Q1) Monthly demand $X_m \sim (\mu_m = 1000, \sigma_m^2 = 250^2)$

a) Annual demand, $X_A$, find $\mu_A$ and $\sigma_A$.

$$X_A = \frac{X_m + X_m + \ldots + X_m}{12 \text{ months in a year}}$$

$$E[X_A] = E[X_m] + E[X_m] + \ldots E[X_m]$$

$$\mu_A = 12 \mu_m$$

$$\mu_A = 12 \times 1000 = 12,000$$

$$\sigma_A^2 = 12 \sigma_m^2$$

$$\sigma_A = \sqrt{12} \sigma_m = \sqrt{12} \times 250 \approx 866.$$  

b) Weekly demand, $X_w$, find $\mu_w$ and $\sigma_w$

$$X_m = \frac{X_w + X_w + X_w + X_w}{4 \text{ weeks in a month}}$$

$$E[X_m] = E[X_w] + \ldots E[X_w]$$

$$\mu_m = 4/\mu_w$$

$$\mu_w = \frac{\mu_m}{4} = \frac{1000}{4} = 250$$

$$\sigma_m^2 = 4 \sigma_w^2$$

$$\sigma_w = \frac{\sigma_m}{2} = \frac{250}{2} = 12.5$$
Q2) Note that, sum of normally distributed random variables is also normally distributed.

a) First find the total monthly demand, then convert it to total annual demand. Monthly demand for type 1.

Total Monthly demand: \( X_m = X_{m1} + X_{m2} + X_{m3} + X_{m4} \)

\[
E[X_m] = E[X_{m1}] + E[X_{m2}] + E[X_{m3}] + E[X_{m4}]
\]

\[
\mu_m = \mu_1 + \mu_2 + \mu_3 + \mu_4
\]

\[
\mu_m = 400 + 500 + 600 + 450
\]

\[
\mu_m = 1950
\]

\[
\text{Var}(X_m) = \text{Var}(X_{m1}) + \cdots + \text{Var}(X_{m4})
\]

\[
\sigma_m^2 = 60^2 + 65^2 + 70^2 + 65^2
\]

\[
\sigma_m = 130.19^2
\]

\( X_m \), total monthly demand is a normally distributed random variable with mean \( \mu_m = 1950 \), st. dev \( \sigma_m = 130.19 \)

Total Annual demand, \( X_A = \overbrace{X_m + \cdots + X_m}^{12 \text{ months in a year}} \)

\[
\mu_A = 12 \mu_m = 12 \times 1950 = 23400
\]

\[
\text{Var}(X_A) = \text{Var}(X_m) + \cdots + \text{Var}(X_m)
\]

\[
\sigma_A^2 = 12 \sigma_m^2
\]

\[
\sigma_A = \sqrt{12} \times 130.19 = 450.99
\]

\( X_A \), total annual demand is a normally distributed random variable with mean \( \mu_A = 23400 \), st. dev \( \sigma_A = 450.99 \)
6) Annual Revenue \( R_A = 50X_{A1} + 40X_{A2} + 30X_{A3} + 45X_{A4} \)

Remember that, \( X_{Ai} \) is the random variable denoting annual demand for the type \( i \).

\( R_A \) is also a normally distributed random variable.

\[
E[R_A] = 50E[X_{A1}] + 40E[X_{A2}] + 30E[X_{A3}] + 45E[X_{A4}]
\]

\[
E[R_A] = 50 \times 4800 + 40 \times 6000 + 30 \times 7200 + 45 \times 5400
\]

\[
E[R_A] = 939,000
\]

\[\times \] \( \text{Var}(R_A) = 50^2 \text{Var}(X_{A1}) + 40^2 \text{Var}(X_{A2}) + 30^2 \text{Var}(X_{A3}) + 45^2 \text{Var}(X_{A4}) \]

We need to calculate \( \text{Var}(X_{A1}), \text{Var}(X_{A2}), \text{Var}(X_{A3}) \) and \( \text{Var}(X_{A4}) \)

We know that \( \text{Var}(X_{m1}) = 60^2 \) (from the table in question)

\[
X_{A1} = \frac{X_{m1} + \ldots + X_{m1}}{12 \text{ months in a year}}
\]

\[
\text{Var}(X_{A1}) = 12 \times \text{Var}(X_{m1}) = 12 \times 60^2 = 43,200
\]

Similarly
\[
\text{Var}(X_{A2}) = 12 \times 65^2 = 50,700
\]
\[
\text{Var}(X_{A3}) = 12 \times 70^2 = 58,800
\]
\[
\text{Var}(X_{A4}) = 12 \times 65^2 = 50,700
\]

Put these into \( \times \) above.

\[
\text{Var}(R_A) = 1344,707,500
\]
$R_A \sim N(\mu = 939,000, 18,566.3^2)$

$P(R_A \geq 1,000,000) = P\left( \frac{R_A - 939,000}{18,566.3} \geq \frac{1,000,000 - 939,000}{18,566.3} \right)$

$= P(z \geq 3.29)$

$= 1 - \Phi(3.29) \approx 0$

0.9995 (from table 1 at the end of the book)
Q3. \( X_d \) = Number of visitors per day.

\( X_d \sim N(\mu=100, \sigma=40) \)

a) First, find the parameters for number of visitors per week.

\[
X_w = \overbrace{X_d + \ldots + X_d}^{7 \text{ days in a week}}
\]

\[
\mu_w = 7 \mu_d = 7 \times 100 = 700
\]

\[
\text{Var}(X_w) = \text{Var}(X_d) + \ldots + \text{Var}(X_d)
\]

\[
\sigma_w^2 = 7 \sigma_d^2
\]

\[
\sigma_w = \sqrt{7 \times 40} = 105.83
\]

\[
P(X_w \geq 450) = P\left( \frac{X_w - 700}{105.83} \geq \frac{450 - 700}{105.83} \right)
\]

\[
= P(2 \geq -2.36) = \Phi(2.36) = 0.9909
\]

b) \( P(X_w \leq 800) = P\left( \frac{X_w - 700}{105.83} \leq \frac{800 - 700}{105.83} \right) \)

\[
= P(2 \leq 0.9449) = \Phi(0.94) = 0.8264
\]
Q4. 

\[
\begin{align*}
26 & \quad 37 & \quad 27 & \quad 30 & \quad \ldots & \quad 38 \\
25 & \quad 20 & \quad 18 & \quad 41 & \quad \ldots & \quad 33 \\
32 & \quad 50 & \quad 37 & \quad 31 & \quad \ldots & \quad 38 \\
\end{align*}
\]

is the sample given in question.

a) \quad \text{Mean} = \overline{X} = \frac{26 + 37 + \ldots + 38}{30} = 31.567

\[
\begin{align*}
s^2 &= \frac{1}{30} \sum_{i=1}^{30} (X_i - \overline{X})^2 \\
&= 100.8747
\end{align*}
\]

\(s = 10.044\)

\[
\begin{array}{cccc}
\text{Center (c)} & \text{Frequency (f)} & \text{Product} \\
0 - 20 & 10 & 6 & 60 \\
21 - 29 & 25 & 7 & 175 \\
30 - 38 & 34 & 10 & 340 \\
39 - 47 & 43 & 5 & 215 \\
48 - 56 & 52 & 2 & 104 \\
\hline
& & & 894 \\
\end{array}
\]

\[
\begin{align*}
\overline{X} &= \frac{894}{30} = 29.8 \\
s^2 &= \frac{1}{29} \sum_{i=1}^{29} f_i (c_i - \overline{X})^2 = 156.786 \\
s &= 12.521
\end{align*}
\]