HW #7 Solution

1. 16 hours/day \times 250 days/year = 4000 hours/year
   \[ K = \$80\text{/hour}\quad i = 10\% \]
   Production capacity

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e) = \frac{4000}{(c)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Cost</td>
<td>$625.00</td>
<td>2500</td>
<td>0.30</td>
<td>8</td>
<td>13333.33</td>
</tr>
<tr>
<td></td>
<td>$382.50</td>
<td>5000</td>
<td>0.15</td>
<td>4.5</td>
<td>26666.67</td>
</tr>
<tr>
<td></td>
<td>$275.00</td>
<td>7500</td>
<td>0.10</td>
<td>2</td>
<td>40000</td>
</tr>
<tr>
<td></td>
<td>$250.00</td>
<td>12000</td>
<td>0.10</td>
<td>2</td>
<td>40000</td>
</tr>
</tbody>
</table>

(a) First verify that problem is feasible

\[ \sum \frac{\lambda j}{p_j} = \frac{2500}{13333.33} + \frac{5000}{26666.67} + \frac{7500}{40000} + \frac{12000}{40000} \]
\[ = 0.1875 + 0.1875 + 0.1875 + 0.3 \]
\[ = 0.8625 < 1 \quad \text{so feasible} \]

<table>
<thead>
<tr>
<th>Setup</th>
<th>Costs (k_j)</th>
<th>Modified holding costs (h_j^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 x 80</td>
<td>640</td>
<td>(625 \times 0.1 \times (1 - 0.1875) = 50.78)</td>
</tr>
<tr>
<td>4.5 x 80</td>
<td>360</td>
<td>(382.5 \times 0.1 \times (1 - 0.1875) = 31.08)</td>
</tr>
<tr>
<td>2 x 80</td>
<td>160</td>
<td>(275 \times 0.1 \times (1 - 0.1875) = 22.34)</td>
</tr>
<tr>
<td>2 x 80</td>
<td>160</td>
<td>(275 \times 0.1 \times (1 - 0.3) = 17.50)</td>
</tr>
</tbody>
</table>

\[ T^* = \sqrt{\frac{2 \Sigma k_j \lambda_j}{\Sigma h_j^* \lambda_j}} \]
\[ = \sqrt{\frac{2 \times 640}{659900}} = 0.06325 \]

\[ q_j = \lambda_j T^* \]
\[ 2500 \times 0.06325 = 158 \]
\[ 5000 \times 0.06325 = 316 \]
\[ 7500 \times 0.06325 = 474 \]
\[ 12000 \times 0.06325 = 759 \]

The plant would use these lot sizes and repeat the sequence every 16 days.
This solution can be implemented only if $T^*$ is at least $T_{\text{min}}$.

\[
T \geq \frac{\sum \bar{S}_j}{1 - \sum (\lambda_j / P_j)} = T_{\text{min}} \quad \Rightarrow \quad \frac{8+4.5+2+2}{1 - 0.8625} = 120 \text{ hours}
\]

\[
= \frac{120}{250 \times 16} = 0.03 \text{ years}
\]

Since $0.06825 \geq 0.03$ so solution can be implemented.

b) $T_j = \frac{8j}{P_j}$

\[
158/13333.33 = 0.01185
\]

\[
316/26666.67 = 0.01185
\]

\[
474/40000 = 0.01185
\]

\[
759/40000 = 0.018975
\]

Total production time = 0.054525

Total idle time = 0.06325 - 0.054525

= 0.008725 \approx 13.8\%

c) Annual holding and setup costs:

\[
G(T) = \sum_{j=1}^{n} \left( \frac{k_j}{T} + h_j \cdot \lambda_j \cdot T / 2 \right)
\]

\[
= \frac{640 + 360 + 160 + 160}{0.06325} + 659900 \times \frac{0.06325}{2}
\]

\[
= $41738.9
\]
2. Storage space = 12000 sq. feet (W)

\( i = 10\% \)

<table>
<thead>
<tr>
<th>Item</th>
<th>Annual Demand</th>
<th>Cost/Order</th>
<th>Unit Price</th>
<th>Square feet/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12500</td>
<td>150</td>
<td>24,000</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>15000</td>
<td>80</td>
<td>34,500</td>
<td>4.10</td>
</tr>
<tr>
<td>3</td>
<td>15000</td>
<td>80</td>
<td>12.80</td>
<td>4.10</td>
</tr>
</tbody>
</table>

a) Eoq:

First calculate the Eoq's and see if the space limit is violated.

\[
E_{oq_i} = \sqrt{\frac{2 \times K_i \times L_i}{2h_i}}
\]

\( h_1 = 2.4 \)
\( h_2 = 3.45 \)
\( h_3 = 1.28 \)

\[
E_{oq_1} = \sqrt{\frac{2 \times 150 \times 12500}{2 \times 2.4}} = 1250
\]

\[
E_{oq_2} = \sqrt{\frac{2 \times 80 \times 15000}{3.45}} = 834
\]

\[
E_{oq_3} = \sqrt{\frac{2 \times 80 \times 15000}{1.28}} = 1369
\]

Total space = 1250 x 5 + 834 x 4 + 1369 x 4

\[
\sum E_{oq_i} w_i = 15062 > 12000
\]

Since \( \frac{w_i}{q_i} \) are not proportional, use Lagrangian method

\[
Q^*_i = \sqrt{\frac{2 k_i \lambda_i}{h_i + 2 \Theta w_i}}
\]

Where \( \Theta \) is such that

\[
\sum_{i=1}^{3} w_i Q^*_i = W
\]

Determine upper and lower bounds on \( \Theta \), assuming equal ratios.

\[
m = W / \sum (Eoq_i \times wi) = \frac{12000}{15062} = 0.7967
\]
The resulting lot sizes are 996, 665, and 1091.
The three values of \( \theta \) that result in these lot sizes are:

\[
\theta_1 : \sqrt{\frac{2 \times 150 \times 12500}{2.4 + 10 \theta}} = 996 \quad \Rightarrow \quad \theta_1 = 0.138
\]
\[
\theta_2 : \sqrt{\frac{2 \times 80 \times 15000}{3.45 + 8 \theta}} = 665 \quad \Rightarrow \quad \theta_2 = 0.247
\]
\[
\theta_3 : \sqrt{\frac{2 \times 80 \times 15000}{1.28 + 8 \theta}} = 1091 \quad \Rightarrow \quad \theta_3 = 0.092
\]

The true value of \( \theta \) will be between 0.092 and 0.247.
Start with \( \theta = 0.17 \left(= \frac{0.092 + 0.247}{2} \right) \)

\[
\Sigma w_i Q_i = 956 \times 5 + (706 + 953) \times 4 = 11416 < 12000
\]
So \( \theta < 0.17 \), try \( \theta = 0.13 \left(= \frac{0.092 + 0.17}{2} \right) \)

\[
Q_1 = 1006 \quad Q_2 = 731 \quad Q_3 = 1017
\]
\[
\Sigma w_i Q_i = 5 \times 1006 + 4(731 + 1017) = 12022 > 12000
\]
So \( \theta > 0.13 \) / try 0.132

\[
\theta_1 = 1004 \quad Q_2 = 730 \quad Q_3 = 1014
\]
\[
\Sigma w_i Q_i = 1004 \times 5 + (730 + 1014) \times 4 = 11996
\]
So, \( Q_1^* = 1004 \quad Q_2^* = 730 \quad Q_3^* = 1014 \)

b) The value of \( \theta \) found in part (a) can be interpreted as the decrease in the average annual cost that would result from adding an additional unit of resource. (See book pg. 224, paragraph 2) The resource here is the storage space.
2c) No storage limit \( c_2 = 12.80 \)

Budget constraint 50000

Calculate EOQ, see if violates budget constraint

From part (a) \( EOQ_1 = 1250 \)
\( EOQ_2 = EOQ_3 = 1369 \)

\[ \sum c_i EOQ_i = 1250 \times 24 + 2 \times 1369 \times 12.80 \]
\[ = 65046.4 > 50000 \]

Since \( \frac{c_i}{h_i} \) are proportional (\( i = 10\% \), we can use simple scaling

\[ Q^*_1 = 1250 \times \frac{50000}{65046} = 961 \]
\[ Q^*_2 = Q^*_3 = 1369 \times \frac{50000}{65046} = 1052 \]