1. (a) In order to rank the skus from the highest to the lowest claim to storage in a forward
pick area, we need to calculate the viscosity ratio $A_i$ using:

$$A_i = \frac{p_i}{\sqrt{f_i}},$$

where $p_i$ is the number of picks per day, and $f_i$ is the sku flow per day. The latter
can be calculated using:

$$f_i = \frac{\text{Volume per case}}{\text{Pieces per case}} \times \text{Demand per day}.$$ 

Using the data provided, we have:

$$f_1 = \frac{2 \times 5 \times 2}{200} \times 2500 = 250 \text{ cases per day},$$
$$f_2 = \frac{3 \times 4 \times 5}{10} \times 1500 = 9000 \text{ cases per day},$$
$$f_3 = \frac{5 \times 2 \times 1}{20} \times 5000 = 2500 \text{ cases per day},$$
$$f_4 = \frac{5 \times 2 \times 3}{100} \times 500 = 150 \text{ cases per day}.$$

From these,

$$A_1 = \frac{75}{\sqrt{250}} = 4.74,$$
$$A_2 = \frac{250}{\sqrt{9000}} = 2.64,$$
$$A_3 = \frac{200}{\sqrt{2500}} = 4.00,$$
$$A_4 = \frac{60}{\sqrt{150}} = 4.90.$$

The claims can be ranked as $4 - 1 - 3 - 2$.

(b) Under equal space allocation, each sku is assigned $\frac{1000}{4} = 250 \text{ ft}^3$.

(c) Under equal time allocation, each sku $i$ is assigned $\frac{f_i}{\sum_{i=1}^{n} f_i}$ percent of the space
capacity. Total flow is given by $250 + 9000 + 2500 + 150 = 11900 \text{ cases per day}$.

Based on this,

$$V_1 = \frac{250}{11900} \times 1000 = 21.01 \text{ ft}^3,$$
$$V_2 = \frac{9000}{11900} \times 1000 = 756.30 \text{ ft}^3,$$
$$V_3 = \frac{2500}{11900} \times 1000 = 210.08 \text{ ft}^3,$$
$$V_4 = \frac{150}{11900} \times 1000 = 12.61 \text{ ft}^3.$$

Note that the ratio $\frac{f_i}{V_i}$, which gives the number of restocks per day, is equal for all
skus, hence the name equal time allocation.
(d) Under optimal allocation, each sku $i$ is assigned $\frac{\sqrt{T_i}}{\sum_{i=1}^{4} \sqrt{T_i}}$ percent of the space capacity. We have $\sum_{i=1}^{4} \sqrt{T_i} = 172.93$. Based on this,

$$V_1 = \frac{\sqrt{250}}{172.93} \times 1000 = 91.43 \text{ ft}^3,$$

$$V_2 = \frac{\sqrt{900}}{172.93} \times 1000 = 548.59 \text{ ft}^3,$$

$$V_3 = \frac{\sqrt{2500}}{172.93} \times 1000 = 289.13 \text{ ft}^3,$$

$$V_4 = \frac{\sqrt{150}}{172.93} \times 1000 = 70.82 \text{ ft}^3.$$

(e) Throughout this part, we will use the formula:

$$B_i = s p_i - c r \frac{f_i}{V_i}$$

to calculate the benefit of including sku $i$ in the forward pick area under each setting. We start with only including the sku with the highest claim from part (a), which is sku 4. This gives:

$$B_4 = 0.15 \times 60 - 8 \times \frac{150}{1000} = $7.80 \text{ per unit.}$$

Next, we consider including the two skus with highest claim, which are 4 and 1. The optimal volume allocation is:

$$V_1 = \frac{\sqrt{250}}{150 + \sqrt{250}} \times 1000 = 563.51 \text{ ft}^3,$$

$$V_4 = \frac{\sqrt{150}}{150 + \sqrt{250}} \times 1000 = 436.49 \text{ ft}^3.$$

Based on these volumes, the total benefit per sku is given by:

$$B_1 + B_4 = 0.15 \times (75 + 60) - 8 \times \left( \frac{250}{563.51} + \frac{150}{436.49} \right) = $13.95 \text{ per unit.}$$

Since this value is higher than the one obtained under only one sku, we proceed with allowing three skus in the forward pick area. Hence, we will be considering skus 4, 1, and 3. In this setting, the optimal volume allocation is given by:

$$V_1 = \frac{\sqrt{250}}{150 + \sqrt{2500} + \sqrt{250}} \times 1000 = 202.56 \text{ ft}^3,$$

$$V_3 = \frac{\sqrt{2500}}{150 + \sqrt{2500} + \sqrt{250}} \times 1000 = 640.54 \text{ ft}^3,$$

$$V_4 = \frac{\sqrt{150}}{150 + \sqrt{2500} + \sqrt{250}} \times 1000 = 156.90 \text{ ft}^3.$$

Based on these volumes, the total benefit per sku is given by:

$$B_1 + B_3 + B_4 = 0.15 \times (75 + 200 + 60) - 5 \times \left( \frac{250}{202.56} + \frac{2500}{640.54} + \frac{150}{156.90} \right)$$

$$= $1.50 \text{ per unit.}$$
Since this unit benefit is lower than the case obtained when two skus are included, the optimal policy is to include 563.51 ft\(^3\) of sku 1 and 436.49 ft\(^3\) of sku 4 in the forward pick area.

2. (a) The from-to chart is constructed by considering the daily flow between each pair of sub-departments. For instance, there is no direct flow from sub-department A to B. From A to C, 100 units of P1 and 50 units of P3 flowing every day, resulting in a total flow of 150 units. There is no flow from A to D, and 250 units of P2 moved from A to E every day.

Proceeding in this manner, we obtain the following from-to chart for the department:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>150</td>
<td>0</td>
<td>0</td>
<td>250</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>100</td>
<td>0</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>300</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(b) In this part, we would like to find the weighted average of the coordinates for each sub-department. We will make use of the following formulas for the \(x\)- and \(y\)-coordinates, respectively:

\[
C^X_j = \frac{\sum_{i=1}^{n_j} x_i}{n_j}, \quad C^Y_j = \frac{\sum_{i=1}^{n_j} y_i}{n_j}.
\]

Here, \(j\) represents the sub-department, \(n_j\) is the number of grids corresponding to sub-department \(j\), and \(x_i\) and \(y_i\) correspond to the centroids of grid \(i\), respectively. We have the following:

\[
C^X_A = \frac{25 + 35}{2} = 30, \quad C^Y_A = \frac{2 \times 5}{2} = 5.
\]
\[
C^X_B = \frac{3 \times 5}{3} = 5, \quad C^Y_B = \frac{5 + 15 + 25}{3} = 15.
\]
\[
C^X_C = \frac{2 \times 15 + 2 \times 25}{4} = 20, \quad C^Y_C = \frac{2 \times 15 + 2 \times 25}{4} = 20.
\]
\[
C^X_D = \frac{15}{1} = 15, \quad C^Y_D = \frac{5}{1} = 5.
\]
\[
C^X_E = \frac{2 \times 35}{2} = 35, \quad C^Y_E = \frac{15 + 25}{2} = 20.
\]

Thus, the centroids are (30, 5), (5, 15), (20, 20), (15, 5), and (35, 20), respectively.

(c) The rectilinear distance between each pair of departments \(i\) and \(j\) is found using the formula:

\[
d_{ij} = |C^X_i - C^X_j| + |C^Y_i - C^Y_j|.
\]

Using the centroids we found in part (b), we obtain the following distance matrix:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>35</td>
<td>25</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>35</td>
<td>0</td>
<td>20</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>20</td>
<td>0</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>20</td>
<td>20</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
<td>35</td>
<td>15</td>
<td>35</td>
<td>0</td>
</tr>
</tbody>
</table>
(d) The material handling costs are calculated by multiplying the flow amounts in part (a) with the corresponding distance in part (c), and multiplying the result by $0.02, which is the material handling cost per unit per meter. The resulting cost matrix is given as follows:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>$75</td>
<td>0</td>
<td>$250</td>
</tr>
<tr>
<td>B</td>
<td>$35</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$210</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>$40</td>
<td>0</td>
<td>$20</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>$120</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(e) 2-opting is performed by exchanging the locations of two sub-departments. This can be done in two different ways: (i) by immediately exchanging a pair of sub-departments with identical sizes, or (ii) by interchanging adjacent sub-departments in the layout, regardless of their size.

As an example to (i), the following figure shows the resulting layout after sub-departments A and E have been exchanged:

```
B    C    C    A
  B    C    C
  B    D    E    E
```

Alternatively, as an example to (ii), we could have exchanged sub-departments C and E in the original layout:

```
B    E    C    C
  B    E    C    C
  B    D    A    A
```

After 2-opting is performed, the comparison is made by repeating parts (b) through (d) for the new layout, that is, finding the new centroids, resulting distances, and the daily total material handling cost. One would then need to compare the total cost to the one found in (d).