ISYE 3104 - Homework #6 Solutions

d : 1400/quartes = 5600/year I = 20%

1. C= $4000/unit, h=I.c=(0.2)(4000)=800/unit year, K=$10,000, b=$1200/unit year

a) \( Q = \sqrt{\frac{2Kd}{h}} \sqrt{\frac{b+h}{b}} = \sqrt{\frac{2(10000)(5600)}{800}} \sqrt{\frac{1200+800}{1200}} = 483 \)

c) \( S = Q \left( \frac{h}{b+h} \right) \approx 193 \)

b) \( H = Q - S = 290 \)

d) \( T = \frac{Q}{d} = 4.3125 \) weeks

e) \( C(Q_1S) = \frac{Kd}{Q} + \frac{(Q-h)^2}{2Q} + \frac{h^2}{2Q} = 231,862 \)

f) With the backordering option, the company backorders some demand to save on the holding costs. Since the max inventory level is lower with backordering compared to no backordering (for the same \( Q \)) the company can order bigger quantities and also save on the fixed cost.

2. K=8000

\[
\frac{C(Q)}{Q} = \begin{cases} 
\frac{4200}{(4200)(400)+Q-400)(4150)} \cdot \frac{Q}{4200)(400)+Q-400)(4150)} = 20000 \frac{Q}{Q} + 4150 = \frac{S_2}{Q} + 4150 , \quad Q \leq 400 \\
(4200)(400)+Q-400)(4150)+4100)(Q-600) \cdot \frac{Q}{50000} + 4100 = \frac{S_3}{Q} + 4100 , \quad 400 < Q \leq 600 \\
\end{cases}
\]

a) \( Q_1 = \sqrt{\frac{2Kd}{h}} = \sqrt{\frac{(2)(8000)(5600)}{(0.2)(4200)}} \approx 327 \)

\( Q_2 = \sqrt{\frac{2(K+S_2)(5600)}{h}} = \sqrt{\frac{(2)(58000)(5600)}{(0.2)(4100)}} \approx 615 \quad \text{not realizable} \)

\( Q_3 = \sqrt{\frac{2(K+S_3)(5600)}{h}} \sqrt{\frac{(2)(58000)(5600)}{(0.2)(4100)}} = 890 \)

\( C(Q_1) = \frac{80000}{327} + (0.2)(4200) \frac{327}{2} + (4200)(5600) = 23,794,343 \)

\( C(Q_3) = \frac{58000}{890} + (0.2) \left( \frac{50000}{890} + 4100 \right) \cdot \frac{890}{2} + (\frac{20000}{890} + 4100) \cdot 5600 = 23,649,845 \)

\( \Rightarrow \) order \( Q_3 = 890 \)

b) \( T = \frac{Q}{d} = 890/5600 = 0.159 \) years

c) \( 23,694,845 \)

3. Company X. You get the discount \( c_2 \) for all the units if your order \( Q \geq B \). However, in case of company Y you get the discount only for additional units above \( B \).