Topics on inventory management

- **Objective**
  - An introduction to the fundamental concepts, tradeoffs and methods in inventory management

- **Topics**
  - Deterministic inventory models
    - Economic Order Quantity (EOQ) model and its extensions
  - Stochastic inventory models
    - Newsvendor model
    - \((Q,R)\) policies
Why do we care?

- At a macro level - the total investments by firms in inventory in the US = 20-25% of GNP.
  - Enormous potential for efficiency increase by controlling inventories
- At a firm level
  - Sales growth: right inventory at the right place at the right time
  - Cost reduction: less money tied up in inventory, inventory management, obsolescence
  - Higher profit

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**Inventory Turnover**

\[
\text{Inventory Turnover Ratio} = \frac{\text{Annual Sales}}{\text{Average Inventory Level}}
\]

<table>
<thead>
<tr>
<th>Industry</th>
<th>Upper quartile</th>
<th>Median</th>
<th>Lower quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric components and accessories</td>
<td>8.1</td>
<td>4.9</td>
<td>3.3</td>
</tr>
<tr>
<td>Electronic computers</td>
<td>22.7</td>
<td>7</td>
<td>2.7</td>
</tr>
<tr>
<td>Household audio and video equipment</td>
<td>6.3</td>
<td>3.9</td>
<td>2.5</td>
</tr>
<tr>
<td>Paper Mills</td>
<td>11.7</td>
<td>8</td>
<td>5.5</td>
</tr>
<tr>
<td>Industrial chemicals</td>
<td>14.1</td>
<td>6.4</td>
<td>4.2</td>
</tr>
<tr>
<td>Bakery products</td>
<td>39.7</td>
<td>23</td>
<td>12.6</td>
</tr>
<tr>
<td>Books: Publishing and printing</td>
<td>7.2</td>
<td>2.8</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Source: Based on a survey conducted by Risk Management Associates (2001)
Why hold inventories?

- Economies of scale
- Uncertainty
  - Demand, supply, lead times
- Variability
- Transportation
- Capacity restrictions – smoothing

We build and keep inventory in order to match supply and demand in the most profitable/cost effective way.

Types of inventory

- Supplier
- raw materials
- components
- work-in-process
- pipeline
- finished goods
- Customer

Inventory location: Warehouse, manufacturing facility, retailer, supplier, in-transit
Costs of inventory

- Physical holding costs:
  - out of pocket expenses for storing inventory (insurance, security, warehouse rental, cooling)
  - All costs that may be entailed before you sell it (obsolescence, spoilage, rework...)
- Opportunity cost of inventory: foregone return on the funds invested
- Operational costs:
  - Delay in detection of quality problems
  - Delay the introduction of new products
  - May increase throughput times

HP DeskJet Printer Case

<table>
<thead>
<tr>
<th>Inventory Driven Cost</th>
<th>Product A</th>
<th>Product B</th>
<th>Product C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component devaluation</td>
<td>2.10</td>
<td>4.20</td>
<td>2.20</td>
</tr>
<tr>
<td>Price protection</td>
<td>7.15</td>
<td>2.30</td>
<td>0.80</td>
</tr>
<tr>
<td>Product return</td>
<td>1.15</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Obsolescence</td>
<td>2.55</td>
<td>0.65</td>
<td>0.40</td>
</tr>
<tr>
<td>Holding cost of inventory</td>
<td>1.30</td>
<td>1.10</td>
<td>0.80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>14.25%</strong></td>
<td><strong>8.85%</strong></td>
<td><strong>4.80%</strong></td>
</tr>
</tbody>
</table>

Source: Callioni et al., Harvard Business Review, March 2005
How do you manage your inventory?
How much do you buy? When?

- Soda
- Milk
- Toilet paper
- Gas
- Cereal
- Cash

What do you consider?

- Cost of not having it
- Cost of going to the grocery or gas station (time, money), cost of drawing money
- Cost of holding and storing
- Price discounts
- How much you consume
- Some safety against uncertainty
Complexity

The Home Depot

- “Our inventory consists of up to 35,000 different kinds of building materials, home improvement supplies, and lawn and garden products.”
- “We currently offer thousands of products in our online store.”
- “We offer approximately 250,000 more products through our special order services.”

Amazon.com

- ~100 million items in stock, $2.76 billion in sales in 2000
- Inventory turnover rate: 17 or 18 times a year; compared to 6 to 8 times in a bricks-and-mortar store

What is an inventory control system?

An inventory control system is a set of rules and procedures for deciding **when** and **how much** to order of each item in order to profitably meet customer demand.

Minimize Costs
Subject to Customer service $\geq$ Desired level
Characteristics of Inventory Systems

- Review time:
  - Is the current level of inventory known (observed) at all times?
    - Continuous review versus periodic review
- Excess demand:
  - How is excess demand handled?
    - Backordering versus Lost sales
- Lead time
- Changing inventory:
  - Does the item deteriorate over time? (e.g., perishable items)
- Planning horizon:
  - How far into the future do we want to consider?
- Items / stages:
  - How many items/stages need to be considered simultaneously in the model
    - Single-item versus multi-item models
    - Single-stage versus multi-stage models

Inputs/Characteristics of Inventory Systems

- Demand
  - External or internal (dependent vs. independent)
  - Deterministic (known) or stochastic (random)
  - Constant or variable
- Costs
  - Holding cost
  - Order cost
  - Penalty cost
Demand – Dependent vs. Independent

Item A: Independent demand
All other items: Dependent (derived) demand

Once you forecast the demand for the end item, you do not need to forecast the demand for components or subassemblies

Bill of Materials (BOM)
### Inventory Control - Demand

#### Variability

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>Constant/Stationary</th>
<th>Variable/Non-Stationary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>Economic Order Quantity (EOQ) – Tradeoff between fixed cost and holding cost</td>
<td>Aggregate Planning – Planning for capacity levels given a forecast</td>
</tr>
<tr>
<td>Stochastic</td>
<td>Lot size/Reorder point (Q,R) or (s,S) models – Tradeoff between fixed cost, holding cost, and shortage cost</td>
<td>Materials Requirements Planning (MRP)</td>
</tr>
</tbody>
</table>

- **Very difficult problem!**
The EOQ model – Basic model

Assumptions:
- Demand is known and constant, \( d \) units per unit time
- Shortages are not allowed
- Lead time is zero
- Costs
  - Fixed/setup cost \( K \)
  - Unit purchasing cost \( c \)
  - Hence, total ordering cost for \( Q \) units is \( K + cQ \)
  - Holding cost is \( h \) per unit per unit time
    - In general, \( h = ic \), where \( i \) is the annual holding rate
- Initial inventory is zero

Example 4.1

- Number 2 pencils at the campus bookstore are sold at a fairly steady rate of 60 per week. The pencils cost the bookstore 2 cents each and sell for 15 cents each. It costs the bookstore $12 to initiate an order, and holding costs are based on an annual interest rate of 25 percent. Determine the optimal number of pencils for the bookstore to purchase and the time between placement of orders. What is the yearly holding and setup cost for this item?
Example with finite production rate

- A local company produces machine tools for industrial clients. The demand for the tools is 2,500 units, whereas the production capacity (rate) is 10,000 units per year. It costs $50 to initiate a production run, each unit costs $2 to manufacture and the cost of holding is based on a 30% annual interest rate. Determine the optimal size of a production run, the length of each production run, and the average annual cost of setup and holding. What is the maximum level of on hand inventory?

EOQ with backorders

- TennisPro, A distributor of tennis equipment, sells 250 tennis rackets per month. It costs the distributor $150 to place an order and each racket incurs a holding cost of $0.75 per month. If there is not enough stock to satisfy the demand at any time, excess demand is backordered at a cost of $2 per racket per month. Determine
  - the optimal economic order quantity,
  - the optimal backorder quantity,
  - the number of orders per month,
  - the length of the order cycle, and
  - the total annual cost.