

Recap

- Last class (February 6, 2003)
 - Battle of the sexes under incomplete information
 - First-price sealed-bid auctions
 - Equilibrium recap
 - One-card poker game
- Today (February 13, 2002)
 - One-card poker game results and analysis
 - One-card poker game analysis
 - Principal-Agent problem

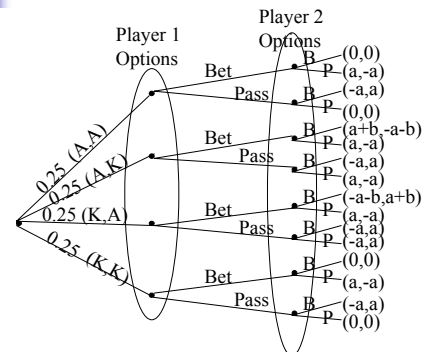
ISYE 6230 one-card poker game

- Total number of games played = 224
- Average = 10.27
- Number of different outcomes:
 - (B,B): 131
 - (B,P): 40
 - (P,B): 38
 - (P,P): 10

ISYE6230 one-card poker strategies

- Everyone bet whenever they had an ace
- Most common strategy: "A: bet, K: pass."
- The second most common strategy: "Always bet."
- Third most common strategy: Start with "A: bet, K: pass" and bet more aggressively later.
 - Alternate between betting and passing if the card is K
 - Always bet
 - Calculate the probability that the opponent has an A and bet depending on that probability

Example: One-card poker



Example: One-card poker (cont.)

- The payoffs depend on players' actions and on card combinations
- Need to compute expected payoffs for each outcome

Example: One-card poker (cont.)

- Payoffs for outcome (B,B) under different card combinations

Card combination	Probability	Payoffs
(A,A)	0.25	(0,0)
(A,K)	0.25	(a+b, -a-b)
(K,A)	0.25	(-a-b, a+b)
(K,K)	0.25	(0,0)

- Expected payoff for player 1: $(0.25)(0) + (0.25)(a+b) + (0.25)(-a-b) + (0.25)(0) = 0$
- Similarly, expected payoff for player 2 is 0
- Expected payoffs for (B,B) = (0,0)

One-card poker in normal form

		Player 2	
		Bet	Pass
Player 1	Bet	0, 0	a, -a
	Pass	-a, a	0, 0

- The unique NE is (B,B)

Principal-Agent Problem

Principal-agent examples

- Restaurant owner – waiter
- Software company – salesman
- Auto manufacturer – customer leasing a car
- Insurance company – insured

Example

- The principal offers wage w
- If the agent accepts the offer
 - Agent can put "high" ($e=25$) or "low" ($e=0$) effort
 - Agent's utility: $U(w,e)=w-e$
- Agent's reservation level of utility: 81
- Principal's payoff
 - \$270, if the agent works hard
 - \$70, if the agent doesn't work hard

First-best contract

- The agent won't accept the job, unless the wage exceeds his reservation utility:
 - $w \geq 81$
- Employing this agent is worthwhile to the principal only if the agent works hard (otherwise, the principal only gets 70)
- For the agent to work hard, his utility from working hard should exceed his reservation utility:
 - $U(w,e) \geq 81$
 - $w - 25 \geq 81 \rightarrow w \geq 106$
- First-best contract: offer \$106 + ϵ to the agent and "trust" that he will work hard

Moral hazard

- First-best contract: Offer the agent $106 + \epsilon$
- What is the problem with this contract?

"Moral hazard": the agent takes a decision or action that affects his or her utility as well as the principal's, the principal only observes the "outcome" (as an imperfect signal of the action taken), and the agent does not necessarily choose the action in the interest of the principal.

Alternative: Offer a contract where the wage depends on the effort level.

Contract conditioned on effort level

- Offer two wage rates:
 - w^H if the agent exerts high effort
 - w^L if the agent exerts low effort
- How to choose w^H so that accepting the offer and working hard is desirable for the agent?
 - $w^H - 25 \geq 81$ participation constraint (individual rationality constraint)
 - $w^H - 25 \geq w^L$ incentive constraint

Contract conditioned on outcome

- Suppose the agent is a salesman representing the principal to a client
- Three possible outcomes based on the effort level of the salesman
 - The client places no order (\$0)
 - The client places a "small" order (\$100)
 - The client places a "large" order (\$400)
- Probabilities for different outcomes under each effort level

	No order (\$0)	Small order (\$100)	Large order (\$400)	Expected order size
High effort	0.1	0.3	0.6	\$270
Low effort	0.6	0.3	0.1	\$70

Contract conditioned on outcome

- A contract where wage depends on the observable outcome
 - No order → agent pays the principal \$164
 - Small order → agent pays the principal \$64
 - Large order → principal pays agent \$236

Contract conditioned on outcome

- Principal's (expected) revenue if the agent works hard: \$270 Expected profit: \$164
- How much does the principal's revenue differ from the expected revenue under each outcome?
 - No order → $0-270 = -270$ $-270+106 = -\$164$
 - Small order → $100-270 = -170$ $-170+106 = -\$64$
 - Large order → $400-270 = 130$ $130+106 = \$236$
- Principal's profit
 - No order → \$164
 - Small order → \$164
 - Large order → \$164

Contract conditioned on outcome

- Agent's choices and (expected) payoffs under each choice (assuming the agent is risk-neutral)
 - Reject the contract and get reservation utility \$81
 - Accept the contract and don't work hard $(0.1)(236)+(0.3)(-64)+(0.6)(-164)-0 = -94$
 - Accept the contract and work hard $(0.6)(236)+(0.3)(-64)+(0.1)(-164)-25 = 81$

The principal always gets \$164!!

Contract with positive wages

- Suppose the agent only accepts positive wages.
- What are the wages w^0 , w^S and w^L corresponding to no order, small order and large order outcomes that maximize the principal's payoff?
- Participation constraint $0.6 w^L + 0.3 w^S + 0.1 w^0 - 25 \geq 81$
- Incentive constraint $0.6 w^L + 0.3 w^S + 0.1 w^0 - 25 \geq 0.6 w^0 + 0.3 w^S + 0.1 w^L$
- Nonnegativity constraint: $w^0, w^S, w^L \geq 0$
- Principal's objective
 Maximize $0.6(400 - w^L) + 0.3(100 - w^S) + 0.1(0 - w^0)$. Equivalently,
 Minimize $0.6 w^L + 0.3 w^S + 0.1 w^0$

Risk aversion

- What if the agent is risk-averse?
 - A person who prefers to get the expected value of a gamble for sure instead of taking the risky gamble is risk averse
 - E.g.: getting \$25 for sure vs. getting \$0 with probability 0.75 and \$100 with probability 0.25
- The agent and the principal may have different "beliefs" about the probabilities of different outcomes under different effort levels

Example – Risk averse agent

- Agent's reservation utility = 10
- Agent's possible actions if accepts the contract: work hard ($e=2$), don't work hard ($e=0$)
- Two possible outcomes: L and H
- Principal offers wages w^L and w^H based on the outcome

Example – Risk averse agent

- Probabilities of H and L outcomes
 - Agent does not work hard
 - H with probability 0.4
 - L with probability 0.6
 - Agent works hard
 - Principal's belief
 - H with probability 0.8
 - L with probability 0.2
 - Agent's belief
 - H with probability 0.7
 - L with probability 0.3

Example – Risk averse agent

- Participation constraint
 $0.3w^L + 0.7w^H - 2 \geq 10$
- Incentive constraint
 $0.3w^L + 0.7w^H - 2 \geq 0.6w^L + 0.4w^H$
- Principal's objective
 $C = \min 0.2w^L + 0.8w^H$

Solution: $w^L = 22/3$ $w^H = 14$ $C = 12.66$

What if the agent also shares the principal's belief?
 $w^L = 8$ $w^H = 13$ $C = 12$