.3 An Illustrative Example

In this section we illustrate our solution approach on an example problem instance. In this small example, we assume that three HSS machines and three POF machines are available, all with cycle times of 10 seconds, and that the minimum and maximum ribbon widths are 120 and 140 inches. Table 2 gives the dimensions and number of plates in each order. For most orders, several snap constructions are possible. The selected standard snap configuration for each order (the one whose width is closest to the average of the maximum and minimum ribbon widths in the shift = 130) is denoted by an asterisk in the "selected snap" column in

Table 2 and listed in Table 3.

First, we assign orders to machine types. Because the number of plates per snap for Orders 1, 5, and 9 is no more than three, we can assign those orders to POF machines. The other orders are assigned to HSS machines.

Next, we create a priority list for each construction algorithm. For WTT, the list is $(O4, O2, O3, O1, O6, O5, O7, O8, O9)$, and for WTS the list is $(O3, O4, O1, O2, O6, O5, O9, O8, O7)$. We then apply the construction algorithms to each list of orders. Figure 14 illustrates the schedules after applying the construction algorithms.

The schedules generated by WTT and WTS have end effects that incur significant cycle time scrap. In Figure 15, we would like to apply push-back to the schedule generated by WTS, but there is not enough empty space to push Order 8 backward. However, during the relocation phase of local improvement, Order 8 gets relocated to the third HSS machine and then gets pushed back with Order 9.

Figure 16 shows a similar situation with the schedule generated by WTT. In Figure 16, Order 8 cannot be pushed backward because there is not enough space, i.e. $\delta_2 > \delta_3$. However, when Order 8 and Order 7 are exchanged, iterative push-back can be successfully applied to remove the end effect.

4 Computational Results

We tested our algorithms on actual shift data from the float glass plant we studied, and compared the yield ratio from our solutions to the yield ratio from theirs. We also tested our algorithms on 250 additional instances randomly generated from actual data. For the tests, our algorithms were implemented in C++ and executed on a Linux machine with two 2.4 GHz Processors and 2GB RAM.

We chose parameters to be close to the environment of the float glass plant: we assumed that four HSS and four POF machines are available, that the cycle time of both POF and HSS offloading machines is 13 seconds, and that the ribbon width varies between 120 and 144 inches. The customer order data was exactly the same as the company's, but cannot be shown here due to confidentiality.

Table 4 shows the quality of the solutions for three actual instances of company order data. The three data sets have 36, 40, and 49 orders. After splitting long orders (as described earlier), the number of orders increases to 55, 46, and 57. In all three instances, the heuristics achieve more than 99.2%-99.6% yield in less than 80 seconds, while the yield ratio of the actual shift schedule previously run by the company was

94.8%-97.4%.

For further validation, we conducted an experiment on randomly generated instances, with each order taken from the pool of actual order data to make them realistic. We generated 250 instances, 50 each with 40, 50, 60, 70, and 80 orders. Tables 5 and 6 report heuristic yield ratios broken down by which pieces of the overall solution approach are employed.

As the results shown in Tables 5 and 6 demonstrate, splitting long orders and using improvement heuristics both have significant positive impact on the overall solution quality. Although balancing has only a small effect on average, its improvement seems to be concentrated on instances where the overall solution quality without balancing is worst. For example, in the instance where our heuristics perform worst, balancing improves the yield ratio by over 3%. Thus, it is also an effective part of our overall solution approach.

Although the two construction algorithms WTT and WTS give very similar average solution quality, their results on individual instances can sometimes vary in yield ratio. Therefore, since the algorithms run quickly relative to the required solution time, we recommended to the manufacturer that when possible, they use both algorithms and select whichever of the two schedules has a higher yield ratio. Table 7 shows the results of using this strategy, which improves performance by 0.1%. This strategy has been implemented in the manufacturing facility.

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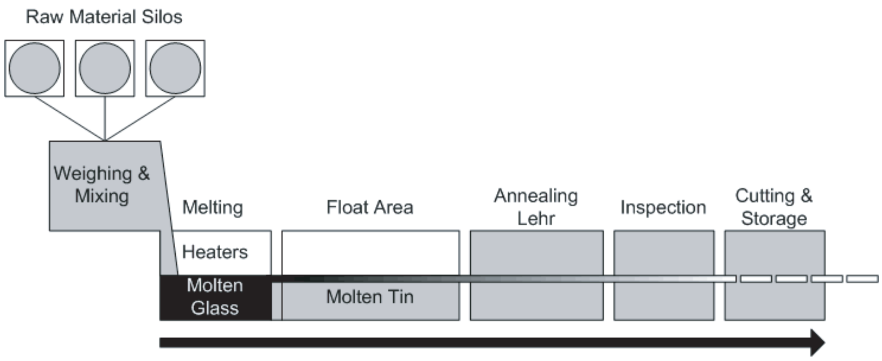


Figure 1: Basic float glass manufacturing process [9].

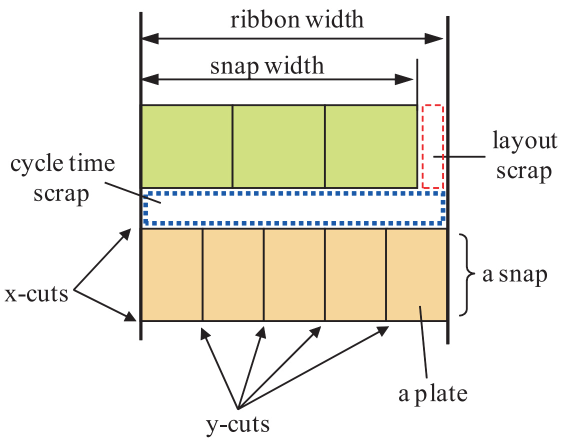
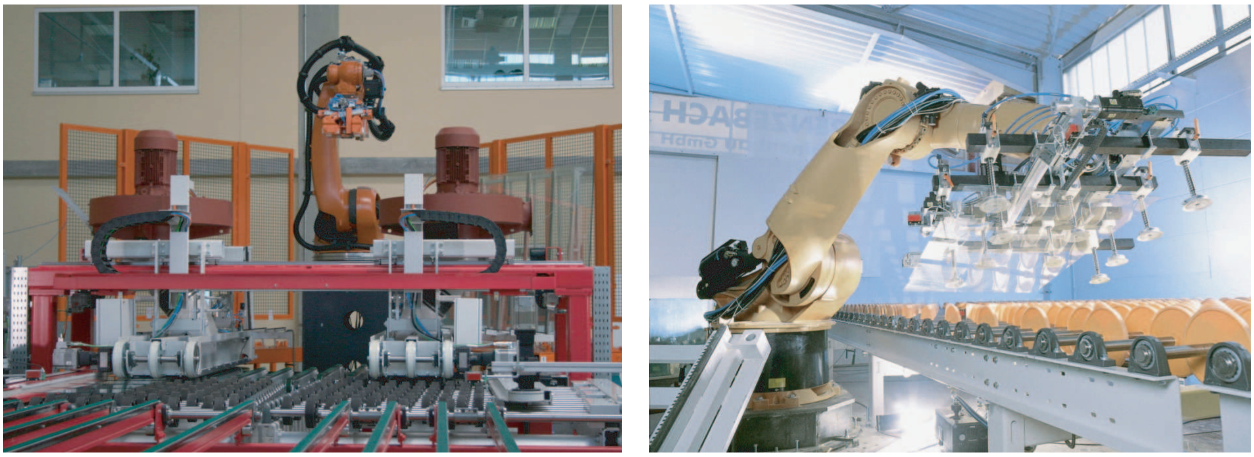
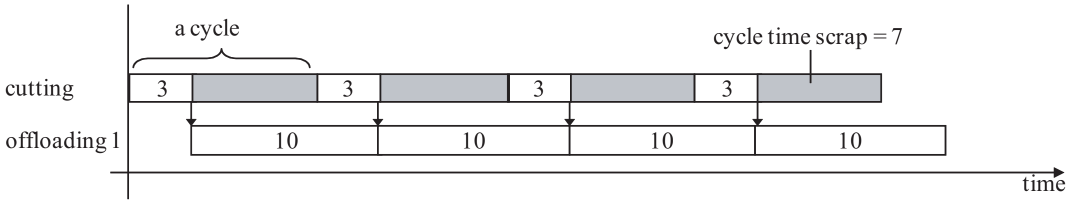


Figure 2: Terminology of a float line.

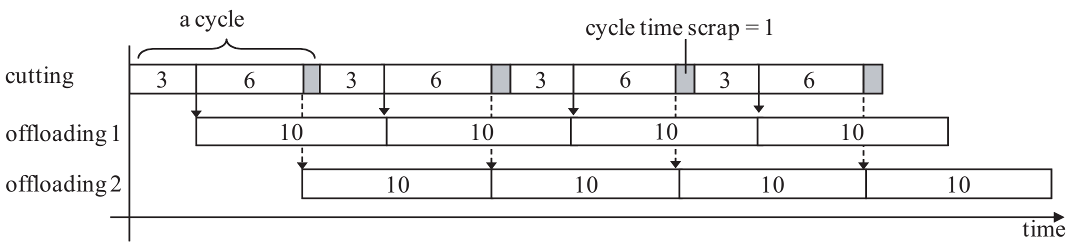


(a) High-speed-stacker (HSS) machine (b) Pick-on-the-fly (POF) machine

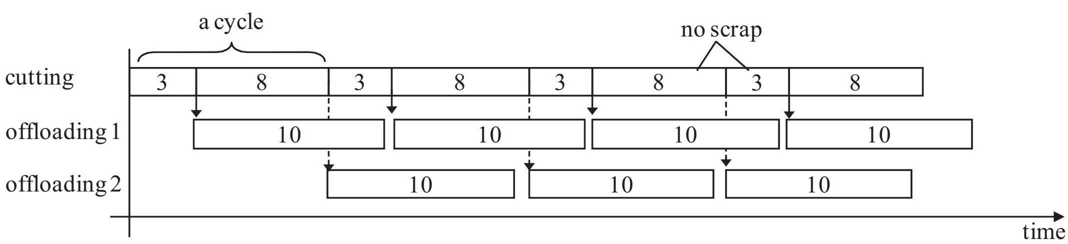
Figure 3: Types of offloading machines [5].



(a) Cutting snaps of a single order consecutively normally incurs significant cycle time scrap.



(b) Interspersing snaps of another order can decrease cycle time scrap.



(c) If interspersed snaps create a cycle at least as long as machines require, there is no cycle time scrap.

Figure 4: Gantt chart showing how cycle time scrap is incurred.

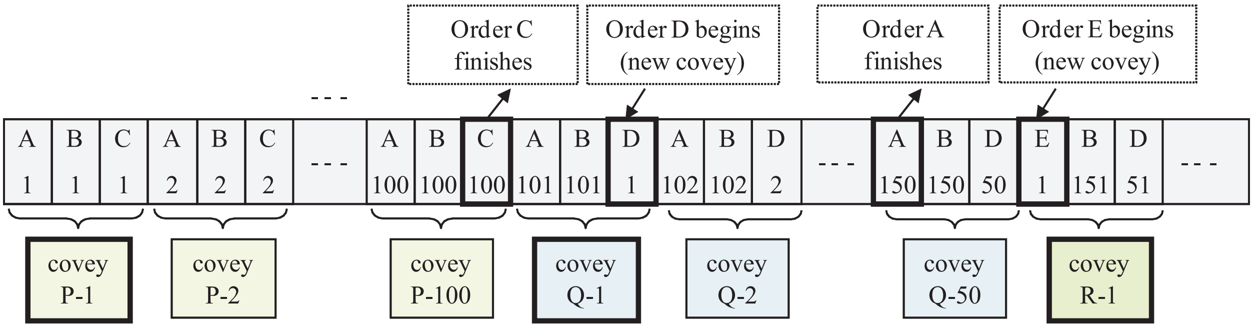


Figure 5: Cutting-based view of coveys.

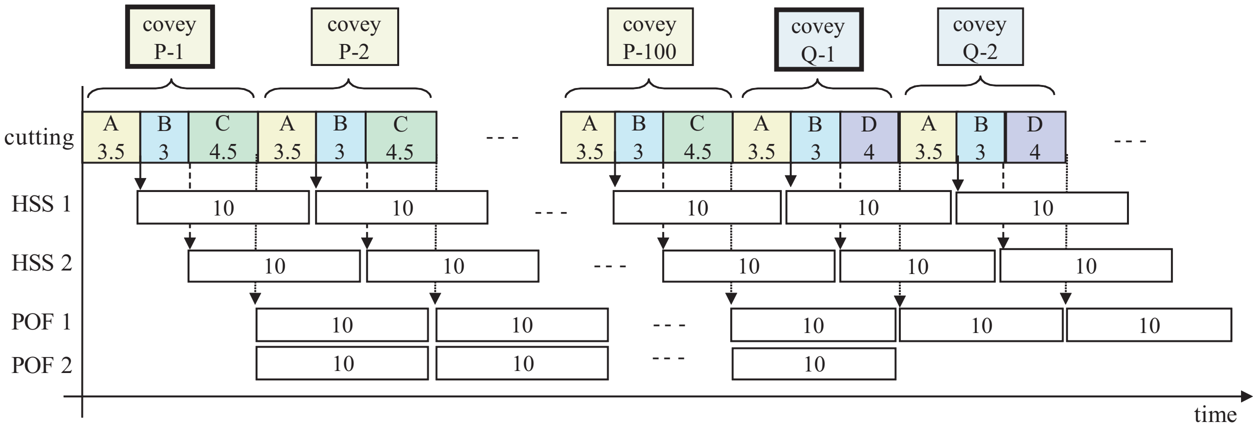


Figure 6: Coveys in Gantt chart.

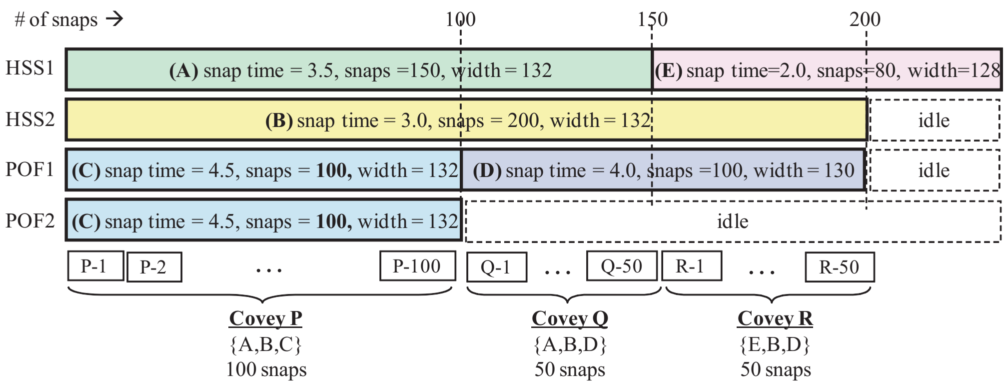
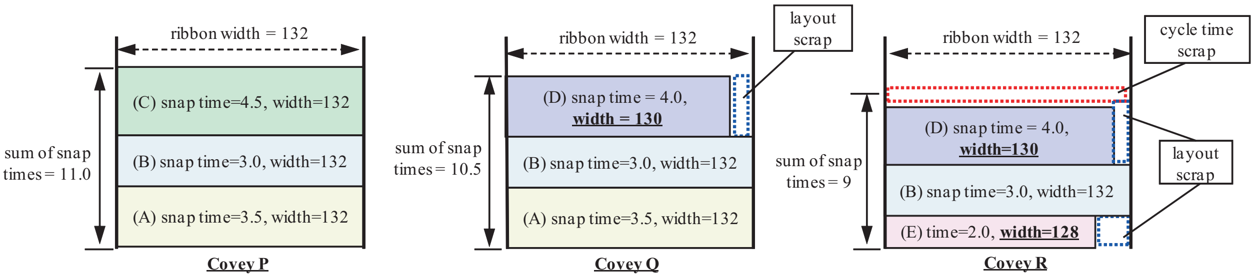


Figure 7: Snap-based view of coveys.

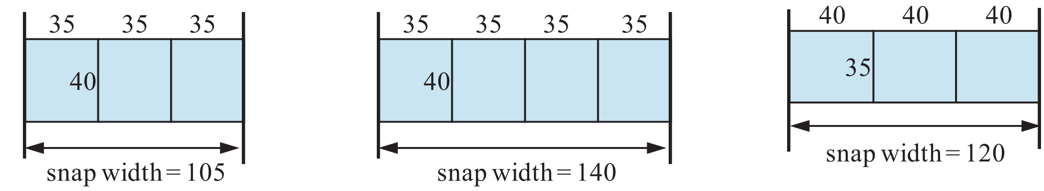


(a) no cycle time scrap and no layout scrap

(b) no cycle time scrap but layout scrap

(c) both cycle time scrap and layout scrap

Figure 8: Scrap according to different covey construction.

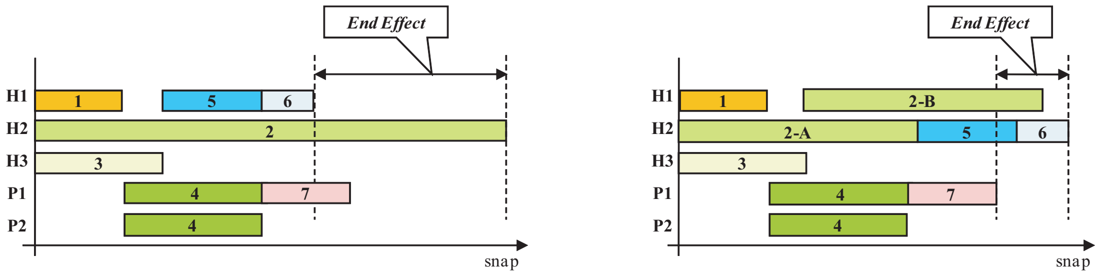


(a) three plates of (35 x 40)

(b) four plates of (35 x 40)

(c) three plates of (40 x 35) after rotation

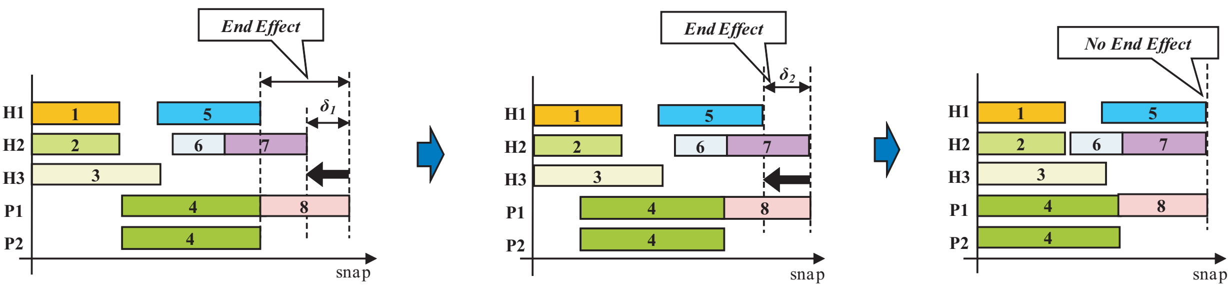
Figure 9: Snap construction alternatives.



(a) Order 2 has a large number of snaps.
→ Large end effect.

(b) Order 2 splits into Order 2-A and Order 2-B.
→ Small end effect.

Figure 10: Splitting long orders.

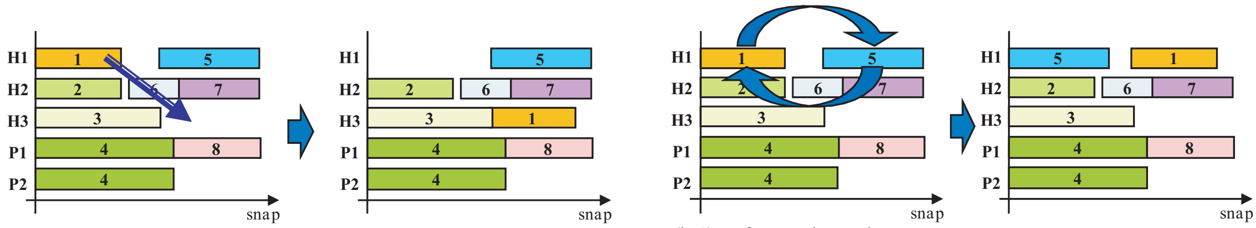


(a) Order 8 is pushed backward by δ_1.
→ Orders 4 and 8 move together

(b) Orders 7 and 8 are pushed backward by δ_2.

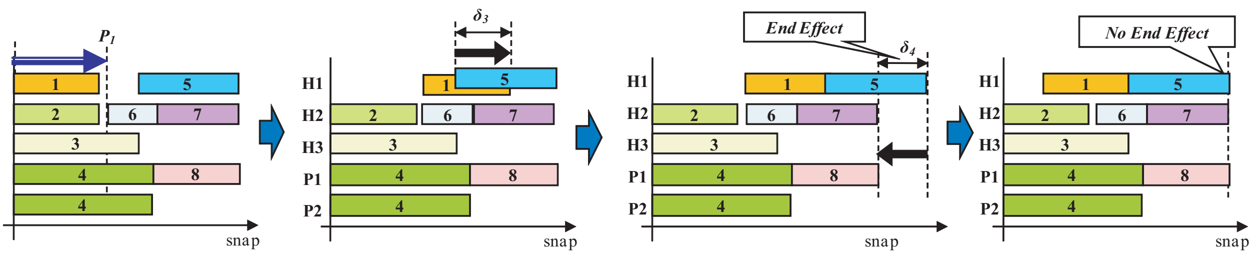
(c) Push-back removes end effect.

Figure 11: Iterative Push-back algorithm.



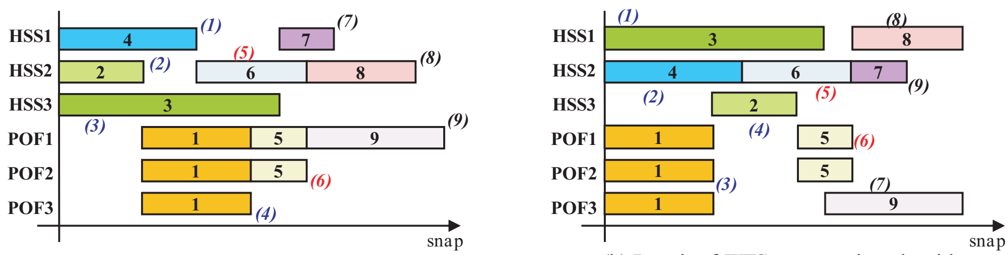
(a-1) Before relocation of Order 1. (a-2) After relocation of Order 1. (b-1) Before exchange between Orders 1 and 5. (b-2) After exchange between Orders 1 and 5.

Figure 12: Examples of relocation and exchange of orders.



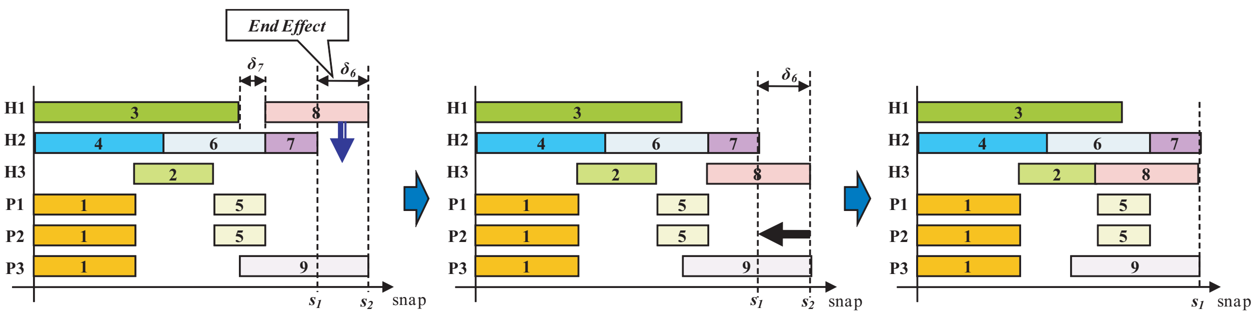
(a) Order 1 is relocated to position P_1. (b) Orders 1 and 5 overlap. → Order 5 is pushed forward by δ_3. (c) To remove end effect, Orders 1 and 5 are pushed backward by δ_4. (d) There is no end effect.

Figure 13: Example of push-forward and push-back algorithms in relocation.



(a) Result of *WTT* construction algorithm. (b) Result of *WTS* construction algorithm.

Figure 14: Result of greedy construction algorithms (small example).

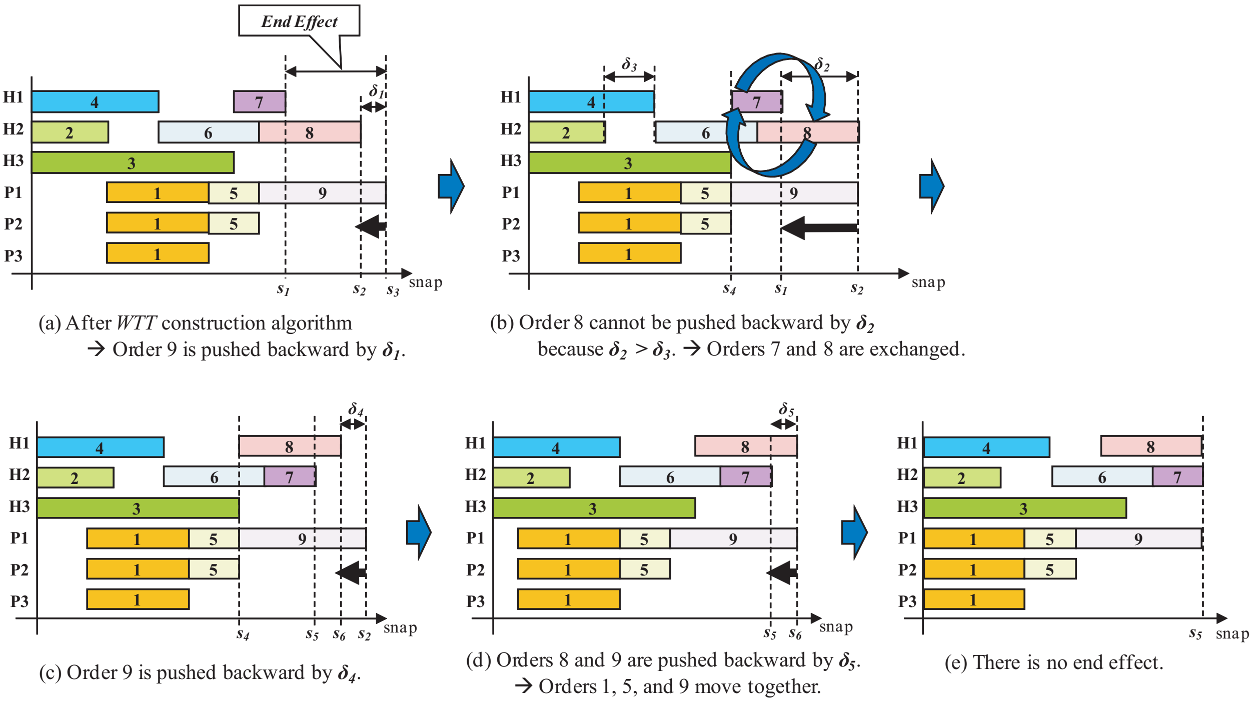


(a) After *WTS* construction algorithm
→ Order 8 cannot be pushed backward.
→ Order 8 is relocated.

(b) Orders 8 and 9 are pushed backward by δ_6.

(c) There is no end effect.

Figure 15: Local improvement after *WTS* algorithm.



(a) After *WTT* construction algorithm → Order 9 is pushed backward by δ_1.

(b) Order 8 cannot be pushed backward by δ_2 because $\delta_2 > \delta_3$. → Orders 7 and 8 are exchanged.

(c) Order 9 is pushed backward by δ_4.

(d) Orders 8 and 9 are pushed backward by δ_5. → Orders 1, 5, and 9 move together.

(e) There is no end effect.

Figure 16: Local improvement after WTT algorithm.

Table 1: Five orders are to be produced.

index	snap time	# of snaps	snap width	plates per snap	offloading machine
A	3.5	150	132	3	HSS 1
B	3.0	200	132	5	HSS 2
C	4.5	100	132	2	POF 1 & 2
D	4.0	100	130	1	POF 1
E	2.0	80	128	7	HSS 1

Table 2: Alternatives for snap construction (small example).

Given customer orders			Alternatives for snap construction			
index	dimension	total plates	width × height	plates	snap width	selected snap
1	44 × 50	600	44 × 50	3	132	*
2	30 × 33	600	30 × 33	4	120	
			33 × 30	4	132	*
3	22 × 45	2,400	22 × 45	6	132	*
			45 × 22	3	135	
4	16.5 × 25	2,000	16.5 × 25	8	132	*
			25 × 16.5	5	125	
5	60 × 65	200	60 × 65	2	120	
			65 × 60	2	130	*
6	32.5 × 35	800	32.5 × 35	4	130	*
			35 × 32.5	4	140	
7	16 × 20	800	16 × 20	8	128	*
			16 × 20	9	144	
			20 × 16	6	120	
			20 × 16	7	140	
8	30 × 32	800	30 × 32	4	120	
			32 × 30	4	128	*
9	55 × 128	250	128 × 55	1	128	*

Table 3: Result of snap construction (small example).

index	total plates	snap width	width × height	plates	# of snaps	snap time
1	600	132	44 × 50	3	200	5.0
2	600	132	33 × 30	4	150	3.0
3	2,400	132	22 × 45	6	400	4.5
4	2,000	132	16.5 × 25	8	250	2.5
5	200	130	65 × 60	2	100	6.0
6	800	130	32.5 × 35	4	200	3.5
7	800	128	16 × 20	8	100	2.0
8	800	128	32 × 30	4	200	3.0
9	250	128	128 × 55	1	250	5.5

Table 4: Computational results on actual data

	Actual shift schedule	Balancing + WTT + local search		Balancing + WTS + local search	
Instance	Yield ratio(%)	Yield ratio (%)	Time (sec)	Yield ratio(%)	Time (sec)
Data Set 1	95.3	99.6	70	99.2	79
Data Set 2	97.4	99.2	26	99.4	23
Data Set 3	94.8	99.3	63	99.5	51

Table 5: Yield ratio(%) for randomly-generated instances, without splitting of long orders

	No splitting of long orders							
	WTT construction				*WTS* construction			
	No balancing		Balancing		No balancing		Balancing	
# of orders	*WTT*	+ local	*WTT*	+ local	*WTS*	+ local	*WTS*	+ local
40	93.3	97.7	93.6	98.0	93.6	97.9	93.5	98.0
50	95.1	98.2	95.2	98.2	96.1	98.3	96.4	98.5
60	95.1	98.6	95.5	98.6	97.0	98.6	96.7	98.6
70	96.2	98.8	96.5	99.0	96.9	98.8	97.3	99.0
80	96.9	98.7	97.2	99.0	97.3	98.9	97.8	99.2
average	95.3	98.4	95.6	98.5	96.2	98.5	96.3	98.6
std. dev.	4.7	1.4	4.6	1.3	4.4	1.3	4.5	1.3
max	99.8	99.8	99.7	99.8	99.8	99.8	99.8	99.9
min	67.5	90.5	72.7	90.5	67.6	90.4	70.6	90.4

Table 6: Yield ratio(%) for randomly-generated instances, with splitting of long orders

	With splitting of long orders							
	WTT construction				*WTS* construction			
	No balancing		Balancing		No balancing		Balancing	
# of orders	*WTT*	+ local	*WTT*	+ local	*WTS*	+ local	*WTS*	+ local
40	97.3	98.8	97.7	99.0	97.6	98.8	98.1	99.0
50	98.2	99.1	98.4	99.2	98.3	99.0	98.6	99.2
60	98.5	99.2	98.6	99.2	98.5	99.1	98.7	99.2
70	98.2	99.1	98.7	99.3	98.2	99.1	98.7	99.3
80	98.1	99.1	98.5	99.3	98.2	99.1	98.7	99.3
average	98.1	99.1	98.4	99.2	98.1	99.0	98.5	99.2
std. dev.	2.1	0.7	1.5	0.4	2.0	0.8	1.3	0.4
max	99.8	99.8	99.8	99.8	99.9	99.9	99.9	99.9
min	86.9	94.3	88.2	97.8	88.1	94.3	90.8	97.5

Table 7: Yield ratio (better of *WTT* and *WTS*) and running time

	Splitting + balancing + construction + local search			
	WTT construction	*WTS* construction	Better of two solutions	
# of orders	Yield ratio (%)	Yield ratio	Yield ratio (%)	Total running time (sec)
40	99.0	99.0	99.1	37
50	99.2	99.2	99.3	83
60	99.2	99.2	99.3	182
70	99.3	99.3	99.4	355
80	99.3	99.3	99.4	614
average	99.2	99.2	99.3	254
std. dev.	0.4	0.4	0.4	301
max	99.8	99.9	99.9	2,131
min	97.8	97.5	97.9	12