1. **(10 points)** A small auto repair shop keeps the most common auto parts in their inventory to speed the repair process and increase their customer satisfaction. The rest or the parts are bought when needed. The demand, cost per unit, and order cost of the stocked parts are shown below:

<table>
<thead>
<tr>
<th>Part</th>
<th>Demand per year</th>
<th>Variable cost</th>
<th>Order cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,220</td>
<td>$35</td>
<td>$150</td>
</tr>
<tr>
<td>2</td>
<td>3,000</td>
<td>$40</td>
<td>$200</td>
</tr>
<tr>
<td>3</td>
<td>1,520</td>
<td>$30</td>
<td>$220</td>
</tr>
<tr>
<td>4</td>
<td>2,670</td>
<td>$25</td>
<td>$130</td>
</tr>
</tbody>
</table>

The shop management uses a 21% annual interest to compute the holding cost.

a. **(3 points)** What is the optimal order quantity for these parts?

**Solution:**

\[
Q_1 = \sqrt{\frac{2K_1 \lambda_1}{h_1}} = \sqrt{\frac{2 \times 150 \times 2220}{.21 \times 35}} = 301
\]

\[
Q_2 = \sqrt{\frac{2K_2 \lambda_2}{h_2}} = \sqrt{\frac{2 \times 200 \times 3000}{.21 \times 40}} = 377.9 \approx 378
\]

\[
Q_3 = \sqrt{\frac{2K_3 \lambda_3}{h_3}} = \sqrt{\frac{2 \times 220 \times 1520}{.21 \times 30}} = 325.8 \approx 326
\]

\[
Q_4 = \sqrt{\frac{2K_4 \lambda_4}{h_4}} = \sqrt{\frac{2 \times 130 \times 2670}{.21 \times 25}} = 363.6 \approx 364
\]

b. **(7 points)** The shop management has established that the shop should never have more than $35,000 invested in the inventory of these parts. Would this policy change your answer in a)? If that is the case, what should be the order size for these parts?

**Solution:**

If the shop follows the orders size in a), they would have a maximum investment in inventory of:

\[301 \times 35 + 378 \times 40 + 326 \times 30 + 364 \times 25 = 44,535\]

So, the solution given in a) cannot be implemented.

Since the interest rate is the same for all products, we have that the optimal solution is just the solution in a) times a factor \(m\) :
\[ m = \frac{C}{\sum_{i=1}^{n} c_i E O Q_i} = \frac{35,000}{(301 \times 35 + 378 \times 40 + 325.8 \times 30 + 363.6 \times 25)} = 0.786 \]

Then \( Q^*_i = \text{round\_down}(m \times EOQ_i) \), and \( Q_1 = 236, Q_2 = 297, Q_3 = 256 \) and \( Q_4 = 285 \) which has a total maximum investment of $34,945. We rounded down because we do not want to exceed the budget constraint. We can increase the last two orders size in one unit: \( Q_3 = 257 \) and \( Q_4 = 286 \) to have an exact maximum investment of $35,000.