Example

- Cost of effort to the agent 10,000a
- Agent’s reservation utility $U=0$
- Agent’s expected payoff: $E[wage – cost of effort]$
- Principal’s expected payoff: $E[output – wage]$

Example (cont)

- Under low effort
  - Agent’s cost=0 → Offer $w=U=0$ to the agent
  - Principal’s expected payoff = $0.5(10,000)+0.5(20,000)-0=15,000$
- Under high effort
  - Principal’s expected revenue = $0.5(20,000)+0.5(40,000)=30,000$
  - Principal’s expected payoff = $E[revenue – wage]$
  - For participation: $E[wage] \geq U+cost\ of\ effort=10,000$
- For the principal to prefer high effort over low $E[wage]\leq 15,000$
**Example: Bonus payment**

- Principal’s proposed contract
  - If the output is low or medium, \( w=0 \)
  - If the output is high, \( w=24,000 \)
- Participation constraint for high effort
  
  \[
  0.5(0)+0.5(24,000)-10,000=2,000>0
  \]

- Incentive constraint
  
  \[
  0.5(0)+0.5(24,000)-10,000=2,000>0.5(0)+0.5(0)=0
  \]

- Principal’s payoff: 18,000  Agent’s payoff: 2,000

- What is the best \( w \) that maximizes the principal’s payoff?

\[
0.5(0)+0.5w-10,000 \geq 0
\]

\( w=20,000 \)

Principal’s payoff: 20,000  Agent’s payoff: 0

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**Example: Revenue sharing**

- Principal’s proposed contract
  - If the output is less than \( x=18,000 \), \( w=0 \)
  - If the output \( R \geq x \), \( w=R-x \)
- Participation constraint
  
  \[
  0.5(20,000-18,000)+0.5(40,000-18,000)-10,000=2,000>0
  \]

- Incentive constraint
  
  \[
  0.5(20,000-18,000)+0.5(40,000-18,000)-10,000=2,000>0.5(0)+0.5(20,000-18,000)=1,000
  \]

- Principal’s payoff: 18,000  Agent’s payoff: 2,000

- What is the best \( x \) that maximizes the principal’s payoff?
Example: Revenue sharing (cont.)

- $x > 20,000$: Participation constraint is not satisfied
  - $0.5(0) + 0.5(40,000-x) - 10,000 = 10,000 - 0.5x < 0$ if $x > 20,000$
- $10,000 < x \leq 20,000$
  - Participation: $0.5(20,000-x) + 0.5(40,000-x) - 10,000 = 20,000 - x \geq 0$
  - Incentive: $0.5(20,000-x) + 0.5(40,000-x) - 10,000 \geq 0.5(0) + 0.5(20,000-x)$
  - $x \leq 20,000 \rightarrow$ To maximize principal’s payoff $x = 20,000$
  - Principal’s payoff: 20,000  Agent’s payoff: 0
- $x \leq 10,000$:
  - Participation: $0.5(20,000-x) + 0.5(40,000-x) - 10,000 = 20,000 - x \geq 0$
  - Incentive: $0.5(20,000-x) + 0.5(40,000-x) - 10,000 \geq 0.5(10,000-x) + 0.5(20,000-x)$
  - To maximize principal’s payoff $x = 10,000$
  - Principal’s payoff: 10,000  Agent’s payoff: 10,000
- Optimal $x = 20,000$

Piece-rate system

- Principal’s proposed contract: $w = \alpha R$ if the output is $R$
- Participation constraint for high effort
  - $0.5(\alpha 20,000) + 0.5(\alpha 40,000) - 10,000 = 30,000\alpha - 10,000 \geq 0 \rightarrow \alpha \geq 1/3$
- Incentive constraint
  - $0.5(\alpha 20,000) + 0.5(\alpha 40,000) - 10,000 = 30,000\alpha - 10,000 \geq 0.5(\alpha 10,000) + 0.5(\alpha 20,000) \rightarrow \alpha \geq 2/3$
- To maximize the principal’s payoff: $\alpha = 2/3$
- Principal’s payoff: 10,000  Agent’s payoff: 10,000
- Is it optimal for the principal to induce high effort under this contract?
Managerial compensation under Cournot competition

- Cournot competition
  - The market price, $P$ is determined by (inverse) market demand:
    - $P = a - Q$ if $a > bQ$, $P = 0$ otherwise.
  - The marginal cost of producing each unit of the good is $c$
  - Each firm decides on the quantity to sell (market share): $q_1$ and $q_2$
  - $Q = q_1 + q_2$ total market demand
  - Both firms seek to maximize profits

Revenue of firm $i$: $R_i = pq_i = (a - Q)q_i$
Profit of firm $i$: $\pi_i = R_i - cq_i$

The owner of firm $i$ offers the following compensation to her manager: $M_i = \mu_i[\alpha_i\pi_i + (1 - \alpha_i)R_i]$  

Managerial compensation under Cournot competition (cont.)

1. The owner of firm $i$ needs to choose $\mu_i$ and $\alpha_i$ to maximize firm profits net of managerial compensation: $\pi^{Owner} = \max \pi_i - M_i$
2. For given $q_j$ and $\alpha_j$, the manager of firm $i$ chooses $q_i$ to maximize $M_i = \mu_i[\alpha_i(R_i - cq_i) + (1 - \alpha_i)R_j] = \mu_i[R_i - \alpha_i cq_i]$
   $M_i = \mu_i[(a - q_i - q_j)q_i - \alpha_i cq_i]$

F.O.C.: $a - 2q_i - q_j - \alpha_i c = 0 \rightarrow q_i = \frac{a - q_j - \alpha_i c}{2}$, $i = 1, 2$
Managerial compensation under Cournot competition (cont.)

In equilibrium:

\[ q_i(\alpha_1, \alpha_2) = \frac{a + \alpha_i c - 2\alpha_i c}{3}, \quad i = 1, 2 \]

\[ Q(\alpha_1, \alpha_2) = \frac{a + \alpha_2 c - 2\alpha_1 c}{3} + \frac{a + \alpha_1 c - 2\alpha_2 c}{3} = \frac{2a - \alpha_1 c - \alpha_2 c}{3} \]

\[ P(\alpha_1, \alpha_2) = \frac{a + \alpha_1 c + \alpha_2 c}{3} \]

Managerial compensation under Cournot competition (cont.)

Stage 1: Owners choose \( \mu_i \) and \( \alpha_i \)

How to choose \( \mu_i \)?

To simplify, assume \( \mu_i = M_i = 0 \)

Owner's objective:

\[ \pi^{\text{Owner}} = [p(\alpha_1, \alpha_2) - c]q_i \]

\[ = \frac{[a + c(\alpha_i + \alpha_j - 3)]}{3} \frac{[a + c(\alpha_j - 2\alpha_i)]}{3} \]

From F.O.C.:

\[ \alpha_i = \frac{6c - a - c\alpha_j}{4c} \]
Managerial compensation under Cournot competition (cont.)

In equilibrium:

\[ \alpha = \alpha_1 = \alpha_2 = \frac{6c - a}{5c} \]

\( \alpha \) increases in \( c \), i.e., owners will compensate managers to place more weight on maximizing profits when costs are high.

\[ q_1 = q_2 = \frac{a - \alpha c}{3} = \frac{2(a - c)}{5} > \frac{a - c}{3} = q_i^c \]

When owners are different than managers, output exceeds the Cournot equilibrium output.

Managerial compensation under Cournot competition (cont.)

Suppose the owner of Firm 2 is also the manager. What is \( \alpha_2 \)?

\[ \alpha_2 = 1. \]

\[ \alpha_1 = \frac{6c - a - c\alpha_2}{4c} = \frac{5c - a}{4c} \]

\[ q_1 = \frac{a + \alpha_2 c - 2\alpha_1 c}{3} = \frac{a - c}{2} \]