Q1

\[ c = 2 \]
\[ K = 70 \]
\[ I = 25\% \]

Order lead time = 1 month.
Annual Demand \( \sim N(\lambda = 100, \sigma^2 = 15^2) \)
\( P = 5 \)

\[
(1) \quad \theta_0 = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2 \times 100 \times 100}{0.25 \times 2}} = 167.
\]

\[
1 - F(R_0) = \frac{\theta_0 h}{\lambda} = \frac{167 \times 0.5}{5 \times 100} = 0.167
\]

\( z \)-value corresponding to a right tail of 0.167 is
\( z = 0.97 \).

Annual Demand \( \sim N(100, 15^2) \)
Lead time = 1 month
Find Demand during lead time \( D_L \).

\[
E(D_L) = \frac{100}{12} = 8.33
\]

\[
\text{Var}(D_L) + \ldots + \text{Var}(D_L) = \text{Var}(D) \quad 12 \text{Var}(D_L) = 15^2
\]

\[
\text{Var}(D_L) = \frac{15}{12}^2 \quad \text{st. dev}(D_L) = \frac{15}{\sqrt{12}}
\]

\[
D_L \sim N(\mu = 8.33, \sigma^2 = 4.33^2)
\]

\[
\sigma^2 + \mu = 4.33 \times 0.97 + 8.33
\]

\[
= 12.53 \text{ E13}
\]

\( z = 0.97 \)

\[
L(z) = 0.0882 \quad n(R) = F(L(z)) = 4.33 \times 0.0882 = 0.382
\]
\[ Q_1 = \sqrt{\frac{2 \lambda \left[ \mu + \rho n(R) \right]}{h}} \]

\[ Q_1 = \sqrt{\frac{2 \cdot 100 \cdot [70 + 5 \cdot (0.382)]}{0.5}} \approx 170 \]

\[ Q_0 = 167 \quad Q_1 = 170 \text{ not close enough. Do more iterations.} \]

\[ 1 - F(R_1) = \frac{Q_1 h}{\lambda} = \frac{170 \cdot 0.5}{5 \cdot 100} = 0.17 \]

2 value corresponding to a right tailed of 0.17 is

\[ z = 0.95 \]

\[ R = r^2 + \mu = 4.33 \times 0.95 + 8.33 \]

\[ = 12.44 \approx 12 \]

\[ z = 0.95 \Rightarrow L(z) = 0.0916 \quad \sigma L(z) = 3.33 \times (0.0916) = 0.397 \]

\[ Q_2 = \sqrt{\frac{2 \lambda \left[ \mu + \rho n(R) \right]}{h}} \]

\[ = \sqrt{\frac{2 \cdot 100 \cdot [70 + 5 \cdot (0.397)]}{0.5}} \approx 170 \]

\[ 1 - F(R_2) = \frac{Q_2 h}{\lambda} = \frac{170 \cdot 0.5}{5 \cdot 100} \]

\[ 2 = 0.95 \Rightarrow R = r^2 + \mu = 12.44 \approx 12 \]

Both Q2 and R2 are within one unit of Q1 and R1, so we can stop here.

\[(Q, R) \Rightarrow (170, 12)\]
b) Safety stock = s
\[ s = R - \mu = 12.1 - 8.33 \approx 4 \text{ units.} \]

c) Holding cost = \( h \left[ \frac{Q}{2} + R - \mu \right] = 0.5 \left[ \frac{170}{2} + 12 - 8.33 \right] \)
\[ = 44.34 \]

Setup cost = \( \frac{KH}{Q} = 70(100) = 4118 \]

Stockout cost = \( \frac{P \times n(\epsilon)}{Q} = 5(100)(0.397) = 1.17 \]

Total average annual cost = 86.69

d) If \( \text{Var}(DL) = 0 \) → use EOQ

Total average annual cost = \( \sqrt{2KH} = \sqrt{2 \times 70 \times 100 \times 0.5} = 83.67 \)

In case of no uncertainty we have less costly plan.

e) \( P(D \leq R) = F(R) = 1 - 0.17 = 0.83 \approx 83\% \)

f) \( \frac{n(R)}{Q} = \frac{0.397}{170} \approx 0.23\% \)

On the average 99.77\% of the demands are satisfied as they occur.
a) \((Q,R)\) Policy is more frequently used in practice, since it includes setup cost for placing an order, and it allows positive lead time. In Multiperiod Newsboy Model, these are not included.

b) One of the assumptions of \((Q,R)\) Policy which may fail in real life is that “the system is continuous review”. In most cases it is not possible to know on-hand inventory level at all times. Many of the inventory systems are under periodic review in practice. (You inspect your inventory level at certain times; at the beginning of each month, every 2 weeks etc)

c) Stock-out cost, \(p\), is the most difficult parameter to estimate. It includes the loss of goodwill cost, which is really hard to know. If a customer requests your product when you’re out of stock, your stock-out cost will not be limited with that customer’s demand. That customer may not come to your company again, or s/he may tell other people about your stock-out situation, which will affect your reputation very badly. All of these should be included in stock-out cost, so it is difficult to estimate.)