1. **(8 points)** A semiconductor manufacturer needs to determine the optimal order quantity for a special type of silicon wafers. The supplier of the wafers has the following pricing policy: for quantities up to 5,000 the supplier charges $3 per wafer; for additional quantities between 5,001 and 10,000 the supplier charges $2.75 per wafer (the discount applies only to the additional quantities); beyond 10,001 units the supplier charges $2.5 per wafer for the additional unit. The manufacturer estimates an annual demand of 25,000 wafers. Order set-up cost is $200, and the holding cost is estimated based on an annual 20% interest rate.

   a. **(7 points)** Find the optimal order quantity.

   **Solution:**
   We have:
   \[
   \lambda = 25,000 \text{ per year} \\
   K = 200 \text{ per order} \\
   I = 0.2 \text{ per year}
   \]

   First we determine the algebraic expression for \(C(Q)\) for each price interval:
   \[
   C_1(Q) = 3Q \text{ for } Q \leq 5,000 \\
   C_2(Q) = (3)(5000) + 2.75(Q - 5000) \text{ for } 5,001 \leq Q \leq 10,000 \\
   C_3(Q) = (3)(5000) + (2.75)(5000) + 2.5(Q - 10,000) \text{ for } Q \geq 10,001
   \]

   Group the common terms and compute an expression for \(C(Q)/Q\):
   \[
   \frac{C_1(Q)}{Q} = 3 \text{ for } Q \leq 5,000 \\
   \frac{C_2(Q)}{Q} = \frac{1250}{Q} + 2.75 \text{ for } 5,001 \leq Q \leq 10,000 \\
   \frac{C_3(Q)}{Q} = \frac{3750}{Q} + 2.5 \text{ for } Q \geq 10,001
   \]

   Then substitute the expressions derived above in the expression for \(G(Q)\). Group the similar terms:
   \[
   G(Q) = \frac{25,000C(Q)}{Q} + \frac{200(25,000)}{Q} + .2\left(\frac{C(Q)}{Q}\right)\frac{Q}{2} \\
   G_1(Q) = \frac{5,000,000}{Q} + .3Q + 75,000 \\
   G_2(Q) = \frac{36,250,000}{Q} + .275Q + 68,875 \\
   G_3(Q) = \frac{98,750,000}{Q} + .25Q + 62,872
   \]
The EOQs are computed as:

\[ Q_1 = \sqrt{\frac{5,000,000}{.3}} = 4,082.5 \text{ (realizable)} \]
\[ Q_2 = \sqrt{\frac{36,250,000}{.275}} = 11,481.2 \text{ (no realizable)} \]
\[ Q_3 = \sqrt{\frac{98,750,000}{.25}} = 19,874.6 \text{ (realizable)} \]

Since only \( Q_1 \) and \( Q_3 \) are realizable, we substitute \( Q \) in the \( G(Q) \) expression for 1 and 3. \( G_1(Q_1) = 77,449.5 \) and \( G_2(Q_2) = 72,812.3 \). Therefore \( Q = Q_3 = 19,874.6 \approx 19875 \) units.

b. (1 point) Find the annual costs (purchasing, set-up and holding costs).

**Solution:**

We found above that it is $72,812.3

Setup = 200(25,000)/19875 = 251.7

Purchasing = 25,000(2.5 + 3750/19875) = 67,217

\( h = (.2)(2.5 + 3750/19875) = .5377 \)

Holding = 19875/2(.5377) = 5,343.8

2. (8 points) The semiconductor manufacturer of problem 1) found another supplier. The new supplier’s pricing policy is as follows. For orders of 5,000 units or less, the supplier charges $3 per waffle; for orders of 5,001 to 10,000, the supplier charges $2.8 per unit; finally for orders above the 10,001 units, the supplier charges $2.7 per unit. Annual demand, set-up and holding costs are the same.

a. (7 points) Find the optimal order quantity.

**Solution:**

We have to find the largest realizable EOQ. We start with the lowest price:

\[ Q_3 = \sqrt{\frac{2(200)(25,000)}{.2(2.7)}} = 4303.3 \text{ (no realizable)} \]
\[ Q_2 = \sqrt{\frac{2(200)(25,000)}{.2(2.8)}} = 4225.8 \text{ (no realizable)} \]
\[ Q_1 = \sqrt{\frac{2(200)(25,000)}{.2(3)}} = 4082.5 \text{ (realizable)} \]
Only Q1 is realizable. We now have to find the annual cost \( G(Q) \) for Q1 and the rest of the breakpoints that take you to the next lower price (5,001 and 10,001).

\[
G_j(Q) = 25,000 c_j + \frac{25,000(200)}{Q} + \frac{.2c_j Q}{2}
\]

\[
G_1(4082.5) = 25,000(3) + \frac{25,000(200)}{4082.5} + \frac{.2(3)4082.5}{2} = 77449.5
\]

\[
G_2(5001) = 25,000(2.8) + \frac{25,000(200)}{5001} + \frac{.2(2.75)5001}{2} = 72400
\]

\[
G_2(10001) = 25,000(2.7) + \frac{25,000(200)}{10001} + \frac{.2(2.5)10001}{2} = 70700.2
\]

Therefore the EOQ is 10,001 units.

b. (1 point) Compare the purchasing, set-up and holding costs of 1) and 2). Which supplier should be chosen based solely on these costs?

**Solution:**

We found above that the new supplier costs are $70,700.2. So based solely on these costs, the new supplier is preferred.

3. (3 points) Assume that in the calculation of an optimal order quantity for an all-units schedule you found the following: \( Q^{(1)}=700 \) units when using a price that applies to all units in an order of 800 units or less; \( Q^{(2)}=795 \) units when using a price that applies to all units in an order of 801 to 900 units; and \( Q^{(3)}=950 \) when using a price that applies to an order quantity above 901. Based on this information only, can you determine the optimal order quantity? Explain.

**Solution:**

Yes. EOQ=950 because it is the largest realizable EOQ and there is not a higher price breakpoint.

4. (3 points) Consider an EOQ of 50 units. Demand is 1,000 units per year, and the lead time is 7 weeks. What is the re-order point? Assume 52 weeks per year.

**Solution:**

We compute the cycle time \( T=50/1000=.05 \) years or 2.6 weeks. This is less than the lead time. So, in order to compute the lead time we compute the ratio of the lead time and cycle time:

\[
7/2.6=2.69 \text{ cycles in the lead time.}
\]

We take the fraction of this number, 0.69 of a cycle time, and multiply times the cycle length to convert it back to years: 0.69(.05)=.0346 years. If we multiply this times the demand rate, we get the reorder point: 0.346(1000)=34.6 \approx 35 \text{ units.}

5. (3 points) Assume that you formulated and solved a linear program for a company’s aggregate planning problem. In the model you added a constraint that limited the on-hand inventory for every month, so that it would not take more than 1,000 ft\(^2\) of storage. After solving the model you got a shadow price of -2 for that constraint, for a given month.

a. (1 points) Was the mentioned constraint binding (i.e., satisfied at equality)? Why?
Solution:
Yes, because the shadow price is greater than zero. Then all the storage space is used in that month, given the optimal solution for the aggregate planning problem.

b. **(2 points)** If the company could rent extra storage space for that given month. How much would be the maximum they should pay for an additional ft²?

Solution:
You could pay up to the shadow price (any value below will actually decrease the cost).