Q.1

\[ P = 6000 \text{/ year} \]
\[ \lambda = 3500 \text{ /year} \]
\[ K = 60 \]
\[ c = 3 \]
\[ I = 0.20 \]

First, find \( h' \)

\[ h' = h(1 - \frac{\lambda}{P}) = Ic(1 - \frac{\lambda}{P}) = (0.2)(3)(1 - \frac{3500}{6000}) \]

a) \( Q^* = \sqrt{\frac{2K\lambda}{h'}} = \sqrt{\frac{2 \times 60 \times 3500}{0.25}} = 1296.15 \text{ motors} \)

b) \( T_{up} = \frac{Q}{P} = \frac{1296.15}{6000} = 0.216 \text{ years} \Rightarrow 78.85 \text{ days} \)

\[ T = \frac{Q}{\lambda} = \frac{1296.15}{3500} = 0.363 \text{ years} \Rightarrow 132.35 \text{ days} \]

\[ T_{down} = T - T_{up} = 0.363 - 0.216 = 0.147 \text{ years} \Rightarrow 53.65 \text{ days} \]
Cycle time: up

Time between production runs ($T$) = 0.363 years

Time devoted to production in each production cycle ($T_{up}$) = 0.216 years

Downtime in each production cycle ($T_{down}$) = 0.147 years

Proportion of uptime = $\frac{0.216}{0.363} = 59.5\%$ of cycle time is uptime.

""" Downtime = $1 - 0.595 = 40.5\%$ of cycle time is downtime.

c) Maximum level of on-hand inv. ($H$)

$$H = \bar{Q} (1 - \frac{1}{P}) = 1296.15 \left(1 - \frac{3500}{6000}\right) = 540.06\ \text{motors}.$$

Maximum $\$ investment = H \cdot c$

$\Rightarrow (540.06)(3) = \$1620.18$

d) Annual cost of holding and setup is:

$$G(\bar{Q}) = \frac{Ks}{\bar{Q}} + \frac{h \cdot \bar{Q}^2}{2} = \frac{60(3500)}{1.36.15} + \frac{0.25(1296.15)}{2}$$

= $162.02 + 162.02 = \$324.04$
Notice that \( G(D) = \left[ 2k \times \frac{h}{c} \right] \)
\[ = \left[ 12.60 \times 500 \times (0.25) \right] \]
\[ = 324.04 \]

Annual Production Cost = 3.0 (3500) = 10,500
Total Annual Cost = 324.04 + 10,500 = 10,824.04

Annual Revenue = 4.5 (3500) = 15,750
Annual Profit = 15,750 - 10,824.04 = $4,925.96

e) \ T = 0.5 \ \text{years} \]
\[ T = \frac{Q}{\lambda} \]
\[ Q = 0.5 \times 3500 = 1750 \ \text{motors} \]

Annual holding + setup cost
\[ = \frac{k \lambda}{2} + h \sqrt{\frac{Q}{2}} \]
\[ = \frac{60 \times 3500}{1750} + 0.25 \sqrt{\frac{1750}{2}} \]
\[ = 120 + 218.75 \]
\[ = 338.75 \]
In part c we find the Annual holding + setup costs in optimal policy as $324.04.
As expected, if a non-optimal policy is used, annual holding + setup costs will increase.

$338.75 >$324.04.
\[ Q(2) = \begin{cases} 
3.5Q & \text{if } 0 \leq Q < 99 \\
2.8Q & \text{if } 100 \leq Q < 249 \\
1.5Q & \text{if } Q \geq 250 
\end{cases} \]

\[ k = 25 \]
\[ \lambda = 750 \text{ /year} \]
\[ I_0 = 0.25 \]
\[ Q(0) = \sqrt{\frac{2k\lambda}{I_0}} = \sqrt{\frac{2(25)(750)}{0.25(2.8)}} = 207.02 \]
\[ Q(1) = \sqrt{\frac{2k\lambda}{Io}} = \sqrt{\frac{2(25)(750)}{0.25(4.8)}} = 231.46 \]
\[ Q(2) = \sqrt{\frac{2k\lambda}{Ic}} = \sqrt{\frac{2(25)(750)}{0.25(1.5)}} = 316.23 \]

So, \( Q(1) \) and \( Q(2) \) are realizable.
\[ G(231.46) = \lambda c_1 + \frac{\lambda K}{Q^{(1)}} + \frac{1}{2} c_1 Q^{(1)} \]

\[ = 750(2.8) + \frac{750 \cdot 25}{231.46} + 0.25 \cdot (2.8) \cdot 231.46 \]

\[ = 2100 + 81.01 \]

\[ = 2181.01 \]

\[ G(316.23) = 750(1.5) + \frac{750 \cdot 25}{316.23} + 0.25 \cdot (1.5) \cdot 316.23 \]

\[ = 1125 + 59.29 + 58.29 \]

\[ = 1243.58 \]

With an order size of \( \approx 316 \), the firm could decrease its annual cost from \$2,250 to \$1,243.58. So, they should consider buying from the Discs Systems Co.