1. a. If firm 2 observes low costs then it faces the following problem:

\[
\max \{(72 - x_1 - x_2)x_2 - 12x_2\}
\]

and FOC yields: \(x_2^L = 30 - x_1/2\). Similarly, for high costs \(x_2^H = 24 - x_1/2\). Thus, \(E[x_2] = 27 - x_1/2\) and solving for \(x_1\) we get: \(x_1 = 22\). This implies: \(x_2^L = 19\), and \(x_2^H = 13\).

When firm 2 does not reveal information then firm 1 faces the following problem:

\[
\max \{E[(72 - x_1 - x_2)x_1 - 12x_1]\} = \max \{(60 - x_1 - E[x_2])x_1\}
\]

FOC implies that Firm 1’s best response is: \(x_1 = 30 - E[x_2]/2\). Finally, the expected profit for firm 1 is: \(E[\Pi_1] = (60 - 22 - 16)22 = 484\), and for firm 2: \(\Pi_2^H = 169\), \(\Pi_2^L = 361\), thus: \(E[\Pi_2] = 265\)

1. b. When firm 2 reveals information then this a full information Cournot game:

When costs are low. Firm 1 faces the same problem as above without expectation, and FOC is now: \(x_1 = 30 - x_2/2\). Firm 2 solves the same problem as above, and the FOC is \(x_2^L = 30 - x_1/2\). Solving we get \(x_1 = x_2 = 20\) and \(\Pi_1^L = \Pi_2^L = 400\).

Use similar analysis to find that when costs are high we \(x_1 = 24\), \(x_2 = 12\), \(\Pi_1^H = 576\) and \(\Pi_2^H = 144\). Thus: \(E[\Pi_1] = 488\), and \(E[\Pi_2] = 272\).

1. c. If firm 2 is revealing the information regardless of its costs it is better off \((272, 265)\). Thus, it is better for Firm 2 to reveal information rather than hide it.

Note that when revealing information Firm 2 is better off when it has low costs, but is worse off when it has higher costs. Now, if it firm 2 first observes the costs and then chooses to reveal (and firm 1 is aware of this procedure), then it will reveal the costs only when it has lower costs. But then firm 1 can implicitly determine whether firm 2 has high costs (since if firm 1 does not reveal its cost it has to be the case that the costs are high). Thus, this case is identical to the case where information is revealed regardless of the costs.

2. There are three pure-strategy equilibriums:

Player 1: Play T if nature is Game 1, Play B otherwise.

Player 2: Play R.

Explanation: If Player 1 plays as above then for Player 2, if he plays R his expected outcome is \(0.5 \cdot 2 + 0.5 \cdot 0 = 1\), while if he plays L his expected outcome is only \(0.5 \cdot 0 + 0.5 \cdot 1 = 0.5\). Thus, Player 2’s best response is R. If Player 2 plays R, then if nature sets at Game 1 then T is a best response for Player 1, and if nature decides Game 2 then B is a best response for Player 1.

Use similar analysis to show that the following are also equilibrium strategies:
Player 1: Play B if nature is Game 1, Play B otherwise.
Player 2: Play R.
Player 1: Play T if nature is Game 1, Play T otherwise.
Player 2: Play L.

3. Since we are looking for a symmetric equilibrium, we are assuming that everyone is playing according to the strategy in the question. The payoff for player $i$, given his value $v_i$ is:

$$\Pi_i(b_i) = \begin{cases} 
  v_i - b_i & \text{if he wins} \\
  0 & \text{if he looses}
\end{cases}$$

The probability to win is the probability that his bid is greater than all other bids. But since everyone else is bidding $b_j = \frac{n-1}{n}v_j$ the we have:

$$Pr(b_i > \max\{b_j\}) = Pr(b_i > \max\{\frac{n-1}{n}v_j\}) = Pr(\frac{n}{n-1}b_i > \max\{v_j\}) =
\left(Pr\left(\frac{n}{n-1}b_i > v_j\right)\right)^{n-1} = F\left(\frac{n}{n-1}b_i\right)^{n-1} = \left(\frac{n}{n-1}b_i\right)^{n-1}$$

Thus, the expected profit is:

$$\Pi_i(b_i) = (v_i - b_i)(\frac{n}{n-1}b_i)^{n-1}$$

F.O.C provides the required condition

$$b_i^* = \frac{n-1}{n}v_i$$