# Supply Chain Coordination in Electronic Markets: Auction and Contracting Mechanisms

Mingzhou Jin · S. David Wu

*Department of Industrial and Systems Engineering, P.C. Rossin College of Engineering*

*200 W. Packer Avenue, H. S. Mohler Laboratory*

*Lehigh University, Bethlehem, PA 18015*

mij2@lehigh.edu · david.wu@lehigh.edu

**Abstract:** In emerging electronic markets, the buyer often faces multiple competing suppliers in an auction setting. While auction typically serves as a price-determination mechanism in this environment, we show that it could also serve as a coordination mechanism for the supply chain. Different forms of auction and market mechanism change the nature of supplier competition, thus the buyer-supplier interaction. The paper establishes analytical results for a two-supplier one-buyer system under four different market schemes. We use complete information analysis to establish a few basic insights, but focus most of the exposition on *asymmetric information* cases. A two-part contract auction is proposed where the buyer announces a price-sensitive order *function*, while the suppliers compete in an ascending bid side payment auction. We show that channel coordination can be achieved if the market intermediary exerts effort to reduce the extend of information asymmetry, while restricting the buyer's profit on the side payments. Insights from the two-supplier one-buyer analysis allow us to rank market schemes by their impact on expected channel efficiency, expected profitability for the buyer, the winning supplier, and expected commission revenue for the market maker. (*Supply Chain Coordination, Contracting, Electronic Markets, Game Theory, Auctions)*

# 1 Introduction

Various forms of electronic marketplaces have emerged to handle business-to-business transactions. Forrester Research predicts that by 2004, 53 percent of business-to-business electronic commerce will constitute goods and services traded through *electronic markets* of corporate buyers and sellers. Despite of the slow-down of eCommerce and other high-tech sectors in 2001, the fundamental transition of industrial supply chains to the realm of electronic commerce is unavoidable, and remains the most critical economic trend in the decades to come. In this paper, we consider an environment for supply chain contracting where multiple suppliers are competing for the buyer's order. While this competition undoubtedly exists in traditional supply chains, in the new environment the competition/contracting processes are not paced over time, but are likely to occur simultaneously. Further, the dynamics of the competition is likely to change due to the increased scope. Freemarkets, a leading business-to-business eCommerce firm, reported in the first quarter of 2000 that some 4,000 suppliers and 47 buyers participated in their electronic market transactions. By the third quarter of 2001, they report direct access to some 150,000 suppliers and 120 major buyers. A main shift in the sourcing dynamics is that the buyer has the option of considering a large number of suppliers with relatively low searching costs. Other leading eCommerce firms such as CommerceOne and Ariba also report a similar phenomenon.

Contracting has been used as a means to coordinating buyers and suppliers in the supply chain to achieve higher system efficiency. The literature on supply chain coordination has its roots in economics, game theory, and more recently, supply chain contracts (c.f., the reviews by Cachon [2001] and Tsay et al. [1999]). Two basic issues in supply chain contracts are *double marginalization* [Spengler 1950] and *information asymmetry*. Since the buyer and the supplier represent different decision entities from different firms, without a deliberate coordination scheme their collective decision could be far from system-efficient. On the other hand, each party owns private information without necessarily the incentives to share, which makes information asymmetry an important consideration. As traditional supply chains are sequential, most researchers focus their analyses on the one-supplier, one-buyer building block. Several studies did examine the situation with competing retailers (buyers), focusing on the characterization of equilibrium behavior (c.f., Bernstein and Federgruen [2000], Carr et al. [1999], and Van Mieghem and Dada [1999]), but very few examine the parallel interaction among competing suppliers.

In this paper, we will focus on supplier competition and asymmetric information in a two-supplier, one-buyer setting. Other researchers also examine the impact of information asymmetry to contracting. Cachon and Lariviere [1999] propose contracts that promote information sharing. Chen [1997] considers the general issue of information decentralization in the supply chain. Corbett and Tang [1999] and Ha [1998] evaluate the value of information (buyer's unit handling cost) for the supplier under various two-part tariff contracts. Lee and Rosenblatt [1986] and Weng [1995] consider the use of quantity discounts to increase the supplier's profit. Also related to our analysis is the work in two-part tariff contract. Under complete information, Katz [1989] shows that the supplier could implement a two-part tariff contract to reach channel coordination while maximizing her own profit. Essentially, the buyer pays the supplier a fixed fee (tariff) then purchases the goods at the supplier's cost. Corbett and Tang [1999] investigate various two-part contracts under a one-supplier, one-buyer setting with the presence of asymmetric information. Their study focuses on supplier-initiated contracts in a retail setting.

Market intermediaries are not considered in previous supply chain contract literature. An intermediary, such as the market maker of an electronic market, is different from the centralized authority who optimizes channel efficiency. Current market makers are typically supported by eCommerce service providers with a profit interest of their own. However, with the right business model, market makers could extract significant benefit from channel efficiency [Wise and Morrison, 2000]. The paper aims to provide insights that allow market makers to compare market schemes and commission revenues, and to evaluate the impact on the suppliers, the buyers, and the supply channel. Resnick, Zeckhauser and Avey [1998] and Belknap and Harker [1999] describe various roles of electronic intermediaries. An *electronic intermediary* implements a particular market mechanism using some performance metric, while monitoring the action of market participants using observable information. The form of electronic intermediation and its underlying market mechanism could influence the efficiency of a particular market, while higher market efficiency sustains and attracts more customers in the long run. Auction is a popular form of market intermediation. Some well-known forms of auctions in one-buyer and multiple-seller environments are listed in Rasmusen et al [1992]. More recently, Beam and Segev [1998] and Herschlag and Zwick [2000] provide surveys on various Internet auctions. This line of literature focuses on different formats of implemented auctions on the Internet, but offer no economic insights in the supply chain context. This latter perspective will be addressed in this paper.

## 2 Two-Supplier, One-Buyer System with Complete Information

We consider a contracting environment with two suppliers and one buyer as depicted in Figure 1. An important feature is that the suppliers are competing to get the buyer's order, as is the case in the electronic markets discussed earlier.

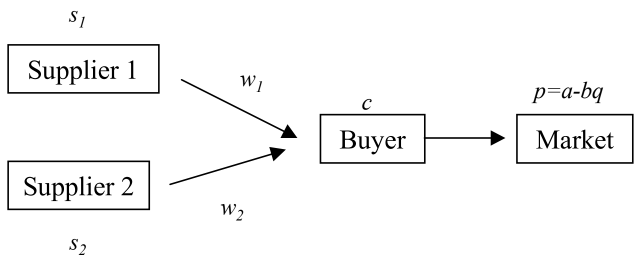


Figure 1. The Two Supplier-One Buyer Contracting Environment

Assume supplier $i$ has a fixed marginal cost $s_i$ and the buyer faces a unit handling cost $c$. Demand in the market is price sensitive as a linear market function: $p = a - bq$ ($p$: retail price, $q$: quantity, $a, b$: positive parameters). To ensure market competitiveness, we impose an additional condition that any supplier $i$'s marginal cost must satisfy $a - c - s_i \geqslant 0$ and $s_i \leqslant \frac{a-c+\min_j s_j}{2}$, otherwise supplier $i$ is considered non-competitive, and could not stay in this market.

Without the loss of generality, throughout our discussion we will consider two suppliers and Supplier 1 has lower cost. The channel coordinated system optimum has order quantity $q_c^* = \frac{a-c-s_1}{2b}$ and profit $\pi_{T,c}^* = \frac{(a-c-s_1)^2}{4b}$. In the following, we consider three different market schemes under complete information: *(c1) catalog auction without contract coordination, (c2) two-part contract initiated by the suppliers,* and *(c3) two-part contract initiated by the buyer.* To compare them with the one supplier case, please refer to the result in the paper of Corbett and Tang [1999].

## 2.1 *Scheme c1*: Catalog Auction without Contract Coordination

We first define *catalog auction* as follows:

1. Each supplier announces her wholesale price $w_i$ for general product categories on an electronic catalog. A supplier may *revise* her posted wholesale price in real-time if any other suppliers offer a lower price, so long as it is still higher than her own cost.
2. The buyer has direct access to the catalogs and will choose the supplier offering the lowest wholesale price.
3. The buyer determines her order quantity $q$.

Thus, without conducting a formal auction, the on-line catalog mechanism described above has the essential features of an auction. Under the above scheme, the buyer's decision problem is as follows:

$$b_{c1} : \max_{q} \pi_{b,c1}(q) = \max_{q}(a - c - bq - w_{c1})q \tag{1}$$

which yield the optimal order quantity $q^*_{c1} = \frac{a-c-w_{c1}}{2b}$. Thus, the supplier's problem is as follows:

$$S_{c1} \quad : \quad \max_{w_{c1}} \pi_{s,c1}(w_{c1}) \tag{2}$$

$$where\, \pi_{s,c1}(w_{c1}) \quad = \quad q^*_{c1}(w_{c1} - s) = \frac{a - c - w_{c1}}{2b}(w_{c1} - s) \tag{3}$$

Because of the competition from Supplier 2, the best strategy for (the lower cost) Supplier 1 is to set her wholesale $w_{c1} = s_2$ which will maximize her profit. Thus the profit for Supplier 1 is as follows:

$$\pi^*_{s_{1,c1}} = \frac{(a - c - s_2) \cdot (s_2 - s_1)}{2b} \tag{4}$$

The profit for the buyer is

$$\pi^*_{b,c1} = \frac{(a - c - s_2)^2}{4b} \tag{5}$$

with the order quantity $q^*_{c1} = \frac{a-c-s_2}{2b}$. Since $s_2 > s_1$, the system can not reach channel coordination and will result in a lower order quantity, which is consistent with the result of *double marginalization* as in the one-supplier, one-buyer system [Corbett and Tang, 1999]. However, due to competition, the buyer's profit increases while the system is closer to channel coordination (comparing to the one-supplier system*)*. Without the need of a contract, the buyer gains more profit by introducing competition among the suppliers. This is consistent with the intuition from the Bertrand model of price competition.

In principal/agent contracting literature [c.f., Fundenberg and Tirole, 1990], it is typically assumed that under complete information the principal (the buyer) will be able to enforce a selling price marginally above the agent's (the supplier) participation constraint defined by $\pi_s^-$(i.e., the minimal acceptable profit for the agent to participate in the transaction). Such an assumption would essentially eliminate all impacts of competition. We did not adopt the same assumption in Scheme $c1$ for the following reasons: first, catalog auction does not provide an *incentive compatible* mechanism that would motivate the agent to act in the interest of the principal. In other words, the principal does not have the means to enforcing a more favorable contract in this context. Second, under the setting of catalog auction, it would be unrealistic for the suppliers to justify a participation threshold that would truly represent the *individual rationality constraint*, as in the simple contract setting.

### 2.2 ***Scheme c2*: Two-Part Contract Auction Initiated by the Suppliers**

We now show that even under a supplier-initiated two-part contract, the buyer could still extract more profit by introducing competition. Consider a two-part contract auction as follows:

1. The suppliers compete in a two-part contract auction where they each proposes a wholesale price $w$, and a side payment $L$ to be charged to the buyer (e.g., the engineering fee).

2. The buyer first chooses the more favorable of the two contracts, then determines her order quantity $q$. The suppliers' decision problem is as follows:

$$S_{c2} \qquad \max_{w,L} \ \pi_{s,c2}(w_{c2}, L_{s,c2}) \tag{6}$$

$$where \qquad \pi_{s,c2}(w_{c2}, L_{s,c2}) = q^*_{c2}(w_{c2} - s) + L_{s,c2} = \frac{a - c - w_{c2}}{2b}(w_{c2} - s) + L_{s,c2} \tag{7}$$

>From previous results, we know that the suppliers should set the wholesale price at cost and extract profits from the side payment. Consider Supplier 2, since she has higher cost $w_{c2} = s_2$,

in order to compete with Supplier 1 she must lower her side payment $L$. However, the most she could do is to set $L = 0$ in which case she received no profit ($\pi_s = 0$). Thus, the buyer would extracted all channel profit, i.e., $\frac{(a-c-s_2)^2}{4b}$. Under complete information, this amount would be the minimum the buyer expects. In order to win the auction, Supplier 1 must offer a discount on the side payment no less than the amount $\frac{(a-c-s_2)^2}{4b}$. Thus, the best strategy for Supplier 1 is to offer the following contract:

$$L_{s,c2} = \frac{(a-c-s_1)^2}{4b} - \frac{(a-c-s_2)^2}{4b} \text{ and } w_{c2} = s_1 \tag{8}$$

Under this scheme, the system will achieve channel coordination, but the supplier can not extract channel profit as in the one-supplier, one-buyer case. Thus, the buyer gain a profit equal to $\frac{(a-c-s_2)^2}{4b}$ which is only possible because of the competition.

### 2.3 *Scheme c3*: Two-Part Contract Auction Initiated by the Buyer

We now propose a buyer-initiated two-part contract auction which also achieves channel coordination:

1. The buyer announces a side payment $L_b$ (e.g., shelf fee) to be charged from the suppliers, and commits to place an order based on a function of the wholesale price: $q = \frac{a-c-w}{b}$.
2. Given the announced order function, the suppliers compete in an auction of wholesale prices, which results in $w$
3. The buyer places order $q$ based on $w$ using the announced order function.

The above scheme achieves channel coordination as the buyer's order function transfers the market demand function to the supplier while taking into consideration the buyer's unit handling cost. In the case where Supplier 1 has a minimal required profit, $\pi_{s_1}^-$ known to the buyer, the buyer can extract the channel profit from the side payment as $L_b = \frac{(a-c-s_1)^2}{4b} - \pi_{s_1}^-$. Supplier 1 will win the auction with wholesale price $w = \frac{a-c+s_1}{2}$ and the resulting order quantity is$\frac{a-c-s_1}{2b}$. However, in reality, the buyer may not know with certainty the suppliers' profit functions, minimal required profits, or marginal costs.

## 3 Two-Supplier, One-Buyer System with Asymmetric Information

The above complete information analysis establishes basic insights for supplier competition in the two-supplier, one-buyer system. For the remainder of the paper, we will focus our attention on the more general case where the suppliers and the buyer hold asymmetric cost information. Each supplier $i$ knows only her own marginal cost $s_i$, and the buyer knows her own unit handling cost $c$. The market still faces a price sensitive demand line $p = a - bq$, where $a$ is only known by the buyer while $b$ is public information. We assume that there is a market intermediary who represents the third-party market maker. The buyer holds a prior probability density function $f(s)$ with supports $\underline{s}$ and $\bar{s}$ over the suppliers' marginal cost $s_i$, and she believes that $s_1$and $s_2$ are independent from each other. Supplier $i$ holds the same prior probability density function $f(s)$ over the *other* supplier's marginal cost $s_j$. This prior distribution function might be offered by the market intermediary, who has access to historic supplier information, and who is willing to share this information for the interest of attracting more customers into the market. We assume that these functions are correct and stationary and have credibility among all players. To streamline the analysis, we make some additional assumptions on the density function as follows. Define random variable $t = s - \underline{s}$, and random variable $T$ as the *larger* number of any two samples of random variable $t$. We assume that $f(s)$ satisfies the inequalities $E[t^2] \geq E[T] \cdot E[t]$ and $2E[T] \leq 3E[t] \leq 2.5E[T]$. These assumptions are satisfied by most distribution functions such as Uniform,Weibull and its special cases. We assume all players in this market to be risk-neutral, so they are only concerned with the expected profit. We again assume that the market imposes a competitiveness requirement, $\bar{s} \leq \frac{a-c+s}{2}$ on all participating players.

We examine four different forms of auction for the interest of achieving supply channel coordination: *(a1) wholesale price auction,* where the suppliers bid wholesale prices for an order with announced quantity, *(a2) catalog auction,* where the buyer determines the order quantity based on suppliers' posted catalog prices, *(a3) two-part contract auction,* where side payments are introduced for channel coordination, but the buyer's retail price and handling cost are observable by the market intermediary, and *(a4)* the same as *a3* except that the retail price and the handling cost are buyer's private information. The focus of our exposition will be to analyze the impact of different auction mechanisms on the interests of all participants, namely, the supplier, the buyer and the market intermediary. All players are risk neutral, expected profit maximizers.

While the profit maximization problem faced by each supplier and buyer are well defined, we will need to specify the profit model for the market intermediary. First of all, we believe that the market intermediary's primary concern should be to create the most efficient possible market environment for the supply channel which, in the long term, attract the most suppliers and buyers. Thus, the market intermediary should have the implicit long-term objective of implementing the channel-coordinated solution. Nonetheless, how the market maker extracts payment from market transactions would have a significant impact to the eCommerce firm's short-term profitability. We consider three commission schemes as follows:

1. *Membership Fee*. The buyers and the suppliers are asked to pay a fee as a member of the network and/or a participant in a market auction. The market maker may charge the participating firms a lump sum membership fee with a certain service package (e.g., participating in the auction with a reduced charge). It should be clear that under this scheme, the market maker has the long-term interest to increase membership in the network, and/or to increase participation in the transactions.
2. *Commission charged by the order quantity*. The intermediary charges a per-transaction commission as a non-decreasing function of the order quantity transacted in the auction. Only the

buyer and the winning supplier are required to pay the commission. Obviously the per unit charge varies depending on the product type. Here, the wholesale price resulting from the auction does not influence the payment.

3. *Commission charged by the transaction value.* The intermediary charges a per-transaction commission by the transaction value, which is a non-decreasing function of the wholesale price and order quantity. The side payment between the supplier and the buyer, if any, is also taken into consideration when calculating the transaction value.

In practice, the market maker may use a combination of the above commission schemes. Freemarkets, for instance, charges a participation fee, as well as a commission that is 1%-3% of the transaction *value*. In the following analysis, we will assume that all suppliers and buyers participating in the auction are charged a fixed membership fee, and we will evaluate separately the effects of the latter two commission schemes under the four auction settings (*a1* to *a4)*. This provides insights from the viewpoint of the market intermediary. The supply chain contracting literature pays little attention to total transaction value as it has no influence on the expected profits for the buyer, the supplier, nor the channel. Thus the market intermediary brings in a unique perspective where both the order quantity and the transaction value are potentially of interest.

### 3.1 The System Optimal Solution

We first establish the system optimal solution as a benchmark. Suppose there is a central agent in the system, and the suppliers and the buyer submit their true costs voluntarily, we may define a channel coordinated system optimal solution that determines which supplier would process the current order with what quantity so as to optimize the system profit. The central agent will assign the order to the supplier with smaller marginal cost, and the maximal profit $\pi^*_T(s_1, s_2)$ will be

$$\pi^*_{T,a}(s_1, s_2) = \frac{(a - c - \min(s_1, s_2))^2}{4b} \tag{9}$$

and the optimal quantity $q^*(s_1, s_2)$ will be

$$q_a^*(s_1, s_2) = \frac{a - c - \min(s_1, s_2)}{2b} \tag{10}$$

The expected maximal profit for the system will be

$$E[\pi_{T,a}^*] = 2\int_{\underline{s}}^{\bar{s}} \frac{(a - c - s)^2}{4b} f(s)[1 - F(s)]ds \tag{11}$$

and the expected optimal quantity with central agent will be

$$E[q_a^*] = \frac{a - c - 2E[s] + 2\eta}{2b} \tag{12}$$

here $\eta = \int_{\underline{s}}^{\bar{s}} sf(s)F(s)ds$ and $E[s] = \int_{\underline{s}}^{\bar{s}} sf(s)ds$, e.g., if the distribution function is uniform, $\eta$ would be $\frac{2\bar{s}+\underline{s}}{6}$.

### 3.2 *Scheme a1:* Wholesale Price Auction

In a buyer-centric electronic market, the most straightforward and widely adopted form of auction would be the *wholesale price auction*, also known as reverse auction. The sequence of events is as follows:

1. The buyer announces her order quantity $q$ with order specifications.
2. The suppliers bid on the wholesale price $w$ through a simultaneous descending bid auction. The market intermediary determines the market wholesale price from the result of the auction.
3. The order is transacted between the buyer and the winning supplier following the announced quantity

Since simultaneous descending bid auction is used, we will find the resulting wholesale price as $w(s_1, s_2) = max(s_1, s_2)$, i.e., the lowest price supplier will be rewarded the second lowest price. Note that from the buyer's perspective $s_1$and $s_2$ are random variables with known density function. The decision problem for the buyer is as follows:

$$B_{a1} \qquad \max_q E[\pi_{b,a1}(q)] \tag{13}$$

$$where \qquad \pi_{b,a1}(q, s_1, s_2) = (p(q) - c - w)q$$

$$= (a - c - bq - max(s_1, s_2))q$$

the optimal quantity $q_{a1}$ to $B_{a1}$is:

$$q_{a1} = \frac{a - c - 2\eta}{2b} \tag{14}$$

Given $(s_1, s_2)$, the profit for the buyer would be

$$\pi_{b,a1}(s_1, s_2) = \frac{[a - c + 2\eta - 2\max(s_1, s_2)](a - c - 2\eta)}{4b} \tag{15}$$

and the optimal expected profit for the buyer will be

$$E[\pi^*_{b,a1}] = \frac{(a - c - 2\eta)^2}{4b} \tag{16}$$

For any distribution function $f(s)$ defined in a bounded interval, we have $2\eta \geqslant E[s]$. Thus, we can find the expected optimal profit for the buyer through $\eta$, which is no less than half of the expected cost of the higher-cost supplier. On the other hand, each supplier $i$ knows her own marginal cost $s$ and the prior distribution of the other supplier's marginal cost. Thus, the expected profit for a specific supplier with marginal cost $s$ is as follows:

$$E[\pi_{s,a1}(s)] = \frac{a - c - 2\eta}{2b} \int_s^{\bar{s}} (x - s) f(x) dx \tag{17}$$

Given $(s_1, s_2)$ , the system's profit function is

$$\pi_{T,a1}(s_1, s_2) = \frac{(a - c - 2\eta)(a - c + 2\eta - 2\min(s_1, s_2))}{4b} \tag{18}$$

The expected value for the system's profit is

$$E[\pi_{T,a1}] = \frac{(a - c - 2\eta)(a - c + 6\eta - 4E[s])}{4b} \tag{19}$$

**Theorem 1** Regardless of the prior distribution on the supplier's cost, the wholesale price auction results in (a) lower expected total profit, and (b) lower expected order quantity, when compared to the system optimum.

The result concerning lower expected profit (part (a)) should be intuitive and is trivial to prove. The result concerning lower expected order quantity (part (b)) follows directly from (12), (14) and the fact that $2\eta \geqslant E[s]$. While the above results seem similar to the double marginalization results under complete information, it is in fact the result of asymmetric information, i.e., with uncertainty on the suppliers' cost, the buyer acts more conservatively and orders less than the optimal amount.

Let's now examine the market intermediary's expected profit. If the intermediary charges commission based on the order quantity with rate $r_1$, her profit is as follows:

$$\pi_{i,a1,2} = r_1 \frac{a - c - 2\eta}{2b} \tag{20}$$

When the commission is determined by the transaction value with rate $r_2$, the expected profit for the intermediary is

$$E[\pi_{i,a1,3}] = r_2 \frac{(a - c)\eta - 2\eta^2}{b} \tag{21}$$

Although *wholesale price auction* is popular in business-to-business eCommerce, the above results suggest that its performance could be improved from the viewpoints of the buyer, the market intermediary, and the system efficiency. We will later compare this result with other auction schemes.

### 3.3 *Scheme a2:* Catalog Auction

We now re-examine the *catalog auction* scheme introduced earlier. This is another common form of buyer-supplier interaction currently exist in eCommerce. Under this scheme, each supplier posts her wholesale price in an on-line catalog for each general product category, and she may *revise* her posted prices in response to competition. Kephert, Hanson and Greenwald [2000] study price dynamics of electronic catalog sales under software agents known as *shopbot* and *pricebots*. Given the seller-posted items and pricing in a variety of electronic catalogs, the buyer may deplete a shopbot (c.f., *addall.com* for book shopping) to compare prices for a particular product on the web, and to find the best prices. On the other hand, the seller may deplete a pricebot, an automated pricing agent, to investigate posted prices by her competitors and adjusts the posted catalog price automatically. The use of software agents such as the pricebot makes on-line catalog posting essentially a form of auction. *Books.com*, for instance, uses pricebot to automatically adjust its listed prices such that they are slightly less than the minimum price offered by *Amazon, Barnes and Noble,* and *Borders*, so long as the price is above the cost.

*Catalog auction* can be viewed as a variant of the *wholesale price auction* as follows:

1. The market intermediary administers a wholesale price auction for general product categories with simultaneous descending bid auction, this determines the wholesale price $w$.

2. The buyer chooses the order quantity $q$ based on the announced wholesale price.

The buyer's decision problem now becomes

$$B_{a2} \qquad \max_q \ \pi_{b,a2}(q, s_1, s_2) \tag{22}$$

$$where \qquad \pi_{b,a2}(q, s_1, s_2) = (p(q) - c - \max(s_1, s_2))q$$

The optimal quantity $q_{a2}(s_1, s_2)$ for the buyer can be expressed as

$$q_{a2}(s_1, s_2) = \frac{a - c - \max(s_1, s_2)}{2b} \tag{23}$$

and the optimal profit for the buyer is

$$\pi^*_{b,a2}(s_1, s_2) = \frac{(a - c - \max(s_1, s_2))^2}{4b} \tag{24}$$

The expected profit for the buyer will be as follows:

$$E[\pi^*_{b,a2}] = \frac{(a-c)^2 - 4\eta(a-c) + 2\int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds}{4b} \tag{25}$$

For supplier $i$ with cost $s$, her expected profit with given parameters $a, b$,and $c$ will be

$$E[\pi_{s,a2}(s)] = \frac{(a-c)\int_s^{\bar{s}}(x-s)f(x)dx - \int_s^{\bar{s}}(x-s)xf(x)dx}{2b} \tag{26}$$

**Theorem 2** When compared to wholesale price auction, catalog auction leads to (a) higher expected profit for the buyer, i.e., $E[\pi_{b,a2}(a,b,c)\ ] \geqslant E[\pi_{b,a1}(a,b,c)]$ , and (b) lower expected profit for the supplier, i.e., $E[\pi_{s,a2}(a,b,c,s)\ ] \leqslant E[\pi_{s,a1}(a,b,c,s)\ ]$. This is true for any given market parameters $a, b$, and $c$.

The formal proof of this theorem can be found in the Appendix. The total profit for the system can be expressed as follows:

$$\pi_{T,a2}(s_1,s_2) = \frac{a - c + \max(s_2,s_1) - 2\min(s_1,s_2)(a-c-\max(s_1,s_2))}{4b} \tag{27}$$

The optimal order quantity decided by the buyer is

$$q_{a2}(s_1,s_2) = \frac{a-c-\max(s_1,s_2)}{2b}$$

The expected order quantity is

$$E[q_{a2}] = \frac{a-c-2\eta}{2b} \tag{28}$$

The expected value for the system's profit is

$$\begin{aligned} E[\pi_{T,a2}(s_1,s_2)] &= 2\int_{\underline{s}}^{\bar{s}}[\int_{\underline{s}}^{s_2} \frac{(a-c+s_2-2s_1)(a-c-s_2)}{4b} f(s_2)ds_2]f(s_1)ds_1 \\ &= \frac{(a-c)(a-c+4\eta-4E[s]) + 2E^2(s) - 2\int_{\underline{s}}^{\bar{s}} s^2 f(s)F(s)ds}{4b} \end{aligned} \tag{29}$$

**Theorem 3** When compared to wholesale price auction, catalog auction yields (a) the same expected order quantity, but (b) higher expected system profit if and only if the following condition is satisfied: $2E[(M - E[s])^2] \geqslant 3E[(M - E[M])^2]$, where $M = max(s_1, s_2)$ and $E[M] = 2\eta$.

The above condition can be easily obtained by comparing (19) and (29). Given the suppliers' cost distribution function, the market efficiency of the *wholesale price auction* and *catalog auction* varies. Unfortunately, neither scheme could achieve channel coordination that maximizes the expected system profit.

**Theorem 4** Regardless of prior distribution on the supplier's cost, catalog auction results in (a) lower expected total profit, and (b) lower expected order quantity, when compared to the system optimum.

Because the wholesale price auction and the catalog auction result in the same expected order quantity (Theorem 3), the first part of theorem is trivial, while the proof for the second part is similar to that of Theorem 1. As to the market intermediary's profit, when the commission is charged based on order quantity and transaction value, we have $E[\pi_{i,a2,2}] = r_1 \frac{a-c-2\eta}{2b}$ and $E[\pi_{i,a2,3}] = r_2 \frac{(a-c)\eta - \int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds}{b}$, respectively. Comparing the market intermediary's profit with *Case a1*, we have the following result:

**Theorem 5** When compared to wholesale price auction, catalog auction generates for the market intermediary (a) the same profit level if commission is charged by order quantity, and (b) the same or less profit if commission is charged by transaction value.

Proof:

$$\begin{aligned} E[\pi_{i,a1,3}] - E[\pi_{i,a2,3}] &= r_2 \frac{\int_{\underline{s}}^{\bar{s}} s(s-2\eta) F(s) f(s) ds}{b} \\ &\geq 0 \end{aligned}$$

Because $\int_{\underline{s}}^{\bar{s}}(s-2\eta)F(s)f(s)ds = 0$ and the term $(s-2\eta)F(s)f(s)$ changes sign at $2\eta$ once from negative to positive. □

### 3.4 Coordination Based on the Two-Part Contract Auction

As shown by Theorems 1 and 4, neither of the above market schemes results in channel coordination . We now propose a two-part contract auction that would achieve channel coordination. The basic contract includes an order charge $w \cdot q$, and a side payment $L$ from the supplier to the buyer (e.g., rebate). The sequence of events is as follows:

*(Two-Part Contract Auction)*

1. The buyer announces an *order function* in $w$ : $q = \frac{(\hat{k}-w)}{b}$, where $\widehat{k}$ is determined by the buyer

2. The suppliers attend a simultaneous ascending bid auction on the side payment $L$.

3. The winning supplier sets the wholesale price $w^*$.

4. The order quantity is determined by the announced order function and the wholesale price $w^*$, i.e., $\frac{(\hat{k}-w^*)}{b}$.

5. The transaction takes place between the buyer and the winning supplier with the final order quantity.

Unlike the wholesale price auction, the buyer's order quantity is now based on the wholesale price, which gives the buyer more flexibility. The market intermediary guarantees that the final order quantity follows the announced order function. This gives the suppliers enough confidence to make their decisions $L$ and $w^*$ by solving a deterministic optimization problem. We separate the decisions of $L$ and $w^*$ so that the auction mechanism itself is simple and the suppliers would still reveal their true cost resulting in channel coordination. To analyze this scheme further, we consider two cases. In the first case, the buyer's retail price $p$ and the unit handling cost $c$ are both observable by the market intermediary. To improve the system efficiency, a restriction on the retail price is imposed. In the second case, the retail price and unit handling cost are private information owned by the buyer, hence no price restriction is possible.

Consider the practicality of the above scheme. First, consider the auction on the supplier's side payment $L$. This is similar in concept to the auction on *shelf fee* offered by large retailers to competing vendors. Further, the notion of using a buyer-announced *order function* is common in fuel procurement contracts in the utility industry [c.f., Bonser and Wu, 2001]. As the demand on energy and the fuel prices could both fluctuate significantly during the contract period, the contract agreement (which has to be specified ex ante) stipulates the order quantity as a function of a certain price index, to be observed over time as the demands unfold. In this context, the two-part contract auction could be augmented such that Steps 1 and 2 take place early on during contract negotiation, while Steps 3 to 5 occur over time as the demands unfold. However, this extension requires more complex dynamic game analysis which we will not address in this paper.

### 3.4.1 *Scheme a3:* Two-Part Contract Auction with Price Restriction

We first consider two-part contract auction under *Scheme a3:* market intermediary knows the buyer's handling costs, and has the authority to impose restriction such that the retail price covers the buyer's handling cost, while additional (buyer's) profit comes from the side payment. This scheme would stop the buyer from manipulating the retail price, and pass on the market demand truthfully to the suppliers (i.e., setting $\hat{k} = a - c$) , and channel coordination could be achieved. In the next section, we will examine the effects when no price restriction can be imposed.

**Theorem 6** If the market intermediary imposes restriction on pricing as $p = w + c$, the two-part contract auction yields channel coordination.

To prove this theorem, first consider the decision problems for the players involved. A supplier with marginal cost $s$ may calculate her maximal attainable profit as $\frac{(\hat{k}-s)^2}{4b}$ after the buyer's order function is announced (i.e., after the buyer announces $\widehat{k}$). Since the supplier who loses the side payment auction will have zero gain, the low-cost supplier must offer a side payment no less than $L(s_1, s_2, \hat{k}, b) = \frac{(\hat{k}-\max(s_1,s_2))^2}{4b}$. This will be the side payment resulting from the auction. As specified in step 3, the winning supplier from the side payment auction will solve her own decision problem to determine the wholesale price $w^*$. Suppose Supplier 1 wins the auction, she will solve the problem:

$$S_{a3} \qquad \max_{w} \ \pi_{s,a3}(w, s_1) = \frac{(w-s_1)(\hat{k}-w)}{b} - L \tag{30}$$

The solution to this problem is

$$w_{a3} = \frac{\hat{k}+s_1}{2} \quad \text{and} \quad q_{a3} = \frac{\hat{k}-s_1}{2b} \tag{31}$$

Since the market intermediary imposes a restriction on pricing as $p = w + c$, the profit maximizing problem for the buyer would be

$$B_{a3} \qquad \max_{\hat{k}} \ \pi_{b,a3}(\hat{k}, b, s_1, s_2) \tag{32}$$

$$where\ \pi_{b,a3}(\hat{k}, b, s_1, s_2) = \begin{cases} \frac{(\hat{k}-\max(s_1,s_2))^2}{4b} & \text{if } \hat{k} \leqslant a-c \\ \frac{(\hat{k}-\max(s_1,s_2))^2}{4b} - \frac{(\hat{k}-a+c)}{b} \cdot \frac{\hat{k}+\min(s_1,s_2)}{2} & \text{if } \hat{k} > a-c \end{cases}$$

When $\hat{k} > a - c$, the buyer will order $\frac{\hat{k}-\min(s_1,s_2)}{2b}$, but she can only sell $\frac{2a-2c-\hat{k}-\min(s_1,s_2)}{2b}$ with wholesale price $\frac{\hat{k}+\min(s_1,s_2)}{2} + c$ and she has to face a loss from the order. The solution for the buyer will be thus $\hat{k} = a - c$. In other words, the buyer will truthfully transfer the market demand function to the suppliers after deducting her unit handling cost. Thus, we get $q_{a3} = \frac{a-c-\min(s_1,s_2)}{2b} = q^*$ so that the system reaches channel coordination. The buyer will receive profit as follows:

$$\pi_{b,a3}(s_1, s_2) = \frac{(a - c - \max(s_1, s_2))^2}{4b} \quad (33)$$

Note that the buyer's profit is the side payment from the winning supplier, and the amount of the profit is equivalent to what she would receive under *catalog auction (a2)*. More interestingly, the supplier would receive higher expected profit under the current scheme. This is shown in the following theorem.

**Theorem 7** Compared to the catalog auction (a2), the two-part contract auction under Scheme a3 yields (a) the same expected profit for the buyer, but (b) higher expected profit for the supplier.

Proof: Part (a) of the theorem is trivial to prove. To prove part (b), suppose Suppliers 1 and 2 have marginal costs, $s_1$,and $s_2$, respectively, and $s_1 < s_2$. The profit for supplier 1 would be

$$\pi_{s1,a3}(s_1, s_2) = \frac{(a - c - s_1)^2}{4b} - \frac{(a - c - s_2)^2}{4b}$$

For the supplier with cost $s$, the expected profit is

$$E[\pi_{s,a3}(s)] = \frac{2(a - c)\int_s^{\bar{s}}(x - s)f(x)dx - \int_s^{\bar{s}}(x^2 - s^2)f(x)dx}{4b} \quad (34)$$

Comparing this with the expected supplier profit $E[\pi_{s,a2}(s)]$ under *catalog auction* , we can verify part (b) knowing the fact that $\int_s^{\bar{s}}(x - s)^2 f(x)dx \geq 0$. □

Recall that *catalog auction (a2)* generates less profits for the supplier than *wholesale price auction (a1)* would. In the following, we show that *Scheme a3* could be more attractive than *a1*for a supplier with cost below a certain threshold.

**Theorem 8** Compared to the wholesale price auction (a1), the two-part contract auction under Scheme a3 yields higher expected profit for the supplier who has a marginal cost $s \leqslant 4\eta - \bar{s}$ .

The proof of this theorem is in the Appendix. Although we can't guarantee all suppliers to have higher expected profit, the suppliers with cost below the stated threshold would benefit. This should benefit the supply chain in the long run. Obviously, the buyer gets more profit under the current scheme than she would under *wholesale price auction*.

Now consider the profit for the market intermediary. If the commission is charged based on order quantity, the intermediary's expected profit is $E[\pi_{i,a3,2}] = r_1 \frac{a-c-2E[s]+2\eta}{2b}$. When the commission is based on the transaction value, the expected profit is

$$E[\pi_{i,a3,3}] = r_2 \frac{(a-c)\eta - \int_{\underline{s}}^{\bar{s}} s^2 f(s)ds}{b} \tag{35}$$

**Theorem 9** Compared with wholesale price auction (a1) and catalog auction (a2), under two-part contract auction (a3) the market intermediary receives higher profit when the commission is based on the order quantity, but receives less profit when the commission is based on the transaction value.

The results of this theorem can be easily obtained by comparing the profit functions, and the fact that $\int_{\underline{s}}^{\bar{s}} s^2 f(s)ds > \int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s)ds$. Although the market intermediary may get less commission when it is based on the transaction value, overall, *scheme a3* is still quite attractive because it creates the best of conditions for the buyer and the suppliers when compared to the previous two schemes. In the long run, markets operating under more favorable conditions are likely to attract more customers, thus generating more transactions and create higher level of membership fee income.

### 3.4.2 *Scheme a4*: Two-Part Contract Auction without Price Restriction

We now consider two-part contract auction where market intermediary has no control over the retail price, while the unit handling cost $c$, and market parameter $a$ are both buyer's private information. We will show that in this case the buyer has incentive to manipulate the order function $(\widehat{k})$ for her own gain, and will not truthfully transfer the market demand function to the supplier. We assume that the slope of the market demand function $b$ is known publicly, but the base price $a$ is private information owned by the buyer. Again we assume that $s_1 \leqslant s_2$.

**Theorem 10** In two-part contract auction under Scheme a4, the buyer has incentive to inflate her unit handling cost $c$, or deflate the base market price $a$ when computing $\hat{k}$ in the order function (Step 1).

The buyer's decision problem is as follows:

$$B_{a4} \qquad \max_{\hat{k}} \; E[\pi_{b,a4}(\hat{k}, s_1, s_2)] \tag{36}$$

$$where \qquad \pi_{b,a4}(\hat{k}) = \frac{(2(a-c) - \hat{k} - s_1)(\hat{k} - s_1)}{4b} + \frac{(\hat{k} - s_1)^2 - (\hat{k} - s_2)^2}{4b}$$

Solving problem $B_{a4}$, we can get

$$\hat{k}_{a4} = a - c + 2E[s] - 4\eta \tag{37}$$

Because $2\eta - E[s] \geqslant 0$, the reported $\hat{k}_{a4}$ is less than the true value $a - c$.

**Theorem 11** Compared to wholesale price auction (a1) and catalog auction (a2), the two-part contract auction under Scheme a4 yields (a) higher system profit , but (b) equivalent expected order quantity.

Part (b) is easy to verify, the expected order quantity is as follows:

$$E[q_{a4}] = E[\frac{a - c + 2E[s] - 4\eta - s_1}{2b}] = \frac{a - c - 2\eta}{2b} \tag{38}$$

The expected profit for the system is

$$E[\pi_{T,a4}] = \frac{(4E[s] - 4\eta)^2 + (a - c)4\eta - 4E[s])(a - c) + 2E[s^2] - 2\int_{\underline{s}}^{\bar{s}} s^2 F(s) f(s) ds}{4b} \tag{39}$$

Comparing this expression with (19) and (29), we find that $E(\pi_{T,a4}) \geqslant E(\pi_{T,a1})$ and $E(\pi_{T,4}) \geqslant E(\pi_{T,a2})$. The proof is given in the Appendix.

**Theorem 12** Compared to the wholesale price auction (a1) and the catalog auction (a2), the two-part contract auction under Scheme a4 yields higher expected profit for the buyer.

This theorem is intuitively obvious. Since the buyer would get the same (more) profit as in the *catalog auction (wholesale price auction)* by truthfully transferring the market demand to the supplier (as in *a3*), the buyer would gain the same or more profit by manipulating $\hat{k}$.

Now consider the impact to the supplier. The wholesale price for the winning supplier would be as follows:

$$w_{a4} = \frac{a - c + 2E[s] - 4\eta + s_1}{2}$$

For the supplier with marginal cost $s$, the expected profit is

$$E[\pi_{s,a4}(s)] = \frac{2(a - c + 2E[s] - 4\eta)\int_{s}^{\bar{s}}(x - s)f(x)dx - \int_{s}^{\bar{s}}(x^2 - s^2)f(x)dx}{4b} \tag{40}$$

To compare the supplier's expected profit under this and other schemes would depend on the distribution function $f(x)$ and the marginal cost $s$, and is more difficult to reach a general conclusion.

By comparing $E[\pi_{T,a4}]$ and $E[\pi_{T,a3}]$, we may conclude that if the market intermediary exerts effort to learn the buyer's unit handling cost $c$ and market parameter $a$, she could bring in extra profit for the system of the amount $\frac{(E[s]-2\eta)^2}{b}$. For the interest of the market intermediary, charging commission based on the order quantity would be no different from the cases under wholesale price auction and catalog auction, i.e., $E[\pi_{i,a4,2}] = r_1\frac{a-c-2\eta}{2b}$. However, when the commission is based on the transaction value, the expected profit of the market intermediary is $E[\pi_{i,a4,3}] = r_2\frac{(a-c+2E[s]-4\eta)\eta-\int_{\underline{s}}^{\bar{s}} s^2 f(s)ds}{b}$, which is obviously less than the commission under *Scheme a3, or* $E[\pi_{i,a3,3}]$. In general, we may conclude that the market intermediary indeed has the incentive to learn the buyer's unit handling cost and to impose price restriction, as it brings more profit for herself and at the same time improves channel efficiency.

# 4 Discussion and Conclusions

In this paper, we examine supply chain coordination in the context of electronic markets. We show that different forms of market intermediation and the extend of information asymmetry change the nature of buyer-supplier interaction. In the following, we will conclude our findings and rank the four market schemes based on the viewpoints from different players.

*Complete Information Cases (Schemes c1 to c3):*

When competition is introduced among the suppliers, the buyer always benefit: in the case without contract coordination (*c1*), the buyer gets more profit since the suppliers face a Bertrand price competition which forces them to set a lower wholesale price then they otherwise would. The buyer even benefits from a supplier-initiated two-part contract auction (*c2*). This is true because the winning (lower cost) supplier must charge a lower side payment then her optimal, otherwise the buyer would choose the competitor. Nonetheless, this latter scheme does achieve channel coordination. In the case of a buyer initiated two-part contract (*c3*), we propose a mechanism where the buyer announces a price-sensitive order *function* and a fee for the supplier, the suppliers in turn determine the wholesale price. We show that this scheme achieves channel coordination.

*Asymmetric Information Case (Schemes a1 to a4):*

1. Most simple price auctions in procurement can be considered as a form of *wholesale price auction* where the buyer specifies features of the order (e.g., order quantity) before the auction, while the suppliers bid on the wholesale price through an auction. There is no intentional contract coordination in this case. We show that the auction results in lower expected order quantity and lower expected system profits when compared to the channel coordinated optimum. While this asymmetric information result seems similar to the complete information case with no contract coordination, rather than double marginalization, it is due to information asymmetry, i.e., with only the suppliers' cost distribution, the buyer acts more conservatively and places a smaller order.
2. Supplier posted catalog is also common in the electronic market. Since the pricing in catalogs could be real-time updated in response to competition, we argue that this is a form of auction: *catalog auction*. In this case, the buyer determines the order quantity using the catalog posted wholesale price (after competition). The Buyer's risk reduces due to the decreased pricing uncertainty, and is expected to gain more profit. The suppliers are expected to earn less profit due to competition. Comparing to the wholesale price auction, catalog auction yields the same expected order quantity, but achieves higher expected system profit. However, the expected order quantity and system profit are both lower than the system optimum.
3. We proposed a *two-part contract auction* where a side payment from the supplier to the buyer (e.g., shelf fee) is to be included as part of the contract. A key feature here is that the buyer announces a price-sensitive *order function* rather than a definite *order quantity*. Knowing this order function, the suppliers participate in a English auction on the side payment. The supplier who offers the highest side payment wins the order, this supplier then set the wholesale price based on the order function and her own cost structure. The final order quantity is determined by the buyer's order function and the wholesale price. The two-part contract auction achieves channel coordination when the market maker exerts efforts to learn the base market price and the buyer's handling cost, while placing restriction on the retail price. However, if the base market price and the buyer's handling cost are private information, the buyer would have incentive to inflate her handling costs or deflate the market price when computing the order function. This has a negative effect to the suppliers' profit, and channel coordination is no longer guaranteed. Nonetheless, in either case the two-part contract auction achieves higher channel efficiency than the commonly seen *wholesale price auction* and *catalog auction*.

*Ranking the Market Schemes*

We now rank the market schemes from the viewpoints of different players, and the system as a whole. Based on channel efficiency (measured in turns of total system profits), we shall rank the four market schemes in the order of *a3, a4, a2*, or *a3, a4, a1*. We could only rank *a2* above *a1* when the special condition stated in Theorem 3 is satisfied. On the other hand, the buyers would prefer the ranking of *a4, (a3* or *a2*), and *a1*. In general, two-part contract auction achieve better system efficiency *and* buyer profitability then the other two auctions. The suppliers would prefer the *wholesale price auction* (*a1*) over the *catalog auction* (*a2*). Of the *two-part contract auction,* the supplier would prefer *a3* over *a4*, while preferring *a3* over *a2*. Furthermore, a low-cost supplier as defined in Theorem 8 would prefer *a3* over *a1*. In general, the supplier's expected profit is more difficult to compare as it depends on the supplier's marginal cost distribution function.

The preference for market intermediary is more complex. For different industry segments, the market maker may have to adopt different market schemes according to business conventions, and to compete with other markets. If we limit our consideration to the expected commission revenue, the market intermediary would prefer *Scheme a3* over the other three when the commission is based on expected order quantity (recall that *a1, a2* and *a4* result in identical order quantity). However, when the commission is determined by the transaction value, the intermediary would rank the market schemes in the order of *a1, a2, a3* and *a4*. If a membership fee is charged, the market maker's interest is beyond transaction-based commissions. Attracting more customers into the market and/or conducting more transactions may have greater influence on the market's long-term success. To this point, we argue that the more efficient the market (the higher the total system profit) is, the more attractive it is for all players, and the more lucrative it will be for the market maker in the long-run. Along these lines, it maybe possible to create business models where the market intermediary is rewarded with a portion of the system profit (e.g., through equity ownership), such that channel coordination becomes a primary concern for market design. These points could be further investigated in future studies.

This paper focuses on a two-supplier one-buyer system, but similar results could be obtained for $n$-suppliers systems. Moreover, we assume the supplier's cost is linear in quantity, but in reality, the marginal cost may be convex or concave. In either case, a single distribution function is not sufficient to describe the supplier's cost under asymmetric information. This would complicate the buyer's order quantity decisions, especially under the wholesale price auction, and catalog auction. However, the buyer's decision under two-part contract auction will not be affected as much, and the system can still reach channel coordination.

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## Appendix

*Proof of Theorem 2.* To prove $E[\pi_{b,a2}(a,b,c)\,] \geq E[\pi_{b,a1}(a,b,c)\,]$

Because $\int_{\underline{s}}^{\bar{s}}(s-2\eta)F(s)f(s)ds = 0$ and $(s-2\eta)F(s)f(s)$ changes from negative to positive exactly once between $[\underline{s},\bar{s}]$, we can assume that $D = -\int_{\underline{s}}^{2\eta}(s-2\eta)F(s)f(s)ds = \int_{2\eta}^{\bar{s}}(s-2\eta)F(s)f(s)ds \geq 0$. Mean Value Theorem implies that

$$\begin{aligned}(E[\pi_{b,a2}\,] - E[\pi_{b,a1}\,])\cdot 2b &= \int_{\underline{s}}^{2\eta} s(s-2\eta)F(s)f(s)ds + \int_{2\eta}^{\bar{s}} s(s-2\eta)F(s)f(s)ds \\ &= D(\alpha-\beta)\end{aligned}$$

here $\alpha \in [2\eta, \bar{s}]$ and $\beta \in [\underline{s}, 2\eta]$, which indicates $\alpha \geq \beta$. Thus, we get the result $E[\pi_{b,a2}(a, b, c)\ ] \geq E[\pi_{b,a1}(a, b, c)\ ]$. $\square$

To prove $E[\pi_{s,a1}(s)\ ] \geq E[\pi_{s,a2}(s)\ ]$ :

Define $G(s) = (E[\pi_{s,a1}(s)\ ] - E[\pi_{s,a2}(s)\ ]) \cdot 2b = \int_{\mathrm{s}}^{\bar{s}}(x - 2\eta)(x - s)f(x)dx$

Obviously when $s \geq 2\eta$, $G(s) \geq 0$. In general, $G(s) = \int_{\underline{s}}^{\bar{s}}(x - 2\eta)(x - s)f(x)dx - \int_{\underline{s}}^{s}(x - 2\eta)(x - s)f(x)dx$

$$
\begin{aligned}
\frac{dG}{ds} &= 2\eta - E[s] \int_{s}^{\bar{s}} (2\eta - x)f(x)dx \\
\frac{d^2G}{ds^2} &= (s - 2\eta)f(s)
\end{aligned}
$$

Since $\frac{d^2G}{ds^2} \leqslant 0$, when $s \leqslant 2\eta$, $G(s)$ is a concave function in $s$ between $[\underline{s}, 2\eta]$. With the assumption that $E[(s - \underline{s})^2] \geq E[M - \underline{s}] \cdot E[s - \underline{s}]$, we know $G(\underline{s}) \geq 0$. So, we get $G(s) \geq 0$, for any $s$ in $[\underline{s}, \bar{s}]$, with the fact that $G(s) \geq 0$ when $s \geq 2\eta$. $\square$

*Proof of Theorem 8.* To prove $E[\pi_{s,a3}(s)] \geq E[\pi_{s,a1}(s)]$

when $s \leq 4\eta - \bar{s}$, clearly $s \leq 2\eta$

$$
\begin{aligned}
\Delta(s) &= (E[\pi_{s,a3}(s)] - E[\pi_{s,a1}(s)]) \cdot 4b \\
&= (\bar{s} - s)(4\eta - \bar{s} - s) - \int_{2\eta}^{\bar{s}} (4\eta - 2x)F(x)dx - \int_{s}^{2\eta} (4\eta - 2x)F(x)dx \\
&\geq (\bar{s} - s)(4\eta - \bar{s} - s) + F(2\eta) \int_{2\eta}^{\bar{s}} (2x - 4\eta)dx - F(2\eta) \int_{s}^{2\eta} (4\eta - 2x)dx \\
&= (\bar{s} - s)(4\eta - \bar{s} - s)(1 - F(2\eta)) \geq 0 \quad \square
\end{aligned}
$$

*Proof of Theorem 11.* To prove $E[\pi_{T,a4}] \geq E[\pi_{T,a2}]$

$$
\begin{aligned}
(E[\pi_{T,a4}) - E(\pi_{T,a2}]) \cdot 2b &= 8\eta E[s] + E[s^2] - 8\eta^2 - 3E^2[s] \\
&= \int_{\underline{s}}^{\bar{s}} (s - 2\eta)(s + 4\eta - 3E[s])f(s)ds
\end{aligned}
$$

$(s+4\eta-3E[s])f(s)$ changes sign from negative to positive only once at $3E[s]-4\eta$ between $[\underline{s}, \bar{s}]$, and $\int_{\underline{s}}^{\bar{s}}(s+4\eta-3E[s])f(s)ds = 4\eta-2E[s] \geq 0$. Let $D = \int_{3E[s]-4\eta}^{\bar{s}}(s+4\eta-3E[s])f(s)ds \geq 0$ and let $d = -\int_{\underline{s}}^{3E[s]-4\eta}(s+4\eta-3E[s])f(s)ds \geq 0$, we have $D \geq d$.

>From $\in [3E[s]-4\eta, \bar{s}]$ and $b \in [\underline{s}$,$3E[s]-4\eta]$. We can find $(b-2\eta) \leq 0$. So,

$$
\begin{aligned}
(a-2\eta)*D-(b-2\eta)*d &\geq (a+E[s]-4\eta)*D-(b+E[s]-4\eta)*D \\
&= (a-b)*D \geq 0 \quad \square
\end{aligned}
$$

To prove $E[\pi_{T,a4}] \geq E[\pi_{T,a1}]$

$$
\begin{aligned}
(E[\pi_{T,a4}]-E[\pi_{T,a1}]) \cdot 2b &= E(s^2)-\int_{\underline{s}}^{\bar{s}} s^2F(s)f(s)ds+4\eta E[s]-2E^2[s]-2\eta^2 \\
&= \int_{\underline{s}}^{\bar{s}}(s-2\eta)(s+2\eta-2E[s])(1-F(s))f(s)ds
\end{aligned}
$$

Because $\int_{\underline{s}}^{\bar{s}}(s+2\eta-2E[s])(1-F(s))f(s)ds = 0$ and $(s+2\eta-2E[s])(1-F(s))f(s)$ change only once from negative to positive at $2E[s]-2\eta$ between $[\underline{s},\bar{s}]$ and $\underline{s} \leq 2E[s]-2\eta \leq \bar{s}$, we define $D = \int_{2E[s]-2\eta}^{\bar{s}}(s+2\eta-2E[s])(1-F(s))f(s)ds \geq 0$

$$
\begin{aligned}
&\int_{\underline{s}}^{\bar{s}}(s-2\eta)(s+2\eta-2E[s])(1-F(s))f(s)ds \\
= \; &(\alpha-\beta) \cdot D \geq 0
\end{aligned}
$$

here $\alpha \in [2E[s]-2\eta, \bar{s}]$ and $\beta \in [\underline{s}, 2E[s]-2\eta]$. $\square$

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