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**Pickup and Delivery with Split Loads**

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Splitting loads such that the delivery of certain loads is completed in multiple trips rather than one trip results in opportunities for a reduction in cost and the number of vehicles used. Several studies have shown the benefit of split deliveries for the Vehicle Routing Problem, in which a vehicle operating out of a depot makes a series of deliveries on each route. In this paper, we quantify the benefit of using split loads for the Pickup and Delivery Problem. A heuristic to solve the Pickup and Delivery Problem with Split Loads is developed and applied to a set of random large scale problem instances, revealing the potential benefit of split loads. This benefit is reduced when the heuristic is applied to a real world trucking industry problem, due to several problem instance characteristics. The benefit of split loads is found to be most closely tied to three characteristics: load size, cost associated with a pickup or delivery, and the frequency with which loads have origins or destinations in common. Prior to a discussion of these results, we define the Pickup and Delivery Problem with Split Loads, and prove that for a set of given origins and destinations the most benefit can occur with load sizes just above one half of vehicle capacity.

**Key words:** vehicle routing; split pickup and deliveries

**History:**

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**Introduction**

As energy costs increase, driver shortages continue and hours of service regulations get tighter, transportation providers are motivated to use the capacity available to them as efficiently as possible. Excess capacity that is not utilized can result in declined service requests or additional costs, as vehicles that would otherwise not be necessary must be dispatched. Trucking firms are eager to eliminate any such occurrences, whether through finding loads for the backhaul portion of a route or consolidating loads from multiple customers onto a common vehicle. Improving capacity utilization through these methods is rather common. However, a more subtle extension of load consolidation is to allow multiple vehicles to service the same load. Splitting loads such that the delivery of certain loads is completed in multiple trips rather than one trip results in opportunities for a reduction in cost and the number of vehicles used. While this generally requires additional visits to a load’s origin and destination, it may eliminate a dedicated trip to deliver the load by
apportioning that load to other vehicles with excess capacity. Several studies have shown the benefit of split deliveries for the Vehicle Routing Problem (VRP), in which a vehicle operating out of a depot makes a series of deliveries on each route ((Dror et al. 1994), (Frizzell and Giffin 1995), (Sierksma and Tijssen 1998), (Archetti et al. 2006a)). In this paper, we apply the use of split loads to the Pickup and Delivery Problem (PDP), in which a vehicle picks up a load from a specific origin and delivers it to its destination. We call this the Pickup and Delivery Problem with Split Loads (PDPSL).

The authors were initially approached by a large 3PL provider (name withheld by request) to analyze one branch of their less-than-truckload pickup and delivery operation. This 3PL provider, operating with an annual revenue over $600 million, utilizes the fleets of various contract carriers to provide dedicated transportation services for a wide range of customers. In utilizing multiple fleets to service one customer, they found that many vehicles were traveling with excess capacity. Split loads were considered for load planning in order to potentially further reduce the inefficiencies associated with this pickup and delivery problem.

This paper introduces the PDPSL into the vehicle routing literature. To the best of the authors’ knowledge, the PDPSL has not been studied in the literature. We define the problem and identify an important structural result, showing that for a given set of pickup and delivery locations the most benefit occurs when load sizes are just above one half of vehicle capacity. We develop a heuristic to solve the PDPSL, with initial testing on random large scale problem instances revealing that significant cost savings can consistently be found with the use of split loads. However, when the heuristic is applied to the 3PL data and the business rules used by the 3PL provider were considered, almost all of the cost savings were eliminated (although there was a reduction in the number of routes used for service). We describe the real world conditions that limit the amount that cost may be reduced through the use of split loads.

The remainder of the paper is organized as follows. Section 2 provides a brief review of the pertinent literature. In Section 3 a problem definition is outlined for the PDPSL. Section 4 describes the potential benefit from split loads and provides a structural result for the problem. Section 5 describes the heuristic we developed. Section 6 presents the results from the computational testing of randomly generated problem instances. Section 7 describes the real world case study and the pertinent results found with these tests. Finally, Section 8 presents our conclusions.

2. Related Work

The two main bodies of routing literature that are relevant to this problem address the Vehicle Routing Problem (VRP) and the Pickup and Delivery Problem (PDP). The PDP is more relevant
to the work presented here; however, split loads have been most extensively applied to the VRP. This special case of the VRP is most commonly referred to as the Split Delivery Vehicle Routing Problem (SDVRP), which occurs when a destination may be serviced by multiple vehicles. The PDPSL is a more complex problem than the SDVRP, primarily because the available capacity of the vehicle changes each time a load is picked up or delivered for the PDPSL, without the vehicle ever returning to a depot. The SDVRP load planning is done with the same fixed capacity prior to a vehicle leaving the depot. With the PDPSL, searching through each instance of available capacity to determine where to insert a split load is significantly more difficult, as all loads that are to be picked up or delivered in between the pickup and delivery of the load to be inserted must be accounted for in evaluating the capacity. Despite the differences, some of the approaches used for solving the SDVRP are applicable to the PDPSL, such as determining how to divide a load that is to be split and finding routing improvements. We now review the pertinent SDVRP literature that has applied techniques similar to those found in this paper.

The SDVRP was first introduced by Dror and Trudeau (1989), who used a heuristic algorithm to find a cost reduction of almost 14% with split deliveries. These results were found by comparing the cost for a set of randomly generated problem instances solved as both a VRP and an SDVRP, with computational times under 30 minutes for the largest problems with 150 customers. Split loads were selected by determining the cost savings found by removing a load from a route and servicing portions of the load on at least two other routes. The loads were divided based on the available vehicle capacity on other routes. It was shown that no two routes can have more than one split in common, greatly limiting the number of splits that had to be analyzed. Dror and Trudeau (1990) later formally formulated the problem as an integer linear program and further developed their heuristic procedure.

Additional work on the SDVRP is found in Ho and Haugland (2004), who present a tabu search heuristic that takes into account time windows and applies four common move operators while simultaneously generating split loads. The split loads are created based on the amount of available capacity on a route when a load is to be placed on the route. This heuristic was used to solve the Solomon (1987) test problems with 100 customers in under 35 minutes. Archetti et al. (2006a) also apply a tabu search based heuristic to the SDVRP. Insertion moves are used for local improvement, with the possibility of inserting a customer into a route without removing it from another route. This heuristic improved upon the solutions found using the Dror and Trudeau heuristic; however, the most complex problem instances with 199 customers required an average of 189 minutes of computational time.
The PDP has been extensively studied, with Savelsbergh and Sol (1995) providing a thorough review of the earlier work on this problem. The PDP is a generalization of the Traveling Salesman Problem (TSP), making it NP-hard in the strong sense. This has focused most current research on heuristic methods. We leave the remainder of the PDP literature review for the section that presents the heuristic developed for this work, as it is most pertinent there.

3. Problem Definition and Assumptions

Prior to discussing the benefits of split loads, we first define the problem and the relevant constraints, describing the graph of the road network, presenting an objective for the problem, and defining a split load.

The PDPSL is defined on a road network composed of edges that connect a set of vertices representing the load origins and destinations. A single vehicle traverses this network, incurring a cost to travel each edge that is equivalent to the distance between the vertices connected by the edge. When the vehicle arrives at an origin it may pickup any portion or all of a load requiring service, as long as capacity constraints are not violated. When the vehicle arrives at a destination, all loads terminating at this location are removed from the vehicle. A location may simultaneously serve as both an origin and a destination. A route is the sequence of origins and destinations that the vehicle follows in servicing all loads. The cost of a route is equivalent to the distance the vehicle travels while completing the required service. The objective of this problem is to minimize the cost of this route.

We assume that there is a single capacitated vehicle to service all requests. A review of the literature on the PDP by Savelsbergh and Sol (1995) indicates that the most studied variant of this problem is the static single vehicle PDP. Also, if multiple vehicles are used without a depot, the number of vehicles is greater than the number of customers, and there are no additional constraints, solving any problem instance is trivial as every load is delivered by an independent vehicle that only services that load. In the real world case presented in Section 7, multiple vehicles are allowed, but additional constraints are applied.

Given the above conditions, a load may be serviced on multiple trips by the vehicle. However, every load that is serviced by more than one trip is not necessarily considered to be split (i.e., a load that has a size greater than vehicle capacity will require at least two trips). The definition of a split load used in this paper is similar to that found in the SDVRP literature (Archetti et al. 2006c). If a load is partitioned into more than the minimum number of divisions for full service, it is considered to be split. For example, if a load is 3.6 truckloads and vehicle capacity is 1, a minimum of four trips are required to fully service the load. If the load is serviced in five or more trips, than it is considered to be split.
4. Evaluating the Benefit of Split Loads

Splitting a load may initially appear to be a cost increasing move. The delivery of a split load requires additional trips between, as well as stops at, the origin and destination of the load. However, there are many instances that result in cost savings through a reduction in total vehicle travel. In this section, the potential benefit from split loads is discussed and the greatest opportunity for cost savings for a given set of origins and destinations is shown to occur when load sizes are just over one half of vehicle capacity.

Figure 1 presents an example of the potential for savings through the use of split loads. Three loads, A, B, and C, each of size 0.6, are to be delivered by a vehicle with a capacity of 1. The distance between the origin of load A, O_A, and its destination, D_A, is X. The distances O_B − D_B and O_C − D_C are also X. The distances O_A − O_B, O_B − O_C, D_A − D_B and D_B − D_C are each 1/X. The distances O_A − O_C and D_A − D_C are 2/X. Assume that the vehicle begins at origin C.

Without split loads, the vehicle must deliver each of the loads individually, as capacity constraints prohibit the combination of any loads on the vehicle. With split loads, the vehicle may pick up load C, split load B by picking up 0.3 truckloads, and deliver both to their respective destinations. It may then return for the other half of load B, pick up load A, and deliver both to their destinations. The result of using split loads is then the elimination of two of the arcs between the origins and destinations with the addition of two trips between the origins and two between the destinations. As X becomes large, the cost of delivering loads is reduced from 5X to 3X through the use of split loads.
loads. While using split loads results in an additional visit to both an origin and a destination, benefit is found through the elimination of two trips between the origins and destinations. It is important to note that most benefit with split loads is found through route length reduction.

As the number of loads to be delivered is increased in this example the savings converge to a constant. Consider two additional loads, \(D\) and \(E\), each of size \(0.6\) with the distances \(O_C - O_D\) and \(D_C - D_D\) equal to \(1/X\), and \(O_C - O_E\) and \(D_C - D_E\) equal to \(2/X\). If \(X\) is large, the minimum cost is \(5X\) with split loads and \(9X\) without split loads. Generalizing the cost with \(k\) loads each of size \(d \leq Q\), where \(Q\) is the vehicle capacity, the cost with split loads is \((2\lceil k d \rceil - 1)X\), while the cost without split loads is \((2\lceil k \lfloor Q/d \rfloor \rceil - 1)X\). If \(d > \frac{Q}{2}\) and \(k\) increases to infinity, the ratio of the two costs goes to \(\frac{Q}{d}\). This ratio grows as \(d\) approaches \(\frac{Q}{2}\), reaching a maximum value of two. The two costs are equivalent for load sizes equal to \(\frac{Q}{m}\), for any integer \(m > 0\). For \(d < \frac{Q}{2}\) and for each \(m > 2\), as the load size decreases from \(\frac{Q}{m-1}\), the ratio of costs approaches \(\frac{m}{m-1}\) until it reaches the size \(\frac{Q}{m}\). Therefore, the maximum cost ratio is reached at just above one half of vehicle capacity. This result may be generalized for any set of origins and destinations.

**Theorem 1** Given the origin and destination locations of a set of \(k\) loads, a vehicle of capacity \(Q\), and a very small value, \(\epsilon\), let \(v(PDPSL)\) be the cost of the optimal PDPSL solution to deliver these loads and \(v(PDP)\) be the cost of the optimal PDP solution. Then the ratio \(\frac{v(PDP)}{v(PDPSL)}\) is maximized when the loads are all of size \(\frac{Q}{2} + \epsilon\), as \(k \to \infty\).

**Proof:** Given a fixed number of loads and set of origin and destination locations, the largest reduction in cost through the use of split loads occurs when the ratio of loads that must be delivered individually in a PDP solution relative to a PDPSL solution is maximized. As this ratio increases, fewer loads may be placed on the vehicle concurrently for delivery with the PDP solution relative to the PDPSL solution. With less opportunity to deliver loads simultaneously, \(v(PDP)\) increases relative to \(v(PDPSL)\). Therefore, the load size that maximizes the ratio of loads that must be delivered individually in a PDP solution relative to a PDPSL solution is also the load size that maximizes the ratio \(\frac{v(PDP)}{v(PDPSL)}\).

Consider any positive integer \(m \geq 2\). If all load sizes are \(\frac{Q}{m} + \epsilon\), we can combine \(m - 1\) loads without splitting, or \(m\) loads with splitting (after removing each \(\epsilon\)). Therefore, the ratio of the number of loads that must be delivered individually between the PDP and PDPSL cases will be \(\frac{\lceil m \rceil}{\lceil m \rceil + 1}\). As \(m\) increases (i.e., as load sizes decrease) and \(k \to \infty\), this ratio decreases.

When \(m = 2\), the ratio of the number of loads that must be delivered individually between the non-split and split cases becomes \(\frac{k}{k + \epsilon k}\). This ratio decreases with an increase in \(\epsilon\) (especially when \(\epsilon > \frac{Q}{k}\), the \(\epsilon\) portions of the loads can no longer be combined onto one vehicle.) Therefore, the
ratio of loads that can be delivered concurrently in a PDP solution relative to a PDP SL solution is maximized when load sizes are equal to $\frac{Q}{2} + \epsilon$. □

This theoretical result is supported experimentally in Section 6. A similar result is shown empirically for the SDVRP in Archetti et al. (2006b).

In the example above, the maximum ratio of costs was found to be two. Determining a bound on this cost ratio for any problem instance is of interest. While a bound of two may be shown to exist for specific simple cases of the PDP SL (e.g., when an optimal PDP SL route has at most one split load on the vehicle at any time), a general bound is more difficult to define. While we are not able to prove this for every instance, we have not found a case where all loads in an optimal PDP SL solution can be delivered complete with a cost that is greater than double that of the PDP SL cost. Therefore, we make the following conjecture.

**Conjecture 2** Given any set of loads requiring service, assume that $v(PDPSL)$ is the cost of the optimal PDPSL solution to deliver these loads and $v(PDP)$ is the cost of the optimal PDP solution. Then,

$$v(PDP) \leq 2v(PDPSL)$$

### 5. PDPSL Heuristic

We have shown by example that there is a benefit from the use of split loads, and this benefit may have theoretical bounds. We will now computationally quantify this benefit for sets of random and real world instances. In order to determine the benefit in realistic routing situations, a solution method is required to solve large scale problem instances. As stated earlier, the Pickup and Delivery Problem is NP-hard, with most research focusing on heuristic solution methods. The PDPSL is a relaxation of the PDP, as the origin and destination of each load must no longer be visited a fixed number of times. However, it is still an NP-hard problem.

**Theorem 3** The PDPSL is NP-hard.

Proof: In order to show that the PDPSL is NP-hard, we reduce the Split Delivery Vehicle Routing Problem, an NP-hard problem (Dror and Trudeau 1990), to the PDPSL. Consider a special case of the PDPSL where all loads have one common origin and the costs to travel the road network satisfy the triangular inequality. The SDVRP can be solved in polynomial time if this case of the PDPSL is solvable in polynomial time. The SDVRP is then reduced to the PDPSL. □

We have developed a heuristic for the solution of the PDPSL that generates an initial feasible PDP solution and then selects loads for splitting using a set of guidelines we established. Traditional
features of the Clarke and Wright Savings Algorithm (CW) (Clarke and Wright 1964) are combined with some common local search techniques to further improve the solution.

Dedicated route segments are initially created for each individual load, after which a load is selected for splitting. Split loads are randomly generated, influenced by the cost of creating the split. A load is randomly selected and its size is compared to all occurrences of excess capacity along the route. If the size of the load is greater than any of these occurrences of excess capacity, it may be split. The load is split such that the portion of the load that is moved has the same size as the excess capacity at the location in the route that it is moved to. That is, if a load of size 0.9 truckloads is to be split, and there are 0.6 truckloads of excess capacity elsewhere on the route, 0.6 truckloads of the load will be delivered along this portion of the route, while 0.3 truckloads are delivered along the original portion of the route. Figure 2 depicts this split load creation, where Route Segment 1 has the excess capacity to be filled and load $B$ is to be split. In this example (not pictured), load $A$ could also be split such that 0.1 truckloads is transported on route 2 and the remaining 0.3 is left on route 1.

A split load is selected for use in the route based on a random number that is generated for each potential load to be split. The cost of creating the split influences the generation of this random number. Any split, regardless of cost, has some likelihood of selection. However, a split load with a lower cost will generate a random number that results in a greater probability of selection. A list of selected split loads is maintained to insure that splits are not returned to their original state with local improvements and to prevent the same split from being repeatedly selected.

Figure 2  Split load creation example in which load $B$ is split and placed on Route Segment 1.
After a load is split, route segments are combined based on cost savings, similarly to the CW Algorithm used for the VRP. The route segments may be combined such that one is placed after another. If capacity constraints allow it, one route segment may also be inserted into another segment. After all possible combinations are made, several local search techniques are utilized to improve the solution, including:

1. Load swaps
2. Load insertions
3. Reordering of origins or destinations.

We now present the algorithm pseudo-code of the heuristic (when a cost is updated, the corresponding solution is updated as well):

**Generate routes:** Create an initial feasible solution and set this as the basic solution with cost \( BASE \). Set \( \text{iter}1 \) and \( \text{iter}2 \) equal to 0.

While (\( \text{iter}1 < \text{ITERMAIN1} \))

\[
\text{Set TEMP equal to BASE.}
\]

**Create Split Loads.** Update \( TEMP \).

Add split load to tabu list and set \( WORK \) equal to \( TEMP \).

While (\( \text{iter}2 < \text{ITERMAIN2} \))

\[
\text{Combine Routes. Update TEMP.}
\]

**Load Swap.** Update \( TEMP \).

**Load Insertion.** Update \( TEMP \).

If \( TEMP < WORK \), set \( BASE \) equal to \( TEMP \), and \( \text{iter}2 = 0 \).

If \( TEMP \geq WORK \), increment \( \text{iter}2 \).

\[
\text{iter1 = iter1 + 1}
\]

\( \text{ITERMAIN1} \) and \( \text{ITERMAIN2} \) are selected to balance performance and computational time. A detailed description of the algorithm can be found at http://www.georgiasouthern.edu/mnowak/publications.htm.

Bent and Hentenryck (2006) develop a similar algorithm for the general PDP, in which a simulated annealing approach is successfully used to decrease the number of routes while a large neighborhood search is used to decrease total travel cost. They were able to improve 76% of the
best solutions for 600 customer benchmark problems, with a computational time under 90 minutes. The tabu search heuristic developed by Nanry and Barnes (2000) is of some interest as the move neighborhoods are similar to those applied in the heuristic presented here. In order to generate solutions, origin and destination locations may be either inserted from one route to another or swapped with an origin-destination pair on another route. However, vehicle capacity constraints may be violated by these moves, with a third type of move inserting individual origins or destinations forward or backward within a route to improve the solution quality or feasibility. This heuristic dramatically reduced the computational time for solving a modified set of benchmark problems for the VRP with time windows. Similarly, Toth and Vigo (1997) presented a parallel insertion heuristic that uses tabu thresholding with several moves like those used in this work. They found significant improvements over hand-made solutions for the Handicapped persons Transportation Problem. Landrieu et al. (2001) also present a tabu search heuristic that uses local improvements. These moves are simpler than those proposed by Nanry and Barnes (2000) or those used in this work, as vertices are swapped or inserted within a route, while respecting precedence and vehicle capacity constraints.

The heuristic presented here was tested on both randomly generated and real world data. We first discuss the results for the randomly generated problem instances.

6. Computational Testing of Random Instances

In order to gain insight into the structural characteristics of the PDPSL and to quantify the average savings of split loads with large scale data, the heuristic was applied to a set of random problem instances. While the Solomon (1987) problems have long existed as a benchmark for testing the VRP with time windows, there is not a set of similar problems for the PDP. Therefore, we generated our own set of problems that are similar in scale to those used for testing the SDVRP. Using the random problem instances, we show that some benefit can be found for almost all problems, with certain instances displaying a cost reduction of over 40% with the use of split loads. The average outcome from testing also indicates that the most benefit is found when load sizes are just over one half of vehicle capacity, supporting Theorem 1 in Section 4.

6.1. Experimental Design

The heuristic was tested on problem sets of three sizes, with 75, 100, and 125 transportation requests. These sizes are similar to the problem sizes used in testing of the SDVRP (Dror and Trudeau 1989, 1990). Each transportation request contains the origin and destination location coordinates, and the fraction of a truckload to be delivered. Coordinates for the pickup and delivery
locations were randomly generated with a uniform distribution over the range \([-40,40]\) for both X and Y coordinates. Each problem size has five origins from which loads could be picked up. The 75 request problem then has 15 destination locations, the 100 request problem has 20 destination locations, and the 125 request problem has 25 destination locations. The networks represent real world situations in which a smaller set of distribution centers services a large number of retail outlets. Testing with a different number of origins found no significant changes to the results (including an equal number of origins and destinations). Every origin-destination combination has a load to be delivered between the two locations. Three different origin and destination location sets were generated for each problem size. The variance was negligible when the different location sets were tested with common load sizes, indicating that the distribution of origins and destinations had little effect on this problem. These data sets are large enough to provide meaningful numerical results; however, larger data sets were tested for the case study presented in Section 7.

The load sizes were also randomly generated. The sizes were all less than or equal to vehicle capacity. This was done in order to determine the load sizes that benefit most from splitting that can otherwise be serviced by a vehicle in one trip without splitting. The case study discussed in Section 7 presents tests run with load sizes greater than vehicle capacity. Without loss of generality, the vehicle had unit capacity. In order to provide insight into the problem, the load sizes were generated within a variety of ranges. Two different range groupings were used when determining load size.

The first set of groupings contained loads generated within narrow ranges of size. The load sizes that provided the most (or least) benefit were evident through the use of narrow load ranges. Eight ranges were used to bound the load sizes, with five different sets of load sizes generated for each range. The ranges indicate the upper and lower bound (inclusive of the bound) on the fraction of the vehicle capacity that the load can occupy, and these were \([0.11-0.2]\), \([0.21-0.3]\), \([0.31-0.4]\), \([0.41-0.5]\), \([0.51-0.6]\), \([0.61-0.7]\), \([0.71-0.8]\), and \([0.81-0.9]\). Load sizes below 0.1 or above 0.9 were not used as splits of these loads rarely occur and any benefit was negligible. The load sizes were randomly generated over each range with a uniform distribution. Each of the three location configurations was matched with each of the five load sets within a load range, resulting in 15 different instances for each load range, and 120 instances overall.

The second set of groupings consisted of loads generated within wider ranges of size. The wider load ranges display the benefit that may be found in a real world setting in which load sizes are more varied. Four ranges were used to bound the load sizes, with 10 different sets of load sizes for each range. The ranges were \([0.1-0.5]\), \([0.5-1.0]\), \([0.3-0.7]\), and \([0.1-1.0]\). The load sizes were randomly
generated over each range with a uniform distribution. Each of the three location configurations was matched with each of the ten load configurations, resulting in 30 different instances for each load range, and 120 instances overall.

The heuristic was used to solve each problem under two scenarios, both with and without split loads. The heuristic without split loads omitted the split load generation step and iterated through the local improvement steps. The heuristic was coded in C and all experiments were run on a 2.4 GHz Xeon processor with a 400Mhz frontside bus and 2 GB RAM.

6.2. Experimental Results

Figure 3 presents the average percentage increase in cost when split loads are not allowed for the 75, 100 and 125 request problem sets with narrow load size ranges. The costs both with and without split loads are based on the distance traveled by the vehicle. With the exception of loads in the range \([0.81-0.9]\), some benefit is found with the use of split loads for every load range. However, more benefit is found with certain load sizes. These numerical results support a theoretical result presented earlier. That is, the most significant benefit with split loads is found when the load sizes are just above one half of vehicle capacity, in the range \([0.51-0.6]\). When splitting is allowed with these load sizes, full truckloads are created through the combination of 1/2 truckloads, with the remainder of the loads delivered on an additional route. The benefit becomes almost negligible in the range \([0.41-0.5]\), as two loads can be simultaneously serviced by the vehicle without any splitting. The loads are large enough that when two unsplit loads are placed on the vehicle, there is little room for a split load to be inserted. The benefit increases in the range \([0.31-0.4]\), as there is more space for split loads when two unsplit loads are simultaneously on the vehicle.

Further decreasing the load sizes results in less of a need for splitting, as unsplit loads can more easily be combined on a capacitated vehicle. However, the benefit in reduced cost for the range \([0.11-0.2]\) is greater than that for the larger load size range \([0.21-0.3]\). The heuristic is able to find more benefit with split loads for smaller load sizes as there are more potential combinations of loads to be placed on the vehicle at the same time, even without splitting. In searching for potential splits to create, the heuristic with split loads may explore more of these combinations. This is reflected in Table 1, which presents the average CPU time in minutes and the average number of splits per route, as the computational time increases for the smallest load sizes, while the number of splits decreases. The trends for the other load ranges follow those of the graph indicating benefit, as the range \([0.41-0.5]\) has the least CPU time required and fewest number of splits per route, while the range \([0.51-0.6]\) has the most time required and greatest number of splits.
The results are slightly different for those load ranges greater than [0.51-0.6]. The range [0.51-0.6] has the second highest benefit as multiple loads can not be combined on a vehicle at any time without split loads. However, the benefit is lower than for the range [0.51-0.6] because there is less capacity available for a split load when a full load is already on a vehicle. The decrease in benefit becomes more dramatic as the load sizes increase, with a benefit of less than 5% for the range [0.71-0.8] and no benefit for the range [0.81-0.9]. The CPU time and number of splits per route follow similar patterns, as they both decrease as load sizes increase above the range [0.51-0.6].

Figure 4 presents a cumulative histogram of the number of problems versus the percentage

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**Table 1** Average CPU time in minutes and average number of iterations for solution with split loads for narrow load ranges.

<table>
<thead>
<tr>
<th>Problem size</th>
<th>75</th>
<th>100</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Range</td>
<td>Average CPU time</td>
<td>Average # of splits</td>
<td>Average CPU time</td>
</tr>
<tr>
<td>0.11 - 0.2</td>
<td>10.2</td>
<td>0.3</td>
<td>24.8</td>
</tr>
<tr>
<td>0.21 - 0.3</td>
<td>10.5</td>
<td>1.3</td>
<td>20.8</td>
</tr>
<tr>
<td>0.31 - 0.4</td>
<td>15.1</td>
<td>8.1</td>
<td>28.2</td>
</tr>
<tr>
<td>0.41 - 0.5</td>
<td>6.6</td>
<td>0.0</td>
<td>13.6</td>
</tr>
<tr>
<td>0.51 - 0.6</td>
<td>25.5</td>
<td>28.4</td>
<td>56.2</td>
</tr>
<tr>
<td>0.61 - 0.7</td>
<td>18.3</td>
<td>21.7</td>
<td>38.8</td>
</tr>
<tr>
<td>0.71 - 0.8</td>
<td>5.8</td>
<td>7.0</td>
<td>10.9</td>
</tr>
<tr>
<td>0.81 - 0.9</td>
<td>4.9</td>
<td>0.0</td>
<td>8.8</td>
</tr>
</tbody>
</table>

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increase in cost when split loads are not allowed for the 125 request problem set with wider load size ranges. The results for the 75 and 100 request problem sets are virtually identical. These histograms are constructed in such a way that the farther a point is to the right on the graph, the greater the number of problems with a higher cost increase. Therefore, the lines farthest to the right indicate those load ranges with the most cost benefit from split loads. The graphs presented are different from the narrow load ranges as the wide load ranges can not be easily adapted to a similar format.

![Figure 4](image_url)  
Figure 4  Cumulative histogram for the number of problems versus the percentage cost increase without split loads for 125 request problem set.

Again, as with the narrow load size ranges, there is a cost benefit from the use of split loads for every load size range. Although the benefit is not as marked as for some of the narrow ranges, some problems were found to have a cost increase upwards of 16%. These results are similar to those found when determining the benefit of split loads for similar loads sizes with the VRP (Dror and Trudeau 1990). Also, the results regarding the most beneficial load sizes support those found with the narrow load size ranges. The range [0.5-1.0] finds the most benefit, while the range [0.3-0.7] finds the second most benefit. These two ranges have the highest concentration of loads just above one half of vehicle capacity. The range [0.1-0.5] has the least benefit, as loads can more easily be combined onto a vehicle without splitting. While the most widespread range [0.1-1.0] has a greater benefit due to the larger load sizes, the load sizes below one half of vehicle capacity temper this benefit. These results indicate that while the most benefit can be found with loads greater than one
half of vehicle capacity, benefit can also be found when load sizes vary over the range of available capacity.

Table 2 further emphasizes the results found in the histograms, as it provides the average percentage cost increase without split loads for each data set, the average CPU time required to obtain the solution, and the average number of splits per route. The results regarding CPU time and number of splits follow those found for the narrow set of load size ranges.

<table>
<thead>
<tr>
<th>Load range</th>
<th>75</th>
<th>100</th>
<th>125</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% cost increase</td>
<td>CPU time</td>
<td># of splits</td>
</tr>
<tr>
<td>0.1-1.0</td>
<td>4.3</td>
<td>7.6</td>
<td>4.7</td>
</tr>
<tr>
<td>0.1-0.5</td>
<td>3.6</td>
<td>9.4</td>
<td>1.6</td>
</tr>
<tr>
<td>0.5-1.0</td>
<td>11.4</td>
<td>11.2</td>
<td>11.0</td>
</tr>
<tr>
<td>0.3-0.7</td>
<td>6.6</td>
<td>11.1</td>
<td>7.6</td>
</tr>
</tbody>
</table>

7. 3PL Case Study

The results from testing with random problem instances indicate that there is a potential for benefit with split loads, particularly with loads over one half of vehicle capacity. However, the random instances do not take into account many real world conditions, such as costs that may be associated with each stop. Also, the random instances are dense as each origin has a load to be delivered to each destination, which is generally not the case. This section presents a case study that considers these additional real world factors.

A large 3PL provider utilized the fleets of various contract carriers to provide dedicated transportation services for a wide range of customers. Each carrier had capacity that was fully dedicated to the customer it was servicing. Multiple fleets were used for several reasons. Certain fleets operated in a specific geographic region, some provided a specific type of service, or simply provided needed capacity. This resulted in a customer receiving service from several transportation providers, many of whom were not directly communicating. In coordinating these services, the 3PL provider found that many vehicles were traveling with excess capacity. It was evident that a more efficient routing procedure was needed, and the possibility of using split loads was introduced. The loads may be split between vehicles operating for a common fleet or separate fleets. The heuristic presented in Section 5 provided a good basis to begin testing these ideas.
7.1. Problem Description
In order to test the suggested changes, the 3PL provider wished to analyze the impact on the services provided for one of their largest customers (name withheld upon request). Data was provided on all of the shipments requiring delivery on a daily basis for this customer. This data covered one week of operations. The shipment information was not expected to change from week to week over a six month planning horizon, so this data was representative of the operations for a significant length of time. In order to apply the heuristic to this problem, several modifications were made to fit the business rules under which the 3PL provider operated. These include the implementation of multiple tours with vehicles of homogeneous capacity, additional costs for one-way trips, a minimum tour cost, a financial and time associated cost for each stop, and a maximum tour length. These alterations are further described in the Appendix. The heuristic was modified to take into account the rule changes, and the data provided by the 3PL provider was analyzed using the following experimental design.

7.2. Experimental Design
The data provided by the 3PL provider included information on 1182 lanes along which loads were moved at least once a week. The heuristic run time for the data from one full day was in excess of 20 hours for the busiest day (Monday). This day required the delivery of over 340 truckloads. While this is more than three times the number of truckloads delivered in the largest randomly generated data set, the significant increase in computational time is primarily a result of the increase in the total number of origins and destinations. The 3PL data had approximately 140 origins and 120 destinations per day, while the largest random instance had 5 origins and 25 destinations. Because of this, the load requests for each day were partitioned into four or five groups similar in total load size to reduce the run time (the full data sets for each day were also run for comparison). The data for Saturday and Sunday was not partitioned as the number of loads requiring service was significantly lower for these days. All experiments were run on a 2.4 GHz Xeon processor with a 400Mhz frontside bus and 2 GB RAM.

7.3. Experimental Results
Table 3 summarizes the results from applying the heuristic to the 3PL data, for both the partitioned sets and the total set. The first column for each day summarizes the test runs with the 3PL data partitioned as described above and the second column contains results for the full data set that was run without any partitioning. The size of the data set is represented by the number of origins and destinations requiring service, and the total number of truckloads to be delivered. On average, only
6% of origin-destination pairs had a request for service. The results include the number of routes used, the computational time required, in minutes, and the percent increase in cost when split loads are not allowed. The difference between the cost with split loads and the cost without is not significant, particularly when considering the additional computational time required when using split loads. Section 7.4 evaluates the business rules and data of the 3PL provider, and determines the impact these have on the significant decrease in the benefit of split loads. The full data set results in less benefit with split loads than the partitioned data because there are fewer split loads tested for the larger set relative to the size of the data set. Although there is little benefit associated with a reduction in cost, the reduction in the number of routes is meaningful. If each route is operated by a unique vehicle, as is often the case with the 3PL provider, eliminating routes can result in a cost savings. There is generally a fixed cost associated with each vehicle in operation or, as with the case presented here, a minimum tour cost associated with each route. Minimizing the number of vehicles or routes is the primary goal for some standard benchmark cases (Li and Lim 2001).

Table 3  Data dimensions and heuristic results for full and partitioned data (Saturday and Sunday were not partitioned)

<table>
<thead>
<tr>
<th>Orig. X Dest. Truckloads</th>
<th>MONDAY 145 X 123 342.6</th>
<th>TUESDAY 141 X 117 331.2</th>
<th>WEDNESDAY 141 X 119 336.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Cost Increase</td>
<td>1.65 0.61</td>
<td>1.64 0.70</td>
<td>1.33 1.03</td>
</tr>
<tr>
<td>Routes</td>
<td>249 270</td>
<td>235 256</td>
<td>238 251</td>
</tr>
<tr>
<td>Time (min.)</td>
<td>95 &lt; 0.2</td>
<td>90 &lt; 0.2</td>
<td>107 &lt; 0.2</td>
</tr>
<tr>
<td>Partitioned</td>
<td>Total</td>
<td>Partitioned</td>
<td>Total</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orig. X Dest. Truckloads</th>
<th>THURSDAY 140 X 115 328.3</th>
<th>FRIDAY 142 X 117 329.1</th>
<th>SATURDAY 24 X 22 31.8</th>
<th>SUNDAY 14 X 12 18.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Cost Increase</td>
<td>1.68 0.78</td>
<td>1.19 0.89</td>
<td>1.33 0.71</td>
<td></td>
</tr>
<tr>
<td>Routes</td>
<td>226 245</td>
<td>236 248</td>
<td>18 22</td>
<td></td>
</tr>
<tr>
<td>Time (min.)</td>
<td>108 &lt; 0.2</td>
<td>114 &lt; 0.2</td>
<td>3 &lt; 0.1</td>
<td></td>
</tr>
<tr>
<td>Partitioned</td>
<td>Total</td>
<td>Partitioned</td>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

7.4. Evaluation of 3PL Provider Rules and Data
The results when applying the heuristic to the 3PL data countered those results found when using the randomly generated data in Section 6.2 as there was little benefit from allowing split loads.
This is a factor of both the characteristics of the 3PL data and the additional rules that were implemented for the 3PL case. We methodically tested each alteration that was made for the 3PL case study by making individual changes to the original model, isolating each modification and determining the effect it had on the results. We initially analyze the effect that the rules have on the benefit of split loads.

In order to make the effect of the rules most visible, the heuristic was tested on data with loads that are most likely to be split. As shown in Section 6.2, these are loads in the range $[0.51-0.6]$. The data used for this test is the same as that used in Section 6.2, and it consists of the 15 problem instances with 75 service requests. The heuristic was initially applied to the data with adherence to all of the 3PL rules. The heuristic was then reapplied while eliminating the rules sequentially. The following rule changes were tested:

- use of a single vehicle, uniting all distinct tours into one continuous route;
- elimination of per stop costs, including the time costs related to pickups and dropoffs;
- elimination of the penalty for a one-way trip in which the vehicle does not return close to its original starting location at the end of a tour;
- elimination of a minimum tour cost.

Table 4 presents the average percentage increase in cost without split loads, under each of the different rule conditions. As these values indicate, the per stop costs are a primary reason for the reduction in the benefit found for using split loads. These costs accounted for the most significant portion of the overall cost for the tests with all 3PL rules, representing 68% of the total cost with split loads and 65% without split loads. The reduction in overall cost when allowing split loads was a result of reducing travel costs rather than eliminating stops. This result is not surprising, as generating a split load forces a route to visit an additional origin and destination. This result is significant, as most real routing situations have at least some cost per stop. The remaining rule changes had little effect on the benefit of split loads.

<table>
<thead>
<tr>
<th>All 3PL rules</th>
<th>No rules</th>
<th>All 3PL rules with:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>single vehicle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no per stop cost</td>
</tr>
<tr>
<td>% Cost increase</td>
<td>22.2</td>
<td>23.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no one-way trip penalty</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no minimum cost</td>
</tr>
<tr>
<td></td>
<td>42.0</td>
<td>42.9</td>
</tr>
<tr>
<td></td>
<td>22.0</td>
<td>19.0</td>
</tr>
</tbody>
</table>

As the per stop costs resulted in the most significant decrease in the benefit from split loads,
these costs were eliminated, and the heuristic was reapplied to the 3PL data. The increase in the benefit of split loads without these costs averaged just below 1%, an almost negligible change. This indicates that most of the benefit is lost due to the characteristics of the data, particularly the range of load sizes and the limited number of origin-destination pairs requiring service. Almost 60% of the loads were below one half of a truckload; therefore, the load sizes of the 3PL data are not conducive to splitting.

To test the effect on the benefit of split loads, the load sizes were modified for each of the partitioned data sets. The load size for each service request was changed to a randomly generated value in the range \([0.51-0.6]\) or \([1.51-1.6]\) (the latter range of loads are initially partitioned such that a full truckload is delivered on one trip and the remaining \([0.51-0.6]\) truckloads are delivered on a second trip). The origins and destinations were not altered. The range within which the load size was selected was determined such that the total number of truckloads for a data set was comparable to the original total. The heuristic was run with the new data with all of the 3PL rules applied.

Table 5 presents the average percentage cost increase without split loads for each weekday (Saturday and Sunday were not included as the results for these days were abnormal due to the small number of service requests), as well as the average of the five weekdays. The benefit of using split loads increases significantly with the altered load sizes. However, it is still well below the 22% benefit found when using the randomly generated instances with loads in the range \([0.51-0.6]\). As stated before, full truckloads are not as likely to be split and the use of loads in the range \([1.51-1.6]\) has an impact on the benefit.

Table 5 Benefit of split loads with the 3PL rules when the 3PL load sizes are altered.

<table>
<thead>
<tr>
<th>Load range</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.51 - 1.6 / 0.51 - 0.6</td>
<td>7.41</td>
<td>7.50</td>
<td>8.18</td>
<td>8.12</td>
<td>7.31</td>
<td>7.58</td>
</tr>
<tr>
<td>0.51 - 0.6</td>
<td>14.67</td>
<td>14.47</td>
<td>14.74</td>
<td>14.27</td>
<td>13.79</td>
<td>14.03</td>
</tr>
</tbody>
</table>

To determine the effect that full truckloads may have, the load sizes were again altered. Each service request was changed to a randomly generated value in the range \([0.51-0.6]\). With the lack of any full truckloads, the total number of truckloads for each data set was on average almost 30% lower than the original total. The average percentage cost increases for the loads in this range are also presented in Table 5. The benefit is almost doubled through the elimination of the full truckloads. However, it is still more than 7% below the benefit found with the randomly generated data sets used in Section 6.2.
This difference can be explained by the limited number of origin-destination pairs requiring service. In the randomly generated problem instances, every origin-destination combination has a load associated with it. Therefore, when a load is split, there is a high probability that load can be placed on another route segment where the origin or destination is shared with another load. This results in a much lower cost for splitting a load as only one other origin or destination must be visited, and in some cases no additional stops must be made. This is not true of the 3PL data. While many origins have loads requiring transportation to multiple destinations (and many destinations receive loads from multiple origins), the frequency with which this occurs is not as regular. Fewer than 37% of origins send loads to more than one destination, with only 12% sending loads to more than three destinations. The values are similar for destinations, with less than 38% of destinations receiving loads from more than one origin and only 16% receiving loads from more than three origins. Therefore, when a split load is created, most instances result in a route visiting an additional origin and destination. There is still a benefit for using split loads, but it is not as significant.

8. Conclusions

Splitting loads with the Pickup and Delivery Problem can reduce cost in many cases. This was shown for almost all sets of randomly generated problem instances, with certain instances displaying a cost reduction of over 40%. Problems with load sizes spread out over wide ranges displayed a cost reduction up to 16%. The most benefit was shown to occur with loads of size just over one half of vehicle capacity, both theoretically and experimentally. However, the benefit from split loads was shown to diminish under certain real world conditions.

The 3PL case study that was presented required the modification of the heuristic based on several business rules. The 3PL data was also different from the randomly generated instances in that the number of origin-destination pairs requiring service was limited (less dense). These changes revealed the problem instance characteristics that are most closely tied to the benefit of split loads: load size, cost associated with a pickup or delivery, and the frequency with which loads have origins or destinations in common. Load sizes just over one half of vehicle capacity were found to have the most benefit. Further, those loads with size close to vehicle capacity or below ten percent of capacity found almost no benefit. As splitting a load often resulted in generating additional stops at origins or destinations, increasing the cost of making a pickup or delivery reduced the benefit of split loads. However, if several loads share a common origin or destination, splitting a load does not necessarily result in the addition of stops to a route.
These results do not indicate that the use of split loads is strictly limited to unrealistic situations. Rather, we have seen that some conditions are conducive to finding benefit from split loads while others are not. Also, although the cost savings were reduced under the 3PL rules, there was a reduction in the number of routes, which may be meaningful. The question for future research is to determine which market sectors exhibit the characteristics that make them most likely to benefit from split loads. This will require specifying those markets that generally incur a low cost for each stop visited, predominantly move load sizes just over one half vehicle capacity, or have loads with many origins or destinations in common. Alternatively, heuristics involving the use of split loads may be applied to real world problems where split loads are a requirement, such as industries utilizing lean manufacturing techniques that require the shipment of small loads. Further, it will be important to study methods for overcoming the various operational hurdles to achieving the potential benefits from the use of split loads.

Appendix. 3PL Provider Business Rules

In order to apply the heuristic to the 3PL provider problem, several modifications were made to fit the business rules under which they operated.

1. **Multiple tours:** While a solution in Section 6 consisted of a continuous route serviced by a single vehicle, the 3PL routing allows for the use of multiple “tours,” or route segments, that can be serviced by more than one vehicle. Each tour begins at an origin and ends at a destination. One vehicle may service multiple tours; however, the 3PL provider is not concerned with the cost associated with traveling from the end of one tour to the beginning of another. Vehicles are assumed to be of homogeneous capacity.

2. **One-way trip costs:** A different rate was charged dependent on whether a tour was considered to be roundtrip or one-way. As is common within the trucking industry, roundtrips are highly encouraged as the vehicle and driver return close to the initial origin (which is viewed as their “home” base) at the end of a route. Based on the 3PL provider guidelines, if the vehicle was within a specified distance of the initial origin at its final destination, the tour was a roundtrip and a discounted mileage rate was used.

3. **Minimum tour cost:** Each tour had a minimum cost in order to recover the fixed costs associated, regardless of length.

4. **Cost per stop:** A standard per stop cost was implemented. This accounted for the various monetary costs associated with the vehicle loading or unloading at a facility.

5. **Driver time per stop:** The 3PL provider assumed that the driver will spend some amount of time at a pickup or dropoff location. These times were treated as additional costs for making a stop.

6. **Maximum tour length:** Based on current U. S. Department of Transportation guidelines on driver hours and the expected distance that could be traveled within the maximum time allowed, each tour had a maximum length.
The remaining restrictions under which the 3PL provider operates were amenable to change based on route requirements and are not considered here.

References

Archetti, C., M. W. P. Savelsbergh, M.G. Speranza. 2006b. To split or not to split: That is the question. To appear in *Transportation Research E*.


