Question 1

We can easily see that the new discount policy is "incremental discount" as opposed to HW #4 Question 2 which was "all-units discount."

\[ C(Q) = \begin{cases} 
3.5Q & \text{for } 0 \leq Q < 100 \\
350 + 2.8(Q-100) & \text{for } 100 \leq Q < 250 \\
770 + 1.5(Q-250) & \text{for } 250 \leq Q 
\end{cases} \]

\[ C(Q) = \begin{cases} 
3.5 \frac{Q}{Q} & \text{for } 0 \leq Q < 100 \\
2.8 + \frac{70}{Q} & \text{for } 100 \leq Q < 250 \\
1.5 + \frac{395}{Q} & \text{for } 250 \leq Q 
\end{cases} \]

Average annual cost function:

\[ \bar{C}(Q) = \frac{C(Q)}{Q} + 25 + \int [C(Q)/Q] dQ \]

\[ x = 750 \]
\[ k = 25 \]
\[ I = 0.25 \]

\[ G_0(Q) = 750(3.5) + 25(750) + 0.25(3.5)Q/2 \]
\[ Q^{(0)} = \frac{\int 2(250750)}{0.25(3.5)} = 207.0 \text{ unfeasible} \]

\[ G_1(Q) = 750(2.8 + \frac{70}{Q}) + 25(750) + 0.25(2.8)Q \]
\[ = 750(2.8) + \frac{95(750)}{Q} + 0.25(2.8)Q + 0.2570(\frac{1}{Q}) \]
\[ Q^{(1)} = \frac{\int 2.95(750)}{0.25(2.8)} = 451.2 \text{ unfeasible} \]
\[ C_2(Q) = 750\left(1.5 + \frac{395}{Q}\right) + 25(750) + 0.25\left(1.5 + \frac{395}{Q}\right)\frac{Q}{2} \]

\[ = 750(1.5) + \frac{420(750)}{Q} + 0.75(1.5)\frac{Q}{2} + 0.75(395)\frac{Q}{2} \]

\[ Q^{(2)} = \sqrt{\frac{2(420)(750)}{0.05(1.5)}} = 1296 \text{ feasible} \]

They should order 1296 units

\[ C_2(Q^{(2)}) = 750\left(1.5 + \frac{395}{1296}\right) + 25(750) + 0.25\left(1.5 + \frac{395}{1296}\right)\frac{1296}{2} \]

\[ = \frac{1383.6}{14.5} \]

\[ = 97.38 \]

\[ C_2(Q^{(2)}) = \$1660.48 \]

\[ 1660.48 < 2250 \]

Yes, the decision to order more and get the discount is reasonable because it resulted in a lower average annual cost.

\[ \text{c) The cost of all-units discount was lower (\$1243.58) than incremental discount (\$1660.48). When incremental discount is used, lot size and time between replenishments increased dramatically (1.93 years). Since HD's can become obsolete quickly, an all-units discount policy which resulted in a lower average annual cost and more frequent replenishments seems to be much more advantageous for the communications firm.} \]
Question 2

Start by finding $Q^*$ from $EOQ = \sqrt{\frac{2KH}{S}}$

$EOQ_1 = \sqrt{\frac{2(\text{500})(\text{800})}{\text{0.20} \times (\text{85})}} \approx 217$

$EOQ_2 = \sqrt{\frac{2(\text{500})(\text{2300})}{\text{0.20} \times (\text{85})}} \approx 536$

$EOQ_3 = \sqrt{\frac{2(\text{500})(\text{1100})}{\text{0.20} \times (\text{50})}} \approx 191$

Where 1 = loveseat 2 = desk 3 = TV

The constraints are:

$\sum w_i Q_i^* = W \Rightarrow \text{space constraint}$

$\sum p_i Q_i^* = C \Rightarrow \text{budget constraint}$

Let's consider space constraint:

$85(217) + 55(536) + 20(191) = 51,745 > 15,725$

The constraint is violated.

We have to use the revised formula $Q_i^* = \frac{2KH_i}{W + \sum w_i L_i}$

By random iteration

Set $Q = 0.5$

$Q_1^* = \sqrt{\frac{2(\text{500})(\text{800})}{\text{0.20} \times (\text{85}) + \text{20} \times (\text{85})}} \approx 89$

$Q_2^* = \sqrt{\frac{2(\text{500})(\text{2300})}{\text{0.20} \times (\text{85}) + \text{20} \times (\text{85})}} \approx 191$

$Q_3^* = \sqrt{\frac{2(\text{500})(\text{1100})}{\text{0.20} \times (\text{50}) + \text{20} \times (\text{50})}} \approx 148$

$85(89) + 55(191) + 20(148) = 24038 > 15,725$

Constraint is still violated $\Rightarrow$ increase $\Theta$ to 1.5

$Q_1^* \approx 54 \quad Q_2^* \approx 115 \quad Q_3^* \approx 135$

$85(54) + 55(115) + 20(135) = 13615 < 15,725$

This time constraint is not violated but there is still space left in the warehouse $\Rightarrow$ decrease $\Theta$ to 1

$Q_1^* \approx 65 \quad Q_2^* \approx 140 \quad Q_3^* \approx 125$

$85(65) + 55(140) + 20(125) = 15,725$ The constraint is active and satisfied

This is the maximum utilization of the warehouse.
Rejection Method

First need to find upper and lower bounds for $\theta$.

Assume equal ratios

\[
\theta = \frac{15,725}{51,945} = 0.3039
\]

Lot sizes are:

- $0.3039(247) = 66$
- $0.3039(336) = 163$
- $0.3039(411) = 58$

Resulting $\theta$'s from these lot sizes are:

$\theta = 0.98 \quad \theta = 0.71 \quad \theta = 7.42$

Lower bound is $\theta = 0.71$, upper bound is 7.42.

Taking the median $\theta = \frac{0.71 + 7.42}{2} = 4.07$

New Q's are:

- $Q_1 = 34$
- $Q_2 = 71$
- $Q_3 = 76$

$85(34) + 85(71) + 20(76) = 8315 < 15,725$

We need to decrease $\theta$; new upper bound is 4.07

$\theta = \frac{0.71 + 4.07}{2} = 2.39$

New Q's are:

- $Q_1 = 44$
- $Q_2 = 92$
- $Q_3 = 94$

$85(44) + 85(92) + 20(94) = 10,680 < 15,725$

We should decrease $\theta$; new upper bound is 2.39

$\theta = \frac{0.71 + 2.39}{2} = 1.58$

Continuing like this, the best $\theta$ for labour the warehouse most efficiently without violating the constraint is found to be $\theta = 1.0$. 


Let's check if the budget constraint is satisfied.
$$85(65) + 40(1408) + 150(128) = 29,875 \leq 30,000$$

This is the most efficient use of our budget with the current warehouse.

b) Let's go back to EOQ's.

$$EOQ_1 = 2117$$
$$EOQ_2 = 536$$
$$EOQ_3 = 191$$

This time we look at the new budget constraint.

$$\frac{85(2117)}{18,445} + \frac{40(536)}{21,440} + \frac{150(191)}{28,650} = 68,535 > 15,000$$

Use scaling method. \( \frac{15,000}{68,535} = 0.2189 \)

$$Q^*_1 = 2117(0.2189) \approx 47$$
$$Q^*_2 = 536(0.2189) \approx 117$$
$$Q^*_3 = 191(0.2189) \approx 41$$

$$\frac{85(47)}{3995} + \frac{40(117)}{4680} + \frac{150(41)}{6150} = 14,825 \leq 15,000$$

We can still increase our lot sizes slightly by ordering one more TV, or 2 more loveseats, so as to use the allowable budget more efficiently.
Question 3

a) \[ T^* = \sqrt{\frac{2 \sum h_j}{\sum \frac{1}{T_j}}} \] \[ \text{where } h_j = h_j(1 - \frac{d_j}{p_j}) \]

\[ h_A' = 0.25(26)(1 - \frac{1800}{28000}) = 6.20 \]
\[ h_B' = 0.25(15)(1 - \frac{2500}{46000}) = 3.55 \]
\[ h_C' = 0.25(20)(1 - \frac{6000}{40000}) = 4.25 \]

Optimal Cycle Time \( T^* \) is calculated as:
\[ T^* = \sqrt{\frac{2(55 + 70 + 150)}{62(1300) + 3.55(2500) + 4.25(6000)}} \]
\[ = 0.1179 \text{ years} \approx 43 \text{ days} \]

b) Optimal Lot Sizes \( Q_j^* = X_j T^* \)
\[ Q_A^* = 1300(0.1179) = 153.97 \approx 153 \text{ units} \]
\[ Q_B^* = 2500(0.1179) = 294.79 \approx 295 \text{ units} \]
\[ Q_C^* = 6000(0.1179) = 707.4 \approx 707 \text{ units} \]

c) Time spent for producing each item \( T_j \cdot Q_j / P_j \)
\[ T_A = \frac{153}{28000} = 0.00546 \text{ yr} \]
\[ T_B = \frac{295}{46000} = 0.00641 \text{ yr} \]
\[ T_C = \frac{707}{40000} = 0.0177 \text{yr} \approx 6.5 \text{ days} \]

d) Total production time (uptime) = \( \frac{3}{5} T^* = T_A + T_B + T_C = 0.0276 \text{ years} \)
Idle time (downtime) = Cycle time - uptime = 0.1179 - 0.0276 = 0.0883 years

0.0883 * 365 = 32.6 days \( \Rightarrow \) The machine is idle.
a) Average annual cost

\[ G(T) = \sum_{j=1}^{\infty} \left( \frac{T_j}{T} + \delta_j \frac{T}{2} \right) \]

\[ = \frac{55 + 90 + 150}{0.1179} + \left[ 6.20 \times (1300) + 3.55 \times (2500) + 4.25 \times (6000) \right] \frac{0.1179}{2} \]

\[ = 2502 + 2502 = \$5004 \]