1. **(10 points)** Consider the auto repair shop in Homework 6. This time there is no budget constraint, but instead, there is a limited space of 12,000 ft\(^3\) to hold these auto parts. The four products consume a space of 10 ft\(^3\), 11 ft\(^3\), 10 ft\(^3\) and 16 ft\(^3\) respectively. The demand, cost per unit, and order cost of the stocked parts are shown below:

<table>
<thead>
<tr>
<th>Part</th>
<th>Demand per year</th>
<th>Variable cost</th>
<th>Order cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,220</td>
<td>$35</td>
<td>$150</td>
</tr>
<tr>
<td>2</td>
<td>3,000</td>
<td>$40</td>
<td>$200</td>
</tr>
<tr>
<td>3</td>
<td>1,520</td>
<td>$30</td>
<td>$220</td>
</tr>
<tr>
<td>4</td>
<td>2,670</td>
<td>$25</td>
<td>$130</td>
</tr>
</tbody>
</table>

The shop management uses a 21% annual interest to compute the holding cost. What should be the order sizes for these parts, considering the space constraint?

**Solution:**

We compute the EOQ first, without considering the constraint:

\[
Q_1 = \sqrt{\frac{2K_1\lambda_1}{h_1}} = \sqrt{\frac{2(150)(2220)}{(21)(35)}} = 301
\]

\[
Q_2 = \sqrt{\frac{2K_2\lambda_2}{h_2}} = \sqrt{\frac{2(200)(3000)}{(21)(40)}} = 377.9 \approx 378
\]

\[
Q_3 = \sqrt{\frac{2K_3\lambda_3}{h_3}} = \sqrt{\frac{2(220)(1520)}{(21)(30)}} = 325.8 \approx 326
\]

\[
Q_4 = \sqrt{\frac{2K_4\lambda_4}{h_4}} = \sqrt{\frac{2(130)(2670)}{(21)(25)}} = 363.6 \approx 364
\]

We compute the volume occupied by these order sizes:

\[
\sum_{i=1}^{4} w_i EOQ_i = 301(10) + 378(11) + 326(10) + 364(16) = 16,252 > 12,000
\]
Since these orders occupy more volume than what is available, we have to adjust our order sizes. Also, since the ratio $w_i/h_i$ is not the same for all the products we cannot just scale down the EOQs but instead we need to find $\theta$.

We are going to use the bisection method, so first we need to get lower and upper bounds for the value of $\theta$. To get these, we use the factor $\frac{12.000}{16.252} = .738$ to scale down the EOQs:

$$Q_1 = 301(.738) = 222.37$$

$$Q_2 = 377.9(.738) = 279.21$$

$$Q_3 = 325.8(.738) = 240.69$$

$$Q_4 = 363.6(.738) = 268.63$$

We find the four $\theta$s that correspond to these order sizes:

$$Q_1 = \sqrt{\frac{2K_1\lambda_1}{h_1 + 2\theta w_1}} = \sqrt{\frac{2(150)(2220)}{(.21)(35) + 2\theta (10)}} = 222.37 \rightarrow \theta = .302$$

$$Q_2 = \sqrt{\frac{2(200)(3000)}{(.21)(40) + 2\theta (11)}} = 279.2 \rightarrow \theta = .318$$

$$Q_3 = \sqrt{\frac{2(220)(1520)}{(.21)(30) + 2\theta (10)}} = 240.69 \rightarrow \theta = .262$$

$$Q_4 = \sqrt{\frac{2(130)(2670)}{(.21)(25) + 2\theta (16)}} = 268.63 \rightarrow \theta = .137$$

So, by selecting the smallest and the largest we find the upper and lower bounds for $\theta$ and that $\theta \in [.137, .318]$. We can now start the bisection search:

<table>
<thead>
<tr>
<th>Min</th>
<th>Max</th>
<th>Theta</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Total Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.32</td>
<td>0.22720637</td>
<td>236</td>
<td>299</td>
<td>248</td>
<td>235</td>
<td>11889.00</td>
</tr>
<tr>
<td>0.14</td>
<td>0.227206</td>
<td>0.18188926</td>
<td>246</td>
<td>311</td>
<td>259</td>
<td>250</td>
<td>12471.00</td>
</tr>
<tr>
<td>0.18</td>
<td>0.227206</td>
<td>0.20454782</td>
<td>241</td>
<td>304</td>
<td>253</td>
<td>242</td>
<td>12156.00</td>
</tr>
<tr>
<td>0.20</td>
<td>0.227206</td>
<td>0.21587709</td>
<td>238</td>
<td>302</td>
<td>250</td>
<td>238</td>
<td>12010.00</td>
</tr>
<tr>
<td>0.22</td>
<td>0.227206</td>
<td>0.22154173</td>
<td>237</td>
<td>300</td>
<td>249</td>
<td>237</td>
<td>11952.00</td>
</tr>
<tr>
<td>0.22</td>
<td>0.221542</td>
<td>0.21870941</td>
<td>238</td>
<td>301</td>
<td>250</td>
<td>238</td>
<td>11999.00</td>
</tr>
<tr>
<td>0.22</td>
<td>0.218709</td>
<td>0.21729325</td>
<td>238</td>
<td>301</td>
<td>250</td>
<td>238</td>
<td>11999.00</td>
</tr>
</tbody>
</table>
Note that if the volume occupied by the new found order sizes is less than 12,000, it means we need to decrease $\theta$, so we update the upper bound (max.) with the previous iteration value of $\theta$. If the total volume is greater than 12,000, then we need to increase $\theta$, so we update the lower bound (min.) with the previous iteration value of $\theta$. If the volume is 12,000 (or we are close enough and/or started to cycle), we have found the optimal (or close to optimal) value of $\theta$.

Here $\theta \approx 0.217$ and $Q_1 = 238$, $Q_2 = 301$, $Q_3 = 250$ and $Q_4 = 238$, with a total volume of 11,999 ft$^3$.

2. **(10 points)** Great Label Co. is a small label manufacturing company that prints digital quality custom labels. Since they are new in the business, they just got their first customer who orders four different labels. The worker performing the setup tasks is paid $18/hour, and other setup costs amount to $20/hr. Currently, they have only one machine to print all the labels. The demand, setup time, cost per unit, and production rate for each label type are shown below:

<table>
<thead>
<tr>
<th></th>
<th>Label 1</th>
<th>Label 2</th>
<th>Label 3</th>
<th>Label 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand per year</td>
<td>25,000</td>
<td>55,000</td>
<td>14,500</td>
<td>10,500</td>
</tr>
<tr>
<td>Cost</td>
<td>$0.5</td>
<td>$0.75</td>
<td>$1</td>
<td>$1.1</td>
</tr>
<tr>
<td>Setup time (hrs.)</td>
<td>1.0</td>
<td>1.5</td>
<td>1.75</td>
<td>2</td>
</tr>
<tr>
<td>Production rate (labels/year)</td>
<td>1,500,000</td>
<td>750,000</td>
<td>555,000</td>
<td>475,000</td>
</tr>
</tbody>
</table>

Assume that holding costs are based on an interest rate of 23% and that the company follows a rotation policy for manufacturing these labels.

a. **(1 points)** Does the single machine have sufficient capacity to satisfy the demand for all the labels? Why?

**Solution:**
Yes. To show this we compute $\sum_{i=1}^{4} \frac{d_i}{p_i} = .0167 + .073 + .026 + .022 = .138 < 1$. This means that it takes (much) less than a year to produce the annual demand of each product. So, it is feasible to produce the labels with one machine.

b. **(3 points)** What is the optimal length of the rotation cycle?

**Solution:**
We have that:

$T_{optimal} = \max(T_{min}, T^*)$

Let’s compute $T^*$, which is the cycle time of a complete rotation (rotate the four labels).

$T^* = \sqrt{\frac{2 \sum_{i=1}^{4} K_i}{\sum_{i=1}^{4} h_i' \lambda_i}} = .1649 \text{ years} = 1.98 \text{ months}$

Where:
To get the set-up costs for each label, we computed the set-up labor cost (hours of set-up times the pay rate) and added the “other set-up costs”.

\[
T_{\text{min}} = \frac{\sum_{i=1}^{4} S_i}{1 - \sum_{i=1}^{4} \lambda_i/P_i} = 7.25 \text{ hours}
\]

Here \( S_i \) is the set-up time per label in hours. No matter what conversion you used from hours to years (for instance, 40 hours per week and 52 weeks per year), 7.25 hours is definitely less than 1.98 months, so \( T^* = .1649 \text{ years} \) is the optimal length of the rotation cycle.

c. (2 points) What is the optimal lot size for each label?

**Solution:**
The optimal lot size is just the demand times the length of the cycle (we have to produce enough to fulfill the demand). For instance:

\[
Q_1 = \lambda_1 T^* = 25,000(.1649) = 4122.95 \approx 4123
\]

Similarly:

\[
Q_2 = 55,000(.1649) = 9070.5 \approx 9071
\]

\[
Q_3 = 14,500(.1649) = 2391.3 \approx 2391
\]

\[
Q_4 = 10,500(.1649) = 1731.64 \approx 1732
\]

d. (2 points) What is the percentage of downtime for the machine?

**Solution:**
We have that \( T_1 \) is equal to \( \sum_{i=1}^{4} Q_i/P_i \) (time when the machine is producing). So, we get that:

\[
\frac{T_2}{T} = \frac{T - \sum_{i=1}^{4} Q_i/P_i}{T} = \frac{.1649 - .0228}{.1649} = .86 \rightarrow 86\%
\]

e. (2 points) What are the total annual setup and holding costs?

**Solution:**
We have that:

\[
\text{Set-up cost of label } i = \frac{K_i \lambda_i}{Q_i}
\]

\[
\text{Holding cost of label } i = \frac{h_i' Q_i}{2}
\]
The costs are shown in the table:

<table>
<thead>
<tr>
<th></th>
<th>Label 1</th>
<th>Label 2</th>
<th>Label 3</th>
<th>Label 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>setup</td>
<td>$230.42</td>
<td>$345.63</td>
<td>$403.23</td>
<td>$460.83</td>
</tr>
<tr>
<td>holding</td>
<td>$233.12</td>
<td>$724.96</td>
<td>$267.82</td>
<td>$214.21</td>
</tr>
</tbody>
</table>

The cost of holding and set-up for the four labels is: $2,880.22 ($1,440.11 each)

3. (10 points) Tech Parts is a company that imports replacement parts for different specialized technologies. Many of these parts are custom-made and imported until the order is made. However, Tech Parts holds inventory of Part X, since it is relatively generic and on high demand, and the order costs are high. The cost of Part X is $1,500, and its demand is steady at 100/month. The order cost is $2,500. Assume that holding costs are based on a rate of 23%.

   a. (1 point) Based on the information above, what is the EOQ for Part X?

   Solution:
   We compute the basic EOQ:
   
   \[ Q = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2(2500)(1200)}{(23)(1500)}} = 131.87 \approx 132 \]

   b. (2 points) Since only Tech Part offers these parts in the U.S.A, customers are willing to wait if the parts are not immediately available. Tech Parts estimates a backorder cost of 150/year per part (book keeping, etc.). Considering the backordering option, what should be the order size?

   Solution:
   From lecture, you derived the optimal EOQ given that backorders are allowed:
   
   \[ Q = \sqrt{\frac{2K\lambda}{h}} \sqrt{\frac{h + b}{b}} = \sqrt{\frac{2(2500)(1200)}{(23)(1500)}} \sqrt{\frac{(23)(1500) + 150}{150}} = 239.6 \approx 240 \]

   c. (1 point) What is the maximum shortage level given b)?

   Solution:
   From lecture, you derived the optimal maximum shortage level S given that backorders are allowed:
   
   \[ S = Q \frac{h}{h + b} = 239.6 \frac{(23)(1500)}{(23)(1500) + 150} = 166.96 \approx 167 \]

   d. (2 points) If each part needs 2 ft³ of storage and if the order size is as computed in b), how much space in ft³ does Tech Parts need to allocate to Part X such that the inventory always fits?

   Solution:
   The maximum inventory level for this inventory model is \(Q-S= 240-167= 73\). Therefore, they need 73(2)=146 ft³.
e. (2 points) What is the percentage of time that customers are waiting for their order of Part X given b)?

**Solution:**
The percentage of time when customers are waiting is the percentage of time there is a backorder.

\[
Time \text{ in backorder} = T_2 = \frac{S}{\lambda} = \frac{167}{1200} = .139 \text{ years}
\]

The cycle time is \(T = \frac{Q}{\lambda} = \frac{240}{1200} = .2 \text{ years}\). Therefore the percentage of time waiting \(T_2/T\) is 70%.

f. (2 points) What are the annual ordering, holding, and backorder costs given b)?

**Solution:**
The costs are given as:

\[
\text{Ordering} = \frac{K\lambda}{Q} = \frac{2500(1200)}{240} = 12,500
\]

\[
\text{Holding} = \frac{h(Q - S)^2}{2Q} = \frac{.23(1500)(240 - 167)^2}{2(240)} = 3830.22
\]

\[
\text{backorder} = \frac{bS^2}{2Q} = \frac{(150)(167)^2}{2(240)} = 8715.31
\]

These three costs sum: $25,045 per year.