Strategic Planning in Fractional Aircraft Ownership Programs

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Abstract
In the fractional ownership model, the partial owner of an aircraft is entitled to certain flight hours per year, and the management company is responsible for all the operational considerations of the aircraft and for making an aircraft available to the owner at the requested time and place. In the recent years although the industry as a whole has experienced significant growth, most of the major fractional jet management companies have been unprofitable. To increase profitability a management company must minimize its operating costs and increase its crew and aircraft utilization. In this paper, we present a methodology for efficiently scheduling the available resources of a fractional jet management company that takes into consideration the details in real world situations. We then discuss several strategic planning issues, including aircraft maintenance, crew swapping, demand increase and differentiation, and analyze their effects on the resource utilization and profitability.

\textbf{Key words}: Aircraft routing, crew scheduling, column generation, set partitioning, decision making
1. Introduction

As partial (time-share or fractional) owners, customers share a given resource, using it for a fraction of the time and at various service levels, depending on the amount they pay. In the fractional jet ownership models, the owners do not compete for time on a particular plane but are entitled to their time whenever they ask for it. Furthermore, the fact that the operational and maintenance issues are taken care of by a management company makes it a convenient option for the owners.

Although fractional ownership of private aircraft has been around since the 1960’s as a business model, it has become increasingly popular over the last ten years. More and more individuals and businesses prefer to become partial owners of an aircraft because this model offers low cost (relative to whole aircraft ownership), flexibility, privacy, and guaranteed availability (with eight hours of advance notice), without the worry of hiring crews or maintaining the aircraft, since the management company provides those services. The fractional owner can fly directly anywhere among 5,500 airports (compared to 500 airports for commercial airlines) at anytime with few check-in or security delays, or lost baggage concerns, a significant benefit relative to commercial airline travel.

The fractional ownership programs provide share sizes from one-sixteenth with 50 flying hours per year to one-second with 400 flying hours per year (Levere 1996; Zagorin 1999). Usually, a partial owner requests a flight, by specifying a departure station, a departure time, an arrival station, and an arrival time, only days or hours ahead of time. The management company must assign a crew and an available aircraft to serve this flight. While scheduling all the requested flights, the management company tries to minimize total operational costs. There are five major operational expenditures: (i) repositioning cost, incurred when an empty aircraft is flown to a requested departure station; (ii) upgrade cost, incurred when a flight is upgraded to a larger aircraft; (iii) transportation cost, incurred when a crew travels to the aircraft or back to the crew base via a commercial airline; (iv) overtime cost, incurred when a crew works an extra day; and (v) charter cost, incurred when additional aircraft must be chartered at a high cost to cover a requested flight. The customer pays for the fuel and crew costs of an actual customer trip, so they are not considered as an operational cost.

Unfortunately, the growth in the demand for fractional aircraft ownership has not translated into profitability for most management companies. In fact, recently only one of the four largest management companies reported profits. We believe that decreasing operational costs and increasing asset (crew and aircraft) utilization will have a positive effect on the profitability of
such businesses. In this paper, we first develop a methodology that will help the fractional management companies in assigning and scheduling aircraft and crews so that all flight requests are covered at the lowest possible cost. Then, to aid with strategic decision-making, we analyze the impact of several tactical and operational strategies on profitability using real operational data. We examine

1. The effect of scheduled and unscheduled maintenance on operational costs.
2. The effect on operational cost and crew and aircraft utilization of (i) allowing the crew to be separated from its initially assigned aircraft during a duty period when the crew’s aircraft goes under long maintenance or when the management company has more crews available than aircraft, (ii) allowing flexibility on the leg departure times, and (iii) increasing demand by introducing a new product, “jet-card”, whereby customers buy flight hours without becoming fractional owners.

In our models, we first assign crews to aircraft in the beginning of a duty, and then assign crews to a sequence of flight legs. This process is called crew pairing or crew scheduling. The crew-pairing problem in the commercial airline industry has been addressed in numerous studies and various solution methods have been developed. The problem is generally formulated as a set partitioning problem (Marsten and Shepardson, 1981). One method that is commonly used to solve set partitioning problems is column generation. Column generation was initially introduced in Dantzig and Wolfe (1960) and there exist a number of papers where it was applied to solve airline crew scheduling problems (see for example Crainic and Rousseau (1987), Lavoie et al. (1988), and Barnhart et al. (1994)). Furthermore, there exist studies that integrate aircraft routing and crew scheduling problems in one model. Cordeau et al. (2001) and Mercier et al. (2003) apply Benders decomposition to simultaneously solve a single type of aircraft routing and crew scheduling problem. Klabjan et al. (2002) propose a solution approach for integrating aircraft and crew pairing by considering time window and plane count constraints in the crew pairing problem and Cohn and Barnhart (2003) incorporate aircraft key maintenance routing decisions within the crew-scheduling model.

For a fractional aircraft ownership program, the crew pairing problem poses a unique situation. Unlike the commercial airlines, the flight legs in a fractional program differ from day to day and week to week, and most are not known in advance. Repositioning requires flying an aircraft without any passengers on board, and repositioning may comprise 35% or more of the total flying. A crew (or an aircraft) starts or finishes its duty at a different location based on the demand
each day. Moreover, fractional programs provide point-to-point service, compared to the commercial airlines. Therefore, the approaches for crew scheduling in commercial airlines can not be directly applied to this particular application.

Keskinocak and Tayur (1998), the first paper in this field, study the fractional aircraft-scheduling problem for a single type of aircraft. They develop and test a zero-one IP for small- and medium-size problems (up to 20 planes and 50 trips) and provide a heuristic for solving larger instances. In their work, the multiple fleet types and crew duty restrictions are not considered. Ronen (2000) presents a decision-support system for scheduling charter aircraft. He develops a set-partitioning model that combines the fleet assignment and routing problems and incorporates maintenance activities and crew availability constraints. Larger scale problems (up to 48 aircraft and 92 trips) in one-day and two-day planning horizons are solved to minimize total cost of scheduling flights, subcontracting flights, and idling aircraft. He includes subcontractor aircraft as a part of the company-owned aircraft but with different cost. Recently, Martin et al. (2002, 2003) extend the methods developed in Keskinocak and Tayur (1998) by including multiple types of aircraft and crew constraints. Their model considers multiple-day planning periods with 10-hour overnight rest between each day. Karaesmen et al. (2005) develop several mathematical models and heuristics that take into account the presence of multiple types of aircraft, scheduled maintenance, and crew constraints. They analyze the efficiency of these models through a computational study by solving daily scheduling problems. Yang et al. (2006) extend this work to multi-day horizons. Most recently, Hicks et al. (2005) develop an integrated optimization system for Bombardier Flexjet (www.flexjet.com), a large fractional aircraft management company. A column generation approach is applied to solve a large-scale mixed-integer nonlinear programming model, which is based on an integer multi-commodity network flow problem. A branch-and-bound approach is used to obtain integer solutions from selected columns, which represent the aircraft itineraries and crew schedules.

In this paper, we develop a scheduling method for a fractional aircraft ownership program, which takes into account the real operational issues, such as crew transportation and overtime. To the best of our knowledge, none of the previous studies consider the overtime costs. The consideration of crew transportation cost only appears in the recent paper by Hicks et al. (2005). According to our study, these two issues make-up up to 15% of the total cost. Although, the scheduling method we present here is very effective with respect to computational time and solution accuracy compared to the algorithms presented in the previous studies, the real contribution of
this paper lies with the insights we were able to provide on the effects of aircraft maintenance, crew swapping, demand increase and differentiation on resource utilization and profitability. Using the scheduling tool we developed we were able to run various scenario analysis on real operational data and assist the fractional management company in making strategic and tactical planning decisions.

2. Problem Description and Basic Terminology

A fractional management company requires that the owners request their flights at least eight hours before their desired departure time. In general, the management company does not change a customer’s request. The fractional management company may operate a non-homogenous fleet with aircraft of different sizes. When an owner requests a flight, the management company is obliged to serve this request with an aircraft that is at least as big as the owner’s aircraft type. That is, the company may provide a larger aircraft without any additional expenses for the owner if it believes that the total operational costs can be decreased by this “complimentary upgrade.” On the other hand, an owner can request an “upgrade” or a “downgrade,” that is a flight request on a larger or a smaller aircraft, respectively. If a requested upgrade or downgrade is approved then the number of flight hours to be deducted from the customer’s account is adjusted with respect to the aircraft type. As a result, the remaining flight hours will be fewer when an upgrade is made and more if a downgrade is made. Moreover, the customer is required to pay the operating expenses associated with the aircraft flown.

We refer to a customer requested flight as a “leg.” A “crew” of two pilots and an aircraft are assigned for flying each leg. If the assigned aircraft is not already at the departure station of the leg, the assigned crew flies the empty aircraft to this station. This empty flight is called a “reposition” move. A “trip” is either a reposition move or a leg. A minimum “turn time,” that is time spent at the gate, is required between two trips.

We define a “duty” as a sequence of flights and related activities, such as briefing and debriefing, within a crew work day. The legality of crew composition and operations is defined by the Federal Aviation Administration (FAA) regulations. According to these regulations, only certain pairs of pilots are allowed to fly a certain aircraft type given their current expertise and training status. The “duty time”, the time span of a duty, cannot exceed fourteen hours and the maximum flying time is limited to ten hours. Furthermore, a ten-hour overnight rest is required to take place between two duties. A crew is notified of any trip assignments (including any
changes to an assigned trip due to owner requests or unscheduled maintenance) at least two hours prior to departure. Also when a pilot travels by a commercial airline, a three-hour minimum connection time, due to the time needed to go through security, check in etc. at commercial airports, before the departure time of the commercial flight is assumed to be incurred. This connection time is also counted as a portion of the duty time.

In general, the pilots work on a schedule in which they will be on-duty for a specified number of days (e.g. one week) followed by an off-duty period (e.g. one week). We denote crew pairs consisting of pilots who are starting their on-duty period as “coming-duty” crews. Coming-duty crews travel from their crew bases to their assigned aircraft locations after their rest period. “Off-duty” crews are the crew pairs that go back to their crew bases at the end of their on-duty period. Sometimes the management company may ask a coming-duty crew to fly to the station where an aircraft is located the day before the crew’s duty-period starts to cover an early morning flight the next day. Also an off-duty crew may arrive to its home base a day after the end of its duty period due to flying a late flight the day before. In both of these cases, the pilots are paid “overtime.” The exchange of a crew with another crew to fly an aircraft is called a “crew-swap” and the days of the week that the coming-duty crew starts its shift and the off-duty crew ends its shift are called “designated duty shift days”.

For the management company aircraft availability is an important issue while scheduling flights. An aircraft is “idle” when it is ready to be assigned to a crew. Aircraft availability is determined with respect to scheduled and unscheduled maintenance events. According to FAA regulations, after a certain number of flight hours, each aircraft goes under scheduled maintenance. In our approach, we do not create the schedules for maintenance but use the maintenance information that is provided by the company. Therefore, we assume that all the planes that need to go under scheduled maintenance during a scheduling period together with the place and duration of the maintenance events are known beforehand. Hence the schedule must assure that each aircraft scheduled to go under maintenance during the planning period arrives at the designated maintenance station at the designated time and remains on the ground during its maintenance period. The crew assigned to fly a plane to its maintenance station either stays until the maintenance is completed or is reassigned to an idle aircraft depending on the duration of the maintenance event. All events that require a plane to be grounded for a period of time due to an unexpected problem are denoted as unscheduled maintenance events. These events may occur anytime in the day without prior knowledge. When an unscheduled maintenance event occurs the
plane stays at the destination station of its last flight until the problem is fixed, and the rest of the legs assigned to the aircraft are reassigned.

Given the demand over a period of time, the quality of the schedules for this period is measured by the total cost incurred by the management company for covering all the demand and the ratio of total repositioning hours to total trip hours. We denote this measure as “reposition ratio.” A good schedule has low overall cost and low reposition ratio. We also define plane utilization to be the total leg hours flown by company’s aircraft divided by the number of available aircraft and use this measure to evaluate strategic decisions.

3. Scheduling Approach

3.1 Basic Assumptions

We first assume that during its duty period a crew stays with one aircraft unless a long maintenance event occurs. Although this assumption provides schedules with low plane utilization, due to the high transportation costs and times incurred when the crews travel by commercial airlines and the increased operational complexity, most fractional management companies prefer to operate with such initial schedules and modify them in an ad-hoc manner if necessary. In our analysis, we will relax this assumption either when an aircraft goes under a long maintenance that lasts more than one day or when an extra crew is available and measure its effects on operational cost and plane utilization. Our modified model treats both the crew whose aircraft goes under long maintenance and the extra crew as special coming-duty crews.

The crew-swaps occur only during designated duty shift days or when an extra crew picks up an aircraft whose crew has already used up its maximum duty or fly time for the day. We also assume that the two pilots who form a crew pair do not split during an entire duty. These assumptions do not divert from the mode the fractional management companies operate in most of the time.

In terms of the cost, we assume that no additional cost or penalty is incurred if an aircraft or a crew idles on the ground. The charter rate is considered to be fixed in the planning horizon. Finally, we only incorporate complimentary upgrades since the fractional management company only incurs extra cost in these cases. The upgrade cost is the extra flying and reposition cost per hour when a leg is covered by an aircraft that is larger than requested.
We formulate the crew and aircraft scheduling problem as a set partitioning problem. We use a column generation method to solve a three-day scheduling problem where at each iteration all the known demand is incorporated. In our rolling horizon approach, after a three-day schedule is determined, the schedule for the first day is fixed and the problem is resolved using the next three-day demand data. We assume that this is a reasonable strategy given that the customer requests can arrive up to eight hours prior to the departure time and in the industry on average 80-90% and 60-75% of the demand is known in advance for the second and third days respectively and this percentage drops significantly after the third day.

3.2 Formulation

In the three-day planning period, the crew-pairing problem is formulated as a set-partitioning problem combined with aircraft and crew constraints. Let $L$ be the set of legs in the three-day planning period and $L'_t$ be the subset of legs that can be covered by fleet type $t$. Let $P$ be the set of available planes at the beginning of the planning period; $T$ be the set of fleet types; $W$ be the set of available crews; and $CP$ be the set of all columns representing the possible pairings. A pairing, consists of a sequence of flights that a crew can feasibly fly. Let $x_j$ be a 0-1 variable indicating if column $j$, corresponding to a feasible pairing and aircraft route, is chosen in the solution or not and $s_k$ be a slack variable indicating whether leg $k$ is covered by a charter or not. Let $c_j$ be the cost of column $j$ and $r_k$ be the charter cost for leg $k$. We formulate the crew-pairing problem as follows:
\[
\text{(Q1) Min } \sum_{j \in CP} c_j x_j + \sum_{k \in L} r_k s_k \\
\text{subject to } \sum_{j \in CP} A_{kj} x_j + s_k = 1 \quad \forall k \in L \quad \pi_k \quad (1) \\
\sum_{j \in CP} B'_{kj} x_j \leq 0 \quad \forall t \in T, \forall k \in L' \quad \lambda'_k \quad (2) \\
\sum_{j \in CP} E_{pj} x_j \leq 1 \quad \forall p \in P \quad \delta_p \quad (3) \\
\sum_{j \in CP} F_{wj} x_j \leq 1 \quad \forall w \in W \quad \rho_w \quad (4) \\
\]

\( A_{kj} \) is 1 if leg \( k \) is included in column \( j \), and 0 otherwise. \( B'_{kj} \) is \(-1\) if a plane is left at the arrival station of the last leg \( k \in L' \) in column \( j \) that is covered by an off-duty crew operating type \( t \) planes, 1 if a plane is picked up at the arrival station of leg \( k \in L' \) by a coming-duty crew operating type \( t \) planes, and 0 otherwise. These inequality constraints allow a plane left by an off-duty crew to either stay on ground or be picked up by a coming-duty crew. \( E_{pj} \) is 1 if plane \( p \), an available aircraft in the beginning of the three-day planning period, is used in column \( j \), and 0 otherwise. \( F_{wj} \) is 1 if crew \( w \) flies the sequence of legs in column \( j \), and 0 otherwise.

Constraints (1) require that each customer leg be flown either by a company aircraft or a charter. \( \pi_k \) is the dual variable associated with the leg cover constraints. Constraints (2) insure that a coming-duty crew picks up an aircraft it can operate at the destination of the last leg the aircraft has flown only if an off-duty crew left it there. \( \lambda'_k \) is the dual variable associated with the aircraft connection constraints. Constraints (3) ensure that an idle aircraft in the beginning of the planning period is picked up only by one crew at any given time or is not used during the planning period. \( \delta_p \) is the dual variable associated with the aircraft availability constraints. Note that these constraints modeling crew-swaps in our model are different from other airline scheduling approaches that appear in the literature. Constraints (4) ensure that a crew is assigned to only one pairing. \( \rho_w \) is the dual variable for the crew constraints.
3.3 Algorithm

Using column generation, we first solve the linear programming (LP) relaxation of the set partitioning problem. Initially, we enumerate all feasible duties that can be legally operated in a day, by using a depth first search algorithm. These duties are made up of customer and repositioning legs and scheduled maintenance events. Then we create an auxiliary network for each available crew pair that is used to identify good pairings. Shortest paths on these auxiliary networks are used to create a set of initial columns that is fed into the initial LP. After solving the initial LP, we update the arc costs on the auxiliary networks using dual information provided by the LP, and solve a pricing problem by finding shortest paths on these networks with the new arc costs. The length of these shortest paths determines if the solution found by the previous LP is optimal and if not what are the profitable columns to be added to the model. When we have an optimal solution for the LP relaxation, we feed all the columns present in the final LP into an integer programming solver. After the integer solution is obtained by solving the IP with all the existing columns in the final LP relaxation, the first day pairing for the three-day planning horizon is fixed and the procedure is repeated for the next three days using the first day information as initial conditions.

We show that the pricing iteration in the column generation process is equivalent to finding a set of shortest paths on appropriate crew networks. For each crew pair, we construct a network \( G=(N, A) \), where the node set \( N \) in each network consists of a source (equivalently a crew) node \( C \) representing the initial location of the crew, nodes \( P \) representing the available aircraft locations, a set of duty nodes representing feasible duties the crew can fly during the planning period, and a sink node.

The arcs emanating from a crew node depend on the crew’s remaining on duty days. If the crew is already on duty and stays with a certain plane, the crew node has only one out-going arc that enters the node of this plane (Figure 1(a) and (c)). If the crew is a coming-duty crew, multiple out-going arcs connect the crew node to all the possible plane nodes and duty nodes where a plane is available from a previously flown leg. These arcs have costs consisting of the cost of the transportation between the crew’s home base and the available plane’s location and cost of overtime if it exists. For example, in Figure 1(b), Crew C2, coming on duty on day 3, can pick up the planes P2 or P3 that are idle at the beginning of the planning period or any other aircraft that is left at the arrival station of a leg completed before day 3. More specifically, in Figure 1(b) the dashed line between C2 and the duty node 1 and the solid lines between node 1 and nodes 30 and
30-39 correspond to the possibility that crew C2 may pick up the plane that flew leg 1 and then feasibly fly leg 30 or legs 30 and 39 on day 3.

In the crew network, an arc between a plane node and the sink node with zero cost represents the fact that the plane stays idle on the ground during the planning period. For example, in Figure 1(b) the dashed line between node C2 and P2 together with the arc between node P2 and the sink implies that crew C2 comes on duty on the third day and picks up plane P2 but stays on the ground without flying any duties during the course of the day. Furthermore, a crew node has an arc going into every duty node if the crew can transport on commercial airlines to pick up the plane and then reposition and serve that duty legally. The cost on the arcs between a crew or a plane node and a duty node is the summation of the transportation or reposition cost, from the crew or plane location to the first departure station in the duty node, and the “duty cost” that is incurred in the duty node. The duty cost includes the total upgrade cost for the legs that are upgraded and the reposition cost incurred while flying a duty.
Day1 duties  Day2 duties  Day3 duties

a. Crew on duty during the three-day planning period

Day1 duties  Day2 duties  Day3 duties

b. Crew comes on duty on the third day.

Day1 duties  Day2 duties

c. Crew goes off duty at the end of the second day.

Figure 1: A partial crew network. Dashed lines represent crew transports to the station where an idle aircraft is located. Solid lines represent feasible connections for covering duties.
A duty node connects to another duty node in the next day if the overnight rest legality constraint is satisfied. The cost on the arcs between two duty nodes is the summation of the reposition cost, from the last arrival station in the first duty to the first departure station in the second duty, and the duty cost of the second duty node. Also, a duty node in the first day can directly connect to any duty node in the third day implying that the crew will be on the ground during the second day between the first and third days’ duties.

Finally, in all but the off-duty crew networks, all duty nodes connect to the sink node directly without cost. This incorporates the possibility that after flying the last leg of the duty, the crew may stay on the ground with its plane for the rest of the planning period. In the off-duty crew networks, the arcs between the duty nodes and the sink have costs consisting of the cost of the transportation between the arrival station of the last leg and the crew’s home base and cost of overtime if it exists.

The crew networks are constructed once in the beginning of each planning period and then the arc costs are updated in the beginning of each pricing iteration of the column generation process. The initial restricted LP consists of columns corresponding to the shortest paths, found with respect to the original costs, together with columns corresponding to charters. The restricted problem determines the dual variables that are passed on to the pricing problem. Given the dual variables for (Q1) at iteration \( i \) the reduced cost for column \( j \) is calculated as follows:

\[
\bar{c}_j^i = c_j - \sum_{k \in L} A_{kj} \pi_k^i - \sum_{p \in A', \tau \in T} B_{kj}' (\lambda^i_{\tau}) - \sum_{p \in P} F_{pj} \delta^i - \sum_{w \in W} F_{wj} \rho^i \quad \forall j \in CP
\]

Determining if there exists profitable pairings reduces to solving a shortest path problem in each of the crew networks when the arc costs are updated using the dual information. Let \( c_{xy} \) and \( \bar{c}_{xy} \) indicate the original and the updated costs of an arc between node \( x \) and node \( y \), respectively. Then at the \( i^{th} \) iteration the updated cost \( \bar{c}_{xy}^i \) are calculated as follows:

\[
\bar{c}_{xy}^i = \begin{cases} 
  c_{xy} - \delta_{xy}^i, & \text{if node } x \text{ is a crew node and } y \text{ is a plane node}, \\
  c_{xy} - \sum_{k \in y} (\lambda^i_{k}), & \text{if node } x \text{ is a crew node and } y \text{ is a duty node}, \\
  c_{xy} - \sum_{k \in y} \pi_k, & \text{if } x \text{ is a plane or duty node and } y \text{ is a duty node}.
\end{cases}
\]

The column generation procedure terminates with an optimal solution for the LP relaxation of the set partitioning problem when no columns with negative reduced costs exist.
3.4 Computational Performance

In our column generation procedure described above, we solve the pricing problem by finding “the shortest path” in each crew network. Therefore, at each iteration at most one profitable column is identified and added to the LP for each crew. After initial computational experiments, we noticed that adding at most one column per crew at each iteration resulted in a large number of pricing iterations we had to go through before an optimal solution was reached. We modified our price-out step so that instead of identifying the shortest path in each crew network, up to $S$ shortest paths are determined. This is efficiently done by finding the shortest path from the source node to every other node within each crew network. The results of a computational study show that adding more than one column has a significant effect in the total run time of the algorithm. Although the effect of adding more columns decreases as $S$ is increased, the best run times, especially for the larger instances, are obtained when up to twenty columns are added per crew. In the rest of our computational study, we add up to twenty columns per crew at each iteration.

We perform a set of experiments to determine the computational efficiency and effectiveness of our solution approach with real data obtained from a fractional management company including the demand and scheduled maintenance. In Table 1, we present the performance of our scheduling tool on different planning horizons and instance sizes. The sizes of the instances are given as the number of planes and legs. In the 2-day and 3-day instances, we run the algorithm once over 2 and 3 days, respectively, with all the demand and scheduled maintenance data. In these multi day instances, the initial conditions for crew and aircraft are assumed to be the same as that of the 1-day problem. The solution times given in the third column are the total times required for solving the LP and the IP. We use the value obtained from solving the LP as a lower bound on the optimal value of the IP. The last column in Table 1 presents the gaps between the value of the integer solutions we obtained and the LP lower bounds, and hence provides upper bounds on the optimality gaps for the solutions we obtain. We observe that as the size of the instances grows and the planning horizon gets longer, the solution time increases from less than one minute to around ten minutes. However, the run time stays within acceptable limits even for operational decision making and the optimality gap never goes above 0.06%.
4. Analysis of Different Tactical and Operational Strategies

The main goal of this study is to identify initiatives that will decrease operational costs by increasing plane utilization. Two major factors affect plane utilization: (i) unavailability of planes to fly a customer leg due to maintenance problems and (ii) idling of the planes on the ground due to the lack of crew who can feasibly operate the plane or to the lack of customer demand. Using data based on the real operational data provided to us by CitationShares (www.citationshares.com), first, we study the impact of scheduled and unscheduled maintenance events on profitability. Then, we investigate two directions for increasing plane utilization: employing different crew swapping strategies and increasing demand and/or modifying demand profiles. For the first direction, we compare the effect of applying different crew swapping strategies. For the second one, we examine the effect of adding flexibility to departure times and analyze the profitability of increasing customer demand while keeping the current resources fixed by introducing a “jet-card.”

4.1 Effect of Scheduled and Unscheduled Maintenance

To incorporate scheduled maintenance, we treat a maintenance event as a special mandatory leg that a specific aircraft has to fly. The arrival/departure location, $S_m$, of the maintenance leg is as-

<table>
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<th>Planning Horizon</th>
<th># Planes, # Legs</th>
<th>Solution Time (Sec)</th>
<th>Optimality Gap (%)</th>
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<td>1-day</td>
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</tbody>
</table>
signed to be the airport where the maintenance is scheduled to take place and the duration of the leg, from time $t_{sm}$ to time $t_{sm}$, is equal to the duration of the maintenance visit. For an aircraft with a scheduled maintenance event due during the current planning horizon we make sure that the aircraft flies this leg by having it arrive at $S_m$ before $t_{sm}$. That is, if the aircraft is already assigned to a crew then we modify the crew network for this crew so that any path in the network includes the maintenance leg, and if the aircraft is not assigned to a crew we force the assignment of the aircraft to the nearest unassigned crew and make the necessary changes to this crew’s network. We note that, our goal with this methodology is not to provide a maintenance plan but to make sure to incorporate the already scheduled maintenance into a plane’s itinerary. Assuming that a plane mainly goes under unscheduled maintenance at the place it breaks down, we determine the start time and location of an unscheduled maintenance leg according to our solution and the maintenance record provided by the company. If a plane needs to go under unscheduled maintenance at $t_{sm,un}$ according to the maintenance record and the plane is flying a trip during this time in our solution, we set the start time of the unscheduled maintenance event as the arrival time of the trip and change the end time accordingly. The start and end station of the maintenance are then set to the arrival station of the trip. To evaluate the effect of unscheduled maintenance, we use the real demand and the maintenance data for a month, in which 937 legs (1,441.1 hours) are covered with 35 available aircraft. During this month 197 scheduled, 104 overnight unscheduled, and 49 mid-day unscheduled maintenance events occur. We solve the scheduling problem with two scenarios: incorporating scheduled and overnight unscheduled maintenance; and incorporating scheduled and all of unscheduled maintenance events that occurred during this month. The designated crew swaps occur twice a week (on Tuesday and Thursday) and crew stays with plane during its duty period.

In Scenario 1, for each three-day planning period, the scheduled and overnight unscheduled maintenance events are added to the demand legs as special legs. In Scenario 2, after an integer solution is obtained for the problem in Scenario 1, we check if any mid-day unscheduled maintenance events exist for the current day. If so, we first fix the pairings for the trips finished before the unscheduled maintenance event occurs, then we add the maintenance leg and re-solve the scheduling problem. The characteristics of the schedule and a break down of operational costs are presented in Table 2. The results indicate that the mid-day unscheduled events, 14% of total number of maintenance requests in the month, increase the overall operational cost by up to 12.5%. This finding suggests that implementing a preventive maintenance program and/or fore-
casting unscheduled maintenance events based on aircraft maintenance history and incorporating this data into the scheduling algorithm are worthwhile directions to consider further.

Table 2: Monthly Schedule and Cost Characteristics under Maintenance Considerations

<table>
<thead>
<tr>
<th></th>
<th>Reposition Hours</th>
<th>Reposition Ratio</th>
<th>Reposition Cost</th>
<th>Upgrade Cost</th>
<th>Transportation &amp; Overtime Cost</th>
<th>Charter Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>732.06</td>
<td>0.337</td>
<td>634,708</td>
<td>29,869</td>
<td>104,951</td>
<td>27,283</td>
<td>796,811</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>786.98</td>
<td>0.353</td>
<td>670,251</td>
<td>36,658</td>
<td>120,538</td>
<td>69,293</td>
<td>896,740</td>
</tr>
</tbody>
</table>

4.2 Effect of Different Crew Swapping Strategies

In this section, we investigate the effect of adopting flexible crew swapping strategies on the operational costs. We evaluate the effect of frequent crew swapping during a duty period by reassigning free crew whose aircraft goes under a long maintenance instead of swapping only on the designated duty shift days.

In all scenarios above, we assume that the number of on-duty crew is the same as that of aircraft and a crew waits at the maintenance station or is sent to its crew base when its aircraft goes under maintenance. Hence there is a one to one correspondence between the number of on-duty crews and available aircraft, which results in the assumption that an available aircraft is assigned to a single crew during the crew’s duty period. This assumption simulates the actual model of operations for most fractional management companies. When the crew is not separated from an airplane during its duty period the cost and the long elapsed time incurred when the crew is transported by commercial airlines is avoided. However under this assumption the utilization of crew and aircraft decreases. Under some circumstances, transporting a crew to fly another aircraft can improve the utilization of crew and aircraft. For instance, FAA regulations require that a crew cannot fly beyond its fly and duty time; its aircraft, however, is idle to be flown by another crew. Therefore, most fractional management companies create their initial schedules with this assumption and modify the schedules later on in an ad hoc manner if assigning a new crew to an aircraft seems to be profitable.

To increase plane utilization by separating crew from the airplane in the middle of a crew’s duty period, we analyze two possibilities: when a plane goes under long maintenance its crew
becomes free to pick up another plane or the management company maintains extra crew. First, we consider crew-swapping opportunities created when an aircraft needs a long maintenance that lasts more than one day hence freeing its crew to be reassigned to another aircraft. Under such a circumstance, a swap may occur between the free crew and a crew who has used up its allowable fly and/or duty hours for the current day. As a result, when one or more aircraft go under long maintenance the number of on-duty crew becomes greater than that of available aircraft and different crews are assigned to operate the same aircraft within a day.

In this setting, the free crew is treated as a special coming-duty crew, whose available days are the number of days remaining in its duty period instead of seven days. This special crew can pick up an idle aircraft and cover some of the legs on the same day the swap occurs. Here, we identify an idle aircraft as an aircraft that has just finished its maintenance or an aircraft whose original crew has finished its duty for the current day. In either case, the free crew transportation time and cost are taken into account. After the swap, the free crew becomes an on-duty crew who can either fly uncovered legs during the current day or the next day’s early legs that cannot be legally flown by the original on-duty crew. In the meanwhile, the original on-duty crew becomes a new free crew that waits for reassignment.

Table 3: Monthly Schedule and Cost Characteristics When Separation of Crew from Aircraft during the Duty Period Is Allowed

<table>
<thead>
<tr>
<th></th>
<th>Reposition Hours</th>
<th>Reposition Ratio</th>
<th>Reposition Cost</th>
<th>Upgrade &amp; Overtime Cost</th>
<th>Charter Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 2</td>
<td>786.98</td>
<td>0.353</td>
<td>670,251</td>
<td>36,658</td>
<td>120,538</td>
<td>69,293</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>766.79</td>
<td>0.347</td>
<td>652,422</td>
<td>38,872</td>
<td>84,704</td>
<td>15,893</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>761.84</td>
<td>0.346</td>
<td>645,385</td>
<td>39,014</td>
<td>83,427</td>
<td>15,893</td>
</tr>
</tbody>
</table>

We demonstrate the effects of this crew swapping strategy with a computational study based on the same monthly data used in Scenario 2. The operational characteristics and costs of the monthly schedule obtained with this model (Scenario 3) are presented in the second row of Table 3. We conclude that the total cost was reduced by 11.7% with the new crew swapping strategy. We particularly note that, this strategy decreases the charter cost by 77% and the transportation and overtime costs by 30%. Furthermore, the number of leg hours flown over the month by a
company aircraft was increased by 0.6% and the sum over the month of the number of aircraft that actually flew at least one leg during a day was decreased by 3%. When we analyze the daily plane utilization rates (that is the ratio of leg hours flown by company aircraft to the number of aircraft that covered at least one leg during the day) we observe improvements of up to 15%.

We next evaluate if even further operational efficiencies can be obtained when the management company keeps extra crew during a planning period. This way more than one crew can be assigned to one aircraft during a day even when long maintenance events do not take place. The extra crew is treated in the same way as the free crew and the model is modified similarly. Row 3 of Table 3 (Scenario 4) presents the results of a computational study on the previous monthly data when only one extra crew is kept and swapping of crew whose aircraft goes under long maintenance is allowed. In this scenario, we assume that the home base for the extra crew is HPN, the most active station for CitationShares. The results indicate that including the extra crew provides a 1% cost improvement over Scenario 3. Hence, we conclude that it will only be profitable to keep an extra crew, if on average the monthly cost for hiring a crew (a pair of pilots) is no more than 1% of the monthly operational cost. In further computational testing, we saw that if more than one extra crew is kept they remain idle.

### 4.3 Effect of Demand Characteristics

In this section, we investigate if modifying demand characteristics and/or increasing the demand while keeping the resources fixed changes the profitability and operational efficiency of a fractional management company. First we notice that even after an extra crew is added the charter cost was not changed between Scenarios 3 and 4. Hence we ask the question if instead of increasing the resources can the management company gain profitability by increasing flexibility of how the demand is served.

We consider allowing a limited flexibility by possibly shifting the departure time of a customer leg by a narrow time interval. The allowed flexibility in the departure times is taken into account in a limited way when all the feasible duties are generated initially, resulting in a larger number of duties being identified. We only consider shifting the departure time of a leg if the shift will enable a crew to fly an extra leg immediately before or after this leg. We will next explain how the shifting procedure is performed via Figure 2. In Figure 2, the solid lines represent the time interval for the customer legs, the dashed lines represent the required reposition and turn times, and the bold arrows represent the shifting of departure times. The letters represent departure and arrival stations. In case (1), our duty generator does not shift the departure times because
the time between the legs A-B and C-D is greater than the required reposition and turn times. On
the other hand, in case (2), to operate both legs E-F and G-H with the same crew, the departure
time of E-F or G-H or both must be shifted. Assuming that when E-F is shifted this does not cre-
ate an illegality for the crew, we create two duties that allow for shifting the departure time of E-
F earlier or G-H later in a minimal way to limit the effect to only one customer leg. In case (3),
the departure time of the second leg K-L is earlier than the arrival time of the first leg I-J. In this
case, if leg K-L can not be covered when leg I-J is moved as early as possible without creating an
illegality for the crew or when K-L is moved as late as possible given the allowable time window
width, we shift the departure time of I-J as early as possible and delay the departure time of leg
K-L as much as needed.

(1) Requested

(2) Requested

E Shifted

G Shifted

(3) Requested

I & K Shifted

Figure 2: An Example Showing How Shifting of Leg Departure Times Are Executed

For the data used in Scenario 3, we were able to show that if only ±15 minutes shifts were al-
lowed on the original departure times, strikingly all the charters in the schedule would have been
avoided and the plane utilization would be increased by 4.6%. We conclude that allowing for
narrow time windows on the customer leg departure times might have a very significant effect in
decreasing operational costs by drastically reducing charters.

Next we investigate the effects of increasing customer demand while keeping the resources
fixed. Recently, a new product, “jet-card,” was introduced by a number of fractional manage-
ment companies. A jet-card is a pre-paid flying card, which targets customers who are not ready
to make the contractual or monetary commitment for partially owning a jet. The program enables
individuals and companies to pre-pay for 25 hours of private flight time on the company-
operated aircraft for each card purchased. Jet-card holders receive most of the same benefits as
fractional owners, including the safe, reliable, flexible and customer-focused service. We assume
that the jet-card holders act as 1/32-share owners and request two-hour trips on average.

While not allowing for any crew swapping except for twice a week on the crew change over
dates, in Scenario 5 we compute the effect of adding new demand legs representing 5%, 10% and
15% increase in the leg hours for the month we have considered previously. The new legs are
randomly selected from the demand data of another month. Note that, in this analysis we take
into account the extra revenue (the hourly flight rate the management company charges) gener-
ated by the new demand. The extra profit earned is calculated by subtracting the cost incurred by
serving the new demand from the extra revenue generated. The second column in Table 4 pre-
sents the extra profit obtained under Scenario 5. We conclude that increasing demand only by
5% is profitable. Next, we assume that these card legs allow for one-hour time-window flexibil-
ity on departures (Scenario 6). Although allowing for leg departure time-windows are not popu-
lar in the industry as a whole yet, our analysis shows that (third column in Table 4) this added
flexibility has a very significant impact in terms of increasing profitability. Table 4 shows that
adding up to 15% demand as jet-card legs becomes profitable with departure time-window flexi-
bility. However, for the same data set, only up to 5% new demand can be handled efficiently and
is profitable without departure time-windows. Furthermore, compared with the average plane
utilizations for Scenario 3, the utilizations in Scenario 6 increase by 4.8%, 9.2%, and 13.6%
when we add 5%, 10%, and 15% card demand, respectively.

<table>
<thead>
<tr>
<th>Percentage increase in demand</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5%</td>
<td>5,499</td>
<td>10,263</td>
</tr>
<tr>
<td>10%</td>
<td>-5,227</td>
<td>13,258</td>
</tr>
<tr>
<td>15%</td>
<td>-9,935</td>
<td>8,974</td>
</tr>
<tr>
<td>20%</td>
<td>-15,857</td>
<td>-6,453</td>
</tr>
</tbody>
</table>
Furthermore, we evaluated the one-hour time window flexibility on the departure times of the card legs in larger data sets, in which there were 5 fleet types and 150 planes. In the monthly data set we used, there were 5004 owners’ legs with 8684.5 flight hours and 636 jet-card legs with 1177.4 flight hours making up 11% of the total demand. Our analysis showed that the operational cost for the whole month decreased by 6.3% if the one-hour time window flexibility was allowed. Furthermore, we simulated the uncertainty in demand by initially only making visible 92%, 85%, and 70% of the demand to the optimizer in the first, second, and third days. We re-ran the optimizer whenever a new demand arrived. The results showed that the one-hour flexibility on the departure time saved at least 1% on the operational cost even without considering its effects on charter costs. We found that the overall cost savings were between 5% and 7% where most cost reductions were due to avoiding charters.

5. Conclusions

In this paper, we propose a methodology for solving the combined crew scheduling and aircraft routing problem for a fractional aircraft ownership management company. Our scheduling approach considers: crew transportation and overtime costs, scheduled and unscheduled maintenance effects, crew rules, and the presence of non-crew-compatible fleets. We analyze several tactical and operational issues that are faced by fractional management companies and demonstrate how these issues affect profitability.

First, we evaluate how scheduled and unscheduled maintenance events affect the total operational costs. Our analysis shows that mid-day unscheduled maintenance has a very significant impact on cost.

Then, we investigate two directions for increasing profitability by improving plane utilization rates: incorporating more flexible crew swapping strategies and increasing and/or modifying customer demand while keeping company resources fixed. The computational results indicate that reassigning the free crew when its aircraft goes under a long maintenance or by maintaining extra crew are profitable options which increase plane utilization rates.

We next examine the effect of increasing and/or modifying customer demand. First, we conclude that incorporating even a few minutes of flexibility on demand departure times significantly reduces the charter costs. Next, we consider the introduction of a new product, 25-hour prepaid jet-card. Our analysis concludes that if one-hour time window flexibility on the departure
times of these card legs is allowed, adding up to 15% demand as jet-card legs becomes profitable.

Allowing flexibility in customer pick up times was not part of the industry operational characteristics but our computational study indicated that this flexibility might have a big effect on profitability or the lack of it. However, changing such characteristics as a single company proved to be very hard from a marketing and sales point of view. Hence we have performed many additional analysis on different types of data sets to convince our industry partners that allowing departure flexibility on at least the jet card legs was crucial. As a result of our analysis, Citation-Shares became the first fractional management company to sell its jet-cards with time-windows. We believe that the impact of these analysis may be very significant given that, the top 4 management companies share about 90% of the market and collectively operate a growing fleet currently numbering over 1000 aircraft strong. We estimate that even a 1% reduction in operating costs across this fleet would result in annual savings of over $20 million, at least $10 million of it in fuel costs.

References


