Recap

- Last class (January 27, 2004)
  - Proof of Friedman’s theorem
  - Repeated Cournot game
  - Wage setting
- Today (January 29, 2004)
  - Repeated game example: Wage setting
  - Extensive form of a game
    - Information sets

Example: Wage setting

- Stage game
  - One firm, one worker
  - The firm offers the worker a wage, w
  - The worker accepts or rejects the firm’s offer
    - Reject: the worker becomes self-employed at wage \( w_0 \)
    - Accept: Work (disutility \( e \)), or Shirk (disutility 0)
      - If the worker works (supplies effort): Output is high=\( y \)
      - If the worker shirks: Output is high with probability \( p \), and low=0 with probability \( 1-p \)
  - The firm does not observe the worker’s effort decision
  - The output of the worker is observed by both parties
Example: Wage setting (cont.)

- Payoffs (Firm, Worker)
  - Work (Supply effort)
    - High output: \((y - w, w - e)\)
  - Shirk
    - High output: \((y - w, w)\)
    - Low output: \((-w, w)\)

- What is the subgame-perfect equilibrium in this stage game?
  - For any \(w \geq w_0\), worker accepts employment and shirks
  - Firm offers \(w = 0\) (or any other \(w < w_0\))

Example: Wage setting (cont.)

- Strategies
  - Firm: Offer \(w = w^*\) in the first stage.
    In stage \(t\),
    - offer \(w = w^*\) if the history of play is high-wage, high-output (all previous offers have been \(w^*\), all previous offers have been accepted, and all previous outputs have been high)
    - otherwise, offer \(w = 0\)
  - Worker:
    - If \(w > w_0\), accept the firm’s offer and supply effort if the history of play, including the current offer, is high-wage, high-output (shirk otherwise)
    - If \(w < w_0\), choose self-employment
Example: Wage setting (cont.)

- Suppose firm offers \( w^* \geq w_0 \)
  - Worker accepts
  - Work (Supply effort)
    \[ V_e = (w^*-e) + \delta V_e \rightarrow V_e = (w^*-e)/(1-\delta) \]
  - Shirk
    \[ V_s = w^* + \delta(pV_s + (1-p) w_0/(1-\delta)) \rightarrow V_s = [(1-\delta)w^*+ \delta(1-p) w_0]/(1- \delta p)(1- \delta) \]
  - Worker should supply effort if \( V_e \geq V_s \)
    \[ w^* \geq w_0 + e + e(1-\delta)/\delta(1-p) \]

If \( p=0 \): \( (w^*-e)/(1- \delta) \geq w^* + w_0 \delta/(1- \delta) \)

Example: Wage setting (cont.)

- When is it the best response for the firm to offer \( w^* \)?
  - From worker’s best response
    \[ w^* \geq w_0 + e + e(1-\delta)/\delta(1-p) \] \hspace{1cm} (1)
  - \( y \geq w^* \)
    \[ y \geq w_0 + e + e(1-\delta)/\delta(1-p) \] \hspace{1cm} (2)

The strategies induce a NE if (1) and (2) hold.

Is this a SPNE?
Example: Wage setting (cont.)

- What are the subgames?
  - Subgames beginning after a high-wage, high-output history
  - Subgames beginning after all other histories

Extensive form of a game

- The set of players
- The order of moves
- The players’ payoffs as a function of the moves that were made
- The set of actions available to the players when they move
- Each player’s information when he makes his move
- The probability distributions over any exogenous events (Nature)
Example 1

Player 1 moves first. After observing player 1’s action, player 2 moves.

Player 1 action set: \{U,D\}  Player 2 action set: \{L,R\}
Player 1 strategies: \{U,D\}
Player 2 strategies: \{(L,L), (L,R), (R,L), (R,R)\}

Normal form representation of extensive-form games

<table>
<thead>
<tr>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(L,L)</td>
</tr>
<tr>
<td>Player 1</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>2,1</td>
</tr>
<tr>
<td>D</td>
<td>-1,1</td>
</tr>
</tbody>
</table>

- Player 2’s strategies correspond to a contingent plan made in advance.
Example 2

Player 1 moves first, player 2 moves next. Player 2 does not know player 1’s action when he chooses his action.

Player 2 moves first, player 1 moves next. Player 1 does not know player 2’s action when he chooses his action.

Example

- Player 1 chooses an action from the feasible set \{L,R\}
- Player 2 observes player 1’s action and then chooses an action from the feasible set \{L',R'\}
- Player 3 observes whether or not the history of actions is \(R,R'\) and then chooses an action from the feasible set \{L'',R''\}
Information set

- An information set for a player is a collection of decision nodes satisfying:
  - The player has the move at every node in the information set
  - When the play of the game reaches a node in the information set, the player with the move does not know which node in the information set has (or has not) been reached
Example (cont.)

Player 2 has two information sets, both singletons. Player 3 has two information sets, one of them is singleton.

Subgame in an extensive form game

- A **subgame** in an extensive form game
  - begins at a decision node $n$ that is a singleton information set (but is not the game’s first decision node)
  - includes all the decision and terminal nodes following $n$ in the game tree (but no nodes that do not follow $n$), and
  - does not cut any information sets (i.e., if a decision node $n'$ follows $n$ in the game tree, then all other nodes in the information set containing $n'$ must also follow $n$, and so must be included in the subgame).
Example

Two subgames, one beginning at each of player 2’s decision nodes

No subgames

Example (cont.)