Designing Mechanisms for the Management of Carrier Alliances

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When cargo carriers form an alliance, sharing network capacity in order to improve profitability, a key issue is how to provide incentive for carriers to make decisions that are optimal for the alliance as a whole. We propose a mechanism that allocates both alliance resources and profits by appropriately setting prices for the resources. Clearly the behavior of an individual carrier within the alliance, and the impact of resource prices on this behavior, is important to understand. We analyze the performance of our mechanism using two different modeling approaches, and find that the behavioral model used can significantly impact alliance recommendations. More specifically, the choice of model impacts the feasibility of routing decisions made by individual carriers, as well as the properties of profit allocations that may be defined by the mechanism. Our proposed behavioral model, which makes use of more realistic control parameters than the alternative model studied, is promising with regard to both considerations. Finally, experimental results for alliances comprised of two and three carriers are analyzed; it is determined that the benefit associated with collaborating increases with network size and fleet capacity, and depending on the characteristics of demand, fleet capacity is a more important factor.

Key words: carrier alliances; asset allocation; decentralized coordination

1. Introduction

Consider a group of independent cargo carriers who each wish to improve their own profitability. They may choose to integrate some portion of their transportation networks in order to make better use of their capacity by delivering more-valuable cargo loads. A group of carriers working together in such a manner is referred to as an alliance. It is reasonable to assume that carriers considering forming an alliance are interested in designing that alliance to function as well as possible, from the standpoint of both profitability and sustainability over time. The challenge in achieving these goals lies in resolving the tradeoff between decisions that are good for the alliance versus decisions that are good for an individual carrier. In this paper we introduce a mechanism to manage the interactions among self-optimizing carriers that addresses the following challenges:

- How can resources be utilized such that the overall system profit is maximized?
- Given that this utilization will not necessarily be optimal for individual carriers in the alliance, what incentives are necessary to encourage carriers to not only participate, but make decisions that lead to system optimal performance?
- How can these incentives be distributed to the carriers, without relying on a centralized decision-maker for proper apportionment?

Our methodology is based on modeling carrier behavior as linear programs. These models are incorporated into a mechanism that manages carrier interactions by setting resource prices such that an appropriate allocation of both alliance resources and profits is attained. As the behavioral models are used to determine the impact of resource prices on carrier behavior, the allocations of resources and profits achieved by the mechanism are heavily dependent on the underlying model employed. Thus it is important to consider the impact of the model selected on the overall performance of the mechanism. In this paper we analyze the performance of our proposed mechanism when two distinct behavioral models are employed, comparing the mechanism output from a practical and
theoretical standpoint. In addition, we conduct experimental analysis to verify that our mechanism performs as predicted, as well as evaluate the potential benefit to be gained by forming alliances among carriers with various network sizes and fleet capacities.

1.1. Air Cargo Alliances
The motivating application for our work is the air cargo industry. Air cargo is assumed to be any freight, excluding mail and passenger baggage, transported using aircraft. More specifically, the focus is on combination carriers, which are those carriers transporting cargo using passenger aircraft. As carriers take steps to improve the profitability of their cargo business, they are increasingly considering collaborations for cargo that are independent of those already established for the passenger industry. The first cargo alliance, SkyTeam Cargo, formed in 2000 and was comprised of the cargo components of Aeromexico, Air France, Delta, and Korean Air (SkyTeam Cargo 2006). These four airlines were already part of the SkyTeam passenger alliance, but SkyTeam Cargo was formed as an independent strategic cargo alliance. Similarly, the WOW Alliance formed in 2002 with the cargo businesses of Star Alliance members Lufthansa, SAS, and Singapore Airlines. Japan Airlines, a carrier from outside the Star Alliance, was added later in 2002 (WOW Alliance 2002).

Cargo alliances among carriers that are not already partners in the passenger business are likely to become more common, since carriers compatible for passenger alliances may not be compatible for a cargo alliance. This is due to differences in flow patterns: passengers typically complete a round trip, resulting in balanced flow, while cargo flow follows unbalanced trade patterns (Zhang and Zhang 2002).

Similar to code sharing in the passenger industry, key decisions involved in the cargo setting include how to share space and revenue among members. An additional consideration in the cargo setting, however, is that of route selection. In contrast to passengers, cargo is relatively insensitive to routing decisions; therefore the decision of how to route cargo through the alliance network becomes a relevant factor in considering collaborations among air cargo carriers. Determining the overall most profitable set of cargo to deliver, and how this cargo should be routed through the combined network, requires a centralized perspective. A centralized model to solve the acceptance and routing problem for the alliance as a whole is presented in Section 2. Since full centralization is generally not an option given the technical and legal challenges associated with integrating the information systems of autonomous carriers, the maximum alliance benefit will only be attained if the participating carriers can be encouraged to make their own acceptance and routing decisions in accordance with the centralized optimal solution. Finding suitable incentives that will influence the behavior of the carriers in an appropriate manner is dependent upon understanding and modeling the decision process of the carrier within the collaborative system. In Section 3 two approaches for modeling the behavior of an individual carrier are formulated and analyzed.

Because an alliance is comprised of a collection of autonomous carriers, the allocation of incentives should be implementable without a centralized distributor. We propose a mechanism that transfers revenue through a collection of prices paid and received as capacity is exchanged; Section 3 includes a description of a methodology used for determining these prices, henceforth referred to as capacity exchange prices, based on the underlying behavioral models discussed. The impact of the behavioral model selected on mechanism performance is characterized in Section 4. Section 5 includes a computational study on alliances comprised of carriers of various network sizes and fleet capacities. Conclusions and insights are presented in Section 6.

1.2. Related Literature on Air Cargo Alliances
There is very little available in the literature relating to air cargo alliances, most likely since alliances among air cargo carriers are a recent development. Most literature concerning air cargo is related to dedicated cargo carriers, cargo operations, or the relationship between the cargo and passenger
industries. For example, the network design problem for dedicated cargo carriers is addressed by (Hall 1989) and (Kuby and Gray 1993), and short-term capacity planning is studied in (Chew et al. 2006). Analysis of airline alliances in the passenger industry is more prevalent, but no existing literature uses a similar methodology or addresses the same questions as in this work. (Park et al. 2001) investigates the impact of international alliances on the passenger market by comparing alliances comprised of airlines with complementary and parallel networks; it is predicted that an alliance that joins complementary networks will be more profitable. In response to a concern that alliances would lead to a situation where major carriers would have a monopoly, (Morrish and Hamilton 2002) finds instead that alliances have merely allowed carriers to preserve, not increase, their narrow profit margins through an increase in load factors and productivity. (G.Bamberger et al. 2004) in fact finds that consumers benefit from the formation of passenger alliances; in the two domestic alliances that were studied fares decreased on the markets impacted by the alliance, in part due to increased competition from rivals competing with the alliance. (Adler and Smilowitz 2007) analyzes potential international alliances among carriers, applying non-cooperative game theory to determine the profitability of an alliance under a given level of competition. The primary issue addressed is the selection of international hubs to maximize the profit of merging airlines.

There is also limited research available on the impact that an alliance in one industry (air cargo or passenger) can have on the other. (Morrell and Pilon 1999) studied a passenger alliance between KLM and Northwest and found that, ultimately, the effects on cargo service were positive. From the other perspective, (Zhang et al. 2004) investigates the effect of an air cargo alliance on the passenger market, finding that cargo service integration can increase outputs in both the cargo and passenger markets.

A widely studied topic in the passenger airline industry that is only recently being applied in the alliance setting is that of revenue management. Literature in this field seeks to maximize revenue through management of seat capacity. (McGill and Ryzin 1999) provides a review of revenue management literature, but is focused primarily on revenue management implemented by a single carrier. (Boyd 1998) describes the technical challenges associated with alliance revenue management; in addition to addressing challenges, (Vinod 2005) discusses how coordination of seat pricing and capacity planning are currently executed in the alliance setting. (Wright et al. 2006) provides a more formal analysis of alliance revenue management mechanisms; a free sale scheme and three types of dynamic trading schemes are discussed. The mechanisms are analyzed to determine their effect on the equilibrium behavior of alliance members and the potential for the mechanism to maximize alliance revenue. Revenue management applied to the air cargo industry is even more limited; differences between the cargo revenue management problem and the passenger yield management problem are discussed in (Kasilingam 1996), as well as complexities in developing additional models to facilitate cargo revenue management.

Outside the airline industry, carrier collaboration has also been studied in the ocean liner shipping industry. (Agarwal and Ergun 2007b) addresses issues related to the formation of alliances in the sea cargo industry; in addition to the distribution of alliance revenue, design of the alliance network is of critical importance. (Slack et al. 2002) demonstrates that alliances among sea cargo carriers lead to increased service frequency and ship size, as well as increased similarity of service routes among carriers. (Song and Panayides 2002) provides a conceptual framework for the application of game theory to alliances in the liner shipping industry. The ability to explain the instability of strategic alliances using cooperative game theory is discussed, as well as the practical limitations of applying game theory to the industry. For an overview of issues related to carrier alliances, including alliances in both the ocean liner and air cargo industries, we refer the reader to (Agarwal et al. 2007).
2. A Centralized Model for Accept/Reject and Routing Decisions

An important motivation for the formation of an alliance among carriers is the recognition by those carriers that the alliance will yield benefit beyond what each carrier can accomplish individually. Given that increasing the revenue earned by the alliance increases the benefit that can be distributed among the participating members, it is reasonable to attempt to determine the set of cargo loads to deliver, and the optimal routing of these loads, that will maximize the alliance profit. This information is obtained by solving a network flow problem from the centralized, or system, perspective; that is, the network and demand from each participating carrier are integrated to create one large pseudo-carrier. The network of a carrier is determined according to the amount of cargo capacity available on each flight leg operated by that carrier; the demand associated with each carrier is presumed to be a set of loads that the carrier must accept or reject for delivery. Note that a freight forwarder can be incorporated into this modeling framework by introducing a carrier with a set of associated loads, but no network capacity.

As the focus of this work is on developing a methodology to manage the interactions among carriers such that alliance-optimal behavior is achieved, several simplifying assumptions are made to improve tractability. First, it is assumed that both cargo loads and flight capacity have single dimension units and are deterministic. Second, it is assumed that origins and destinations for loads correspond to airports, which implies that door-to-door pick-up and delivery services are not considered. Third, costs incurred by operating the network are ignored, as it is assumed that the flight schedule for an individual carrier is motivated by the passenger industry and is therefore fixed. Finally, load splitting is permitted in order to obtain a standard multi-commodity flow linear program.

Let $N$ denote the set of carriers, and $E$ the set of legs operated by each carrier $i \in N$. The set $A$ contains all airports covered by the legs in $E$. Given a planning horizon of $T$ time periods, let $V$ denote the set of nodes $(a,t)$ for each $a \in A$ and $t = 1..T$. Each leg $e \in E$ has capacity $k_e$. Each carrier has a load set $L$ in which an individual load $(o,d,i)$ is characterized by an origin $o$ and destination $d$. The size and per unit revenue of load $(o,d,i)$ are $d_{(o,d,i)}$ and $r_{(o,d,i)}$, respectively.

The centralized goal is to find a flow $f$ of loads such that the system revenue is maximized; this is accomplished by solving the following multi-commodity flow problem:

$$\begin{align*}
(C) : \quad & \text{max} \quad \sum_{(o,d,i) \in L} r_{(o,d,i)} f_{(o,d,i)} \\
\text{s.t.} \quad & \sum_{(u,v) \in E} f_{(u,v)} - \sum_{(v,w) \in E} f_{(v,w)} \leq 0 \quad \forall v \in V, \forall (o,d,i) \in L \\
& \sum_{(o,d,i) \in L} f_{e} \leq k_e \quad \forall e \in E \\
& \sum_{(o,d,i) \in L} f_{e} \leq d_{(o,d,i)} \quad \forall (o,d,i) \in L \\
& f_{e} \geq 0.
\end{align*}$$

(1) reflects the centralized goal of maximizing the amount of revenue earned from delivering loads; the flow variable $f_{(o,d,i)}$ represents flow on a fictitious edge from the destination $d$ to the origin $o$ of load $i$, which is introduced to account for the amount of load $(o,d,i)$ that is delivered. (2) are flow balance constraints, enforcing that every unit accepted for shipment must be appropriately routed through the network. (3) are capacity constraints for each flight, while (4) ensure that the amount of a load delivered does not exceed its size. Let $f^*$ be the optimal solution to $C$; $f^*$ dictates the optimal accept-reject decision for each load, as well as the optimal routing for the set of accepted loads, for the alliance as a whole.
3. Modeling the Individual Perspective

In order for the alliance to earn as much profit as possible, carriers must make their accept-reject and routing decisions in accordance with \( f^* \). We seek to provide a structure to encourage the exchange of capacity among carriers, as carriers will clearly need incentive to allow their capacity to be used by other carriers so that \( f^* \) can be achieved. A natural way to provide this incentive is by establishing a system in which carriers receive payments in exchange for capacity used by other carriers. Recall that these payments are called capacity exchange prices. If \( c_e \) is the capacity exchange price on leg \( e \), the net profit from capacity exchanges for carrier \( i \) is then given by:

\[
s^i = \sum_{e \in E} c_e \left( \sum_{(o,d,j) \notin L^i} f^*_{e}^{(o,d,j)} - \sum_{e \notin E^i} c_e \left( \sum_{(o,d,i) \in L^i} f^*_{e}^{(o,d,i)} \right) \right). \quad (5)
\]

\( s^i \) can be thought of as a side payment provided to carrier \( i \) to compensate \( i \) for the value of capacity being used by other carriers. If \( s^i \) is negative, then carrier \( i \) can be thought of as a net consumer of capacity value.

Let \( q^i \) be the revenue carrier \( i \) earns by delivering loads in accordance with \( f^* \):

\[
q^i = \sum_{(o,d,i) \in L^i} p^{(o,d,i)} f^*_{(d,o,i)}. \quad (6)
\]

Then the net profit \( x^i \) earned by carrier \( i \) is given by \( x^i = q^i + s^i \), and \( x^i \) is carrier \( i \)'s allocation.

Ensuring that \( x^i \) is sufficient to encourage carrier \( i \) to participate, and that the capacity exchange prices are such that carrier \( i \) will abide by the centralized solution, requires an understanding of how capacity exchange prices impact the decisions of individual carriers. In this section we formulate two models for the behavior of an individual carrier. The goal in establishing these models is not to analyze the decision process of a carrier, but to gain insight into how to select capacity exchange prices that encourage carriers to make centrally optimal accept-reject and routing decisions. A critical consideration is the fact that a carrier does not operate in isolation; a carrier must consider the use of capacity by other carriers when making routing decisions. The two models presented differ in how this consideration is incorporated.

3.1. Limited Control Model

We developed the Limited Control model to represent realistic restrictions on the decisions available to an individual carrier participating in an alliance. In the model, the use of capacity by other carriers is acknowledged by limiting carrier \( i \)'s use of capacity on each flight. Pre-determining capacity allotments is a realistic approach given current industry practice; a carrier typically dedicates space on each flight to specific partnering carriers and freight forwarders. Intuitively, the capacity available on a flight can be partitioned according to the centralized solution \( f^* \); if carrier \( i \) uses \( k^i_e \) units of capacity on leg \( e \) in \( f^* \), then the individual model for carrier \( i \) will restrict carrier \( i \) to at most \( k^i_e \) units of capacity on leg \( e \). More specifically, we use the following rules to determine the amount of capacity allotted to each carrier:

- For each edge \( e \) utilized at full capacity in \( f^* \) (\( \sum_{(o,d,i) \in E} f^*_{e}^{(o,d,i)} = k_e \)), allot \( \sum_{(o,d,i) \in E^i} f^*_{e}^{(o,d,i)} \) to each carrier \( i \).
- For an edge \( e \) that is not utilized at full capacity (\( \sum_{(o,d,i) \in E} f^*_{e}^{(o,d,i)} < k_e \)), allot \( \sum_{(o,d,i) \in E^i} f^*_{e}^{(o,d,i)} \) to each carrier \( i \) such that \( e \notin E^i \). Allot \( k_e - \sum_{(o,d,i) \in E^i} f^*_{e}^{(o,d,i)} \) to the operating carrier \( k \).
- Ground edges, assumed to have infinite capacity, are not subject to capacity allotments.
Given an allotment of capacity $k^i_e$ on every leg $e \in E$, the Limited Control model for carrier $i$ is as follows:

\[
(LC^i) : \quad \text{max} \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{(o,d,i)} - \sum_{e \in E^i} \sum_{(o,d,i) \in L^i} c_e \left( \sum_{(o,d,i) \in L^i} f^{(o,d,i)} \right)
\]

s.t. \[\sum_{(u,v) \in E} f^{(o,d,i)} - \sum_{(v,w) \in E} f^{(o,d,i)} \leq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i\] \[\sum_{(o,d,i) \in L^i} f^{(o,d,i)} \leq k^i_e \quad \forall e \in E\] \[f^{(o,d,i)} \leq d^{(o,d,i)} \forall (o,d,i) \in L^i\] \[f^{(o,d,i)} \geq 0.\]

Like the Centralized model $C$, the Limited Control model is a multi-commodity flow LP. The value of (7) is equal to the total revenue earned from delivered loads minus the sum of capacity exchange prices paid. Note that this value is a lower bound on $x^i$, the actual profit allocated to carrier $i$, because it excludes exchange prices that will be paid to carrier $i$.

Given a model to represent the behavior of an individual within the alliance, we seek capacity exchange prices $c_e$ such that the optimal solution to the individual model for carrier $i$ will correspond to the centralized optimal solution $f^*$. This can be accomplished using inverse optimization. As a solution $f^*$ must be optimal when it satisfies primal feasibility, dual feasibility, and complementary slackness conditions, the inverse problem for a carrier is formulated using the dual of his individual problem. The dual of the Limited Control model for carrier $i$ is as follows:

\[
(DLC^i) : \quad \text{min} \sum_{e \in E} k^i_e \alpha^i_e + \sum_{(o,d,i) \in L^i} d^{(o,d,i)} \beta^{(o,d,i)}
\]

s.t. \[\pi^i_v^{(o,d,i)} - \pi^i_u^{(o,d,i)} + \alpha^i_{(u,v)} \geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i\] \[\pi^i_v^{(o,d,i)} - \pi^i_u^{(o,d,i)} + \alpha^i_{(u,v)} \geq -c_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i\] \[\pi^i_{u,v}^{(o,d,i)} - \pi^i_{d,v}^{(o,d,i)} + \beta^i_{(o,d,i)} \geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i\] \[\pi^i_{(o,d,i)} \geq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i\] \[\alpha^i_{(u,v)} \geq 0 \quad \forall (u,v) \in E\] \[\beta^i_{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L^i\]

where $\pi^i$, $\alpha^i$, and $\beta^i$ are the dual variables associated with the flow balance constraints, capacity constraints, and demand constraints, respectively, for carrier $i$. Constraints (12)-(13) correspond to each flow variable $f_{(u,v)}^{(o,d,i)}$, while (14) corresponds to the variables $f_{(o,d,i)}^{(o,d,i)}$.

The inverse problem for carrier $i$ based on the Limited Control model, $InvLC^i$, is formed by modifying the constraints of $DLC^i$ in order to ensure that complementary slackness conditions will be satisfied for $(\pi^i, \alpha^i, \beta^i)$ and $f^*$. For each variable that is positive in the centralized optimal solution $f^*$, the corresponding dual constraint must hold with equality. In addition, the following constraints are included in $InvLC^i$:

\[
\alpha^i_e = 0 \quad \forall e \in E: \sum_{(o,d,i) \in L^i} f^{*(o,d,i)}_{e} < k^i_e
\]

\[
\beta^i_{(o,d,i)} = 0 \quad \forall (o,d,i) \in L^i: f_{(d,o,i)}^{*(o,d,i)} < d^{(o,d,i)}
\]

\[
e^i_e \geq 0 \quad \forall e \in E.
\]
(17) invokes the complementary slackness condition for (9) in $LC^i$; when carrier $i$ does not use his full allotment of capacity on a leg, the dual variable corresponding to that leg must equal 0. Similarly, (18) enforces the complementary slackness condition for constraint (10) in $LC^i$; when a load is not fully delivered, the dual variable corresponding to that load must equal 0. Intuitively, it makes sense to restrict the capacity exchange prices $c_e$ to non-negative values; this assumption is reflected in (19).

As the parameters of interest, the capacity exchange prices $c_e$, appear only in the constraints of the inverse problem ($InvLC^i$), it follows that our interest in the inverse problem is in finding a set of prices $c_e$ and dual variables $\pi^i, \alpha^i, \beta^i$ that will make the set of constraints in the inverse problem feasible. Any vector $c_e$ that, together with $(\pi^i, \alpha^i, \beta^i)$, satisfies the constraints of $InvLC^i$ will make $f^*$ optimal for $LC^i$. Thus, to ensure $f^*$ is optimal for every carrier, we must find one common vector $c_e$ and dual vectors $(\pi^i, \alpha^i, \beta^i)$ that satisfy $InvLC^i$ for every carrier $i$. Let $InvLC$ be the constraint set created by combining the constraints of $InvLC^i$ over all carriers $i$.

**Theorem 1.** A feasible solution $(\pi^i, \alpha^i, \beta^i, c_e)$ to $InvLC$ is guaranteed to exist.

**Proof.** Associate dual variables $f^{(u,v)}$ and $y^{(o,d,i)}$ with constraints (12)-(13) and (14), respectively. Now consider the dual of $InvLC$ when an objective function of $\text{min} \ 0$ is added:

$$\max \sum_{(o,d,i) \in L} r^{(o,d,i)} y^{(o,d,i)} \quad (20)$$

s.t. \[
\sum_{(u,v) \in E} f^{(o,d,i)}_{(u,v)} - \sum_{(v,w) \in E} f^{(o,d,i)}_{(v,w)} \leq 0 \ \forall (o,d,i) \in L, v \in V : v \notin \{o,d\} \quad (21) \\
y^{(o,d,i)} - \sum_{(o,w) \in E} f^{(o,d,i)}_{(o,w)} \leq 0 \ \forall (o,d,i) \in L \quad (22) \\
\sum_{(v,d) \in E} f^{(o,d,i)}_{(v,d)} - y^{(o,d,i)} \leq 0 \ \forall (o,d,i) \in L \quad (23) \\
\sum_{(o,d,i) \in L} y^{(o,d,i)} \leq 0 \ \forall (u,v) \in E, \forall i \in N \quad (24) \\
\sum_{i \in N} \sum_{(u,v) \notin E^i} f^{(o,d,i)}_{(u,v)} \leq 0 \ \forall (u,v) \in E \quad (26) \\
\]

Equations (21)-(23) are associated with $\pi^{(o,d,i)}$, (24) with $\alpha^{(o,d,i)}$, (25) with $\beta^{(o,d,i)}$, and (26) with $c_{(u,v)}$. Assuming all load revenues $r^{(o,d,i)}$ are non-negative, (25) implies that the objective function (20) is bounded. Furthermore, the solution $y = f = 0$ is clearly feasible. As the dual is bounded and feasible, $InvLC$ must also be feasible. $\square$

### 3.2. Strict Control Model

The second model we discuss is based on a model utilized by (Agarwal and Ergun 2007b) for one component of their work in the liner shipping industry. In this alternative model, the flow variables for all the loads in the system, including loads associated with other carriers, are included
in the model for carrier \( i \). Thus the use of capacity by other carriers is acknowledged explicitly through their associated flow variables. As the model for each carrier includes the entire set of flow variables present in the Centralized model \( C \), the flow balance, capacity, demand, and non-negativity constraints for each carrier’s model are exactly as in \( C \). The objective function for carrier \( i \) is as follows:

\[
\max \sum_{(o,d) \in L^i} f^{(o,d,i)}_i + \sum_{e \in E^i} (c_e \sum_{(o,d) \notin L^i} f^{(o,d,j)}_e) - \sum_{e \notin E^i} (c_e \sum_{(o,d,i) \in L^i} f^{(o,d,i)}_e).
\] (27)

The second term of the objective function (27) reflects the capacity exchange prices received by carrier \( i \) as other carriers use capacity operated by carrier \( i \); therefore when capacity exchange prices are high enough, carrier \( i \) is encouraged to leave capacity open for use by other carriers. We refer to this model as the Strict Control model (Strict\(^i\)) because it implies mathematically that a single carrier has full control over the decisions of other carriers.

As with the Limited Control model, we find capacity exchange prices under the Strict Control model by employing inverse optimization. The inverse problem for the Strict Control model for carrier \( i \), InvStrict\(^i\), can be obtained from InvLimited\(^i\) by adding the following constraints:

\[
\begin{align*}
\pi^{i}{}_{v,(o,d,j)} & - \pi^{i}{}_{u,(o,d,j)} + \alpha^{i}{}_{u,v} \geq c_{u,v} \quad \forall (o,d,j) \notin L^{i}, (u,v) \in E^{i} \quad (28) \\
\pi^{i}{}_{v,(o,d,j)} & - \pi^{i}{}_{u,(o,d,j)} + \alpha^{i}{}_{u,v} \geq 0 \quad \forall (o,d,j) \notin L^{i}, (u,v) \notin E^{i} \quad (29) \\
\pi^{i}{}_{a,(o,d,j)} & - \pi^{i}{}_{a,(o,d,j)} + \beta^{i}{}_{a,(o,d,j)} \geq 0 \quad \forall (o,d,j) \notin L^{i} \quad (30)
\end{align*}
\]

In addition, \( \pi^{i}{}_{v,(o,d,i)} \) and \( \beta^{i}{}_{a,(o,d,i)} \) are constrained over \( (o,d,i) \in L \), rather than only over the loads associated with carrier \( i \) as in (15), (16), and (18). Finally, (17) is replaced with the following constraint:

\[
\alpha^{i}{}_{e} = 0 \quad \forall e \in E : \sum_{(o,d,i) \in L} f^{*(o,d,i)}_e < k_e
\]

Once again, we combine the constraints InvStrict\(^i\) over all carriers and search for a feasible set of capacity exchange prices. Such a set will guarantee that \( f^{*} \) is optimal for the individual problems Strict\(^i\).

Like InvLimited\(^i\), a feasible solution to InvStrict\(^i\) must exist. This can be confirmed by examining a result in (Agarwal and Ergun 2007a), in which a multi-commodity flow game with multiple owners on an edge is studied. It is proved that edge prices must exist that satisfy a problem formulated by aggregating the inverse problem of each owner. As the carrier alliance game is a simplified version of the multi-commodity flow game in (Agarwal and Ergun 2007a), the result follows.

4. **Comparison of Limited and Strict Control Models**

Having established two distinct models for the behavior of an individual carrier within the alliance and the methodology for obtaining capacity exchange prices using each of the models, we now focus on the characteristics of resource and profit allocations obtained using each model. What are the advantages and disadvantages of the Limited and Strict Control models? Qualitatively, it can be argued that the Limited Control model offers a more realistic view of the decisions available to an individual carrier. Quantitatively, this section focuses on analyzing the allocations obtained under each model to determine their respective ability to ensure alliance optimal behavior is attained. This analysis will be conducted from the perspectives of centralized feasibility and cooperative game theory.
4.1. Centralized Feasibility

In the following discussion we prove that when using the Strict Control model, the aggregated individual solutions may be infeasible from the centralized perspective. This result is counterintuitive, as one might expect that as the level of control represented in a given model increases, the ability to produce the desired action (in this case, behavior consistent with the alliance optimal solution) would also increase. Instead we find that it is exactly this increased control on the part of an individual carrier that leads to behavior inconsistent with the centralized solution.

**Theorem 2.** Given a set of capacity exchange prices feasible for InvStrict, it is possible that optimal solutions exist for Strict \(i\) that create infeasibility in the centralized setting.

*Proof.* Consider the simple system depicted in Figure 1 in which carrier \(A\) operates a leg with origin \(o\), destination \(d\), and capacity 2. Each carrier has one associated load, described in Table 1. For example, load \((o,d,A)\) represents a load associated with carrier \(A\), with ready time and origin location corresponding to node \(o\), and delivery deadline and destination corresponding to node \(d\).

![Figure 1 Alliance Network](image)

Table 1 Load Descriptions

<table>
<thead>
<tr>
<th>Load</th>
<th>Per-Unit Revenue ((r(o,d,k)))</th>
<th>Size ((d(o,d,k)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((o,d,A))</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>((o,d,B))</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The centralized optimal solution is to deliver one unit of each load. That is, \(f_{o,d,i}^*(o,d,A) = f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 1\). The only value of \(c(o,d)\) that is feasible for InvStrict is \(c(o,d) = 1\). The objective function for Strict \(B\) is \(f_{o,d,i}^*(o,d,B) = c(o,d)\) \(f_{o,d,i}^*(o,d)\), and since \(f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d)\) in any feasible solution to Strict \(B\), all feasible solutions to Strict \(B\) have an objective function value of 0. Consider the following basic solutions: (1) Basic variables: \(f_{o,d,i}^*(o,d,A) = f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 1, s^7 = 1, s^4 = s^5 = s^6 = 0\). (2) Basic variables: \(f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 2, s^1 = s^2 = s^3 = s^4 = 0, s^6 = 1\). Each of these solutions is in fact a basic feasible solution. It follows that each of these solutions is an optimal solution that may be obtained by using a standard solver such as CPLEX.

Assuming carrier \(i\) will implement only those decisions pertaining to his associated loads, the aggregate alliance solution is formed by retaining the optimal value of \(f_{o,d,i}^*(o,d)\) from Strict \(i\); the optimal value of \(f_{o,d,i}^*(o,d,j)\) from Strict \(j\) is ignored. The unique optimal solution to Strict \(A\) is \(f_{o,d,i}^*(o,d,A) = f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 1\); therefore the contribution of carrier \(A\) to the aggregate solution is \(f_{o,d,i}^*(o,d,A) = f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 1\). The contribution of carrier \(B\) based on Solution 1 is \(f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 1\); Solution 1 therefore leads to an aggregate solution which is in fact the centralized optimal solution. The contribution of carrier \(B\) based on Solution 2 is \(f_{o,d,i}^*(o,d,B) = f_{o,d,i}^*(o,d) = 2\), which implies an aggregate solution in which load \((o,d,A)\) and both units of load \((o,d,B)\) are delivered. This aggregate solution is infeasible from the centralized perspective because it requires three units of capacity on leg \((o,d)\). □

Due to the possibility of multiple optimal solutions for Strict \(i\), there is no guarantee that carrier \(i\) will behave in accordance with the centralized solution, even when there is a single set of exchange prices \(c\) that is feasible for InvStrict. The inability to ensure centralized feasibility is clearly a practical limitation of the Strict Control model; in contrast, we show in the following theorem that the Limited Control model can in fact ensure that centralized feasibility is maintained.
Theorem 3. Any solution obtained using the Limited Control model is feasible from the centralized perspective.

Proof. A flow balance constraint for load \((o,d,i)\) and node \(v\) is contained in \(LC^i\), \(\forall(o,d,i) \in L, v \in V\). Because a solution feasible for \(InvLC\) satisfies the flow balance constraints (8) \(\forall i \in N\), all flow balance constraints (2) in the Centralized model \(C\) are satisfied. Similarly, the demand constraint \(f^{(o,d,i)}_{(d,o,i)} \leq d^{(o,d,i)}\) is contained in \(LC^i\), \(\forall(o,d,i) \in L\), and a feasible solution for \(InvLC\) satisfies the demand constraints (10) \(\forall i \in N\). Consequently, the demand constraints (4) are satisfied. Non-negativity must be satisfied by all variables in \(LC^i\), \(\forall i \in N\); non-negativity is therefore satisfied in the centralized case as well. Finally, the Limited Control model employs capacity restrictions \(k^e_i\) for each carrier \(i\) and leg \(e\) that are constructed to ensure that \(\sum_{i \in N} k^e_i \leq k_e\). We therefore conclude that (3) must also be satisfied, and every aggregate solution obtained using the Limited Control model will be feasible from the centralized perspective. \(\Box\)

We have shown that from the practical standpoint of ensuring feasibility of an aggregate solution, it is advantageous to use the Limited Control model. This conclusion was reached by evaluating the optimal solution for individual carriers, assuming they are already participating in an alliance. In the following discussion we evaluate the potential of each model to ensure that all carriers will in fact choose to participate.

4.2. Comparing Allocations Using Cooperative Game Theory

In this section we compare the allocations, rather than acceptance and routing decisions, obtained using each model. It is important to understand how an allocation is perceived by alliance members. Is the allocation for carrier \(i\) enough to convince him to participate in the alliance? The concepts of cooperative game theory provide a framework for measuring and comparing the benefits of various allocations.

In a cooperative game, rational agents attempt to maximize their individual benefit in a setting in which cooperation among agents is allowed. An alliance in which carriers make and receive payments for the use of capacity fits into the structure of a cooperative game with transferrable payoffs, or a game in which participants are allowed to exchange utility among each other; in the carrier alliance setting payoffs take the form of money and are transferred via capacity exchange prices. An outcome of a cooperative game is described by an allocation of benefits to each participant; in the carrier alliance game the allocation \(x^i\) is comprised of direct revenue from delivering loads \(\left(q^i\right)\) plus the net sum of capacity exchange prices paid and received \(\left(s^i\right)\). Of particular interest is the notion of the core, which is the set of allocations that are (i) budget-balanced, meaning that all benefits are allocated, and (ii) stable, meaning that no subset of participants can benefit by leaving the alliance. Let \(v(S)\) be the total profit that a subset of carriers \(S\) can earn on their own; that is, \(v(S)\) is the optimal objective function value when the centralized problem \(C\) is solved for the subset \(S\). The core is defined as follows:

\[
\sum_{i \in N} x^i = v(N) \quad (31)
\]

\[
\sum_{i \in S} x^i \geq v(S) \quad \forall S \subset N \quad (32)
\]

where (31) is the budget-balance condition and (32) is the stability condition. We call the subset of stability equations (32) in which \(|S| = 1\) rationality constraints, as they ensure that each individual carrier will earn at least as much in the alliance as they could earn operating alone.

Basic cost allocation methods are discussed in (Young 1985); a more detailed discussion about allocation methods and the core of a cooperative game is available in (Owen 2001). Key observations from these works include that the core of a cooperative game is often empty, and the core of
a game may contain many allocations. Production games based on linear programming models were studied in (Owen 1975); flow games were later specifically considered in (Kalai and Zemel 1981) and (Derks and Tijs 1985). In (Kalai and Zemel 1981), networks with a single commodity and capacitated edges owned by players were studied. It was shown that such problems have a nonempty core. (Derks and Tijs 1985) extended these results into the multi-commodity flow arena by showing that the result applied to networks with many commodities, but one common source and sink. (Agarwal and Ergun 2007a) consider multiple sources and sinks and multiple owners on an edge.

The properties of a core allocation are clearly desirable for a carrier alliance. We will ultimately prove in the following discussion that while a core allocation can be obtained regardless of the individual behavioral model employed, a greater number of core allocations are feasible under the Limited Control model. We begin by analyzing the relationship of allocations obtained using the Strict Control model with respect to the set of core allocations.

First, any set of capacity exchange prices feasible for Inv\text{Strict} will define an allocation in the core of the carrier alliance game. A result in (Agarwal and Ergun 2007a) proves that for a simple multi-commodity flow game where every edge has a unique owner, edge prices that satisfy the aggregation of the owners’ inverse problems yield an allocation in the core of the multi-commodity flow game. The result applies to Inv\text{Strict} because the carrier alliance game in which the Strict Control model is employed is equivalent to the simple multi-commodity flow game studied. However, as we prove in the following theorem, every core allocation is not feasible under the Strict Control model.

**Theorem 4.** The set of feasible capacity exchange prices for the Strict Control model might exclude some core allocations.

**Proof.** Consider the example described in Figure 2 and Table 2. The time-expanded network illustrated in Figure 2 includes two legs, (1, 3) and (2, 4), operated by carrier A; the capacity on each of these legs is one. The ground edges (1, 2) and (3, 4) have unlimited capacity.

![Figure 2 Alliance Network for Theorem 4 Example](image)

<table>
<thead>
<tr>
<th>Demand</th>
<th>Per-Unit Revenue</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 3, A)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(2, 4, A)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>(1, 4, B)</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>(2, 4, C)</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Loads for Theorem 4 Example

The side payment for carrier B is as follows: \( s^B = -c_{(1,3)}f_{(1,3)}^{(1,4,B)} - c_{(2,4)}f_{(2,4)}^{(1,4,B)} \). Since \( d_{(1,4,B)} = 1 \), it must be true that \( f_{(1,3)}^{(1,4,B)} + f_{(2,4)}^{(1,4,B)} \leq 1 \), which implies \( s^B \geq \min\{-c_{(1,3)}, -c_{(2,4)}\} = -\max\{c_{(1,3)}, c_{(2,4)}\} \). Solving Inv\text{Strict} with an objective function of \( \max c_{(1,3)} \), an optimal objective function value of 3 is attained. Solving Inv\text{Strict} with an objective function of \( \max c_{(2,4)} \) also yields an optimal objective function value of 3. As the maximum feasible value of either leg’s capacity exchange price is 3, the minimum value of \( s^B \) under the Strict Control model is -3.

It can be easily verified that the allocation \( x^A = 7, x^B = 1, x^C = 1 \) is a core allocation, since it satisfies equations (31) and (32). As \( v^A = 0, v^B = 6, \) and \( v^C = 3 \), in order to obtain this allocation the capacity exchange prices must lead to the following side payments: \( s^A = 7, s^B = -5, \) and \( s^C = -2 \). This contradicts the minimum attainable value of \( s^B \), implying that this core allocation cannot be obtained using the Strict Control model. □
We will now characterize the relationship between the core of the carrier alliance game and allocations obtained using the Limited Control model. An important step in accomplishing this is establishing the relationship of allocations obtained using the Limited Control model to those obtained using the Strict Control model, which is described in the following theorem:

**Theorem 5.** The set of allocations that may be obtained using the Strict Control model is a subset of the set of allocations obtained using the Limited Control model.

The set of feasible capacity exchange prices for the Limited Control model defines the constraint set

\[
\text{InvLC} \quad \pi^i_{o,d,i} - \pi^k_{u,d,i} + \alpha^i_{(u,v)} \geq c_{(u,v)} \quad (33)
\]

where each equation holds with equality if \( f^{(o,d,i)}_{(u,v)} > 0 \). In InvLC, there is exactly one constraint corresponding to \( f^{(o,d,i)}_{(u,v)} \), which is constraint (33). Similarly, consider \( f^{(o,d,i)}_{(d,o,i)} \). In InvStrict there are again \( n \) constraints corresponding to \( f^{(o,d,i)}_{(d,o,i)} \):

\[
(\text{contained in InvStrict}^i) \quad \pi^i_{o,d,i} - \pi^k_{u,d,i} + \beta^i_{(o,d,i)} \geq 0 \quad (36)
\]

where each equation holds with equality if \( f^{(o,d,i)}_{(d,o,i)} > 0 \). In InvLC there is only one constraint corresponding to \( f^{(o,d,i)}_{(d,o,i)} \), which is (36). We conclude that the constraint set InvLC is a subset of the constraint set InvStrict, which implies that any solution that is feasible for InvStrict must also be feasible for InvLC. It follows directly that any allocation obtained under InvStrict can also be obtained under InvLC.

To demonstrate that the set of allocations that may be obtained using the Limited Control model can contain allocations that cannot be obtained using the Strict Control model, we return to the example used in the proof of Theorem 4. The set of capacity exchange prices \( c_{(1,3)} = 5, c_{(2,4)} = 2 \) is feasible for InvLC, and results in the allocation \( x^A = 7, x^B = 1, x^C = 1 \). However, it was demonstrated in the proof of Theorem 4 that this particular allocation cannot be obtained using the Strict Control model. It follows that the set of allocations obtained using the Strict Control model must be a subset of the set of allocations obtained using the Limited Control model.

Because of Theorem 5 we know that it is possible to obtain a core allocation using the Limited Control model. But is one guaranteed a core allocation? We in fact show in the next theorem that non-core allocations may be obtained when using the Limited Control model.

**Theorem 6.** The set of feasible capacity exchange prices for the Limited Control model defines a set of allocations that may contain allocations outside the core.

The capacity exchange prices \( c_{(1,3)} = c_{(2,4)} = 0 \) are feasible for InvLC. \( c_{(1,3)} = c_{(2,4)} = 0 \) implies \( s^A = 0, s^B = 0, s^C = 0 \). The resulting allocation, \( x^A = 0, x^B = 6, x^C = 3 \), clearly does not satisfy the set of stability equations 32, as \( v(A) = 4 \), and is therefore not contained in the core. □
That we may obtain an allocation outside the core when employing the Limited Control model is at first disconcerting, as an allocation in which some subset of members is actually receiving less profit than they could earn on their own is clearly undesirable. However, we can ensure that such an allocation is not obtained by the addition of stability constraints to InvLC. The number of stability constraints of type (32) required is \( 2^{|N|} - 1 \). Based on the relatively small number of carriers participating in an air cargo alliance (for example, the SkyTeam Cargo alliance is currently comprised of 8 carriers, while the WOW alliance is comprised of 4 carriers), the total number of stability constraints will not be prohibitively large. However, if the enumeration of all stability constraints does become a concern, one possibility is to incorporate stability constraints for subsets of size \( m \) or smaller, where \( m < |N| \). This is reasonable under the assumption that carriers have limited information about other carriers participating in the alliance. An important consequence of Theorem 5 is the following corollary, which implies that it is possible to guarantee a core allocation when using the Limited Control model.

**Corollary 1.** InvLC remains feasible when enhanced with stability constraints, and all feasible solutions define a core allocation.

There are some instances, however, for which the set of allocations that may be obtained using the Limited Control model is in fact a subset of the core of the carrier alliance game. Theorem 7 characterizes conditions that are necessary in order for this to be the case; these conditions are especially interesting because they imply that in order for the Limited Control model to produce an allocation that is guaranteed to be in the core of the carrier alliance game with transferrable payoffs, the carrier alliance game with non-transferrable payoffs must have a non-empty core as well. (A game with non-transferrable payoffs corresponds to an alliance in which the allocation payoffs, the carrier alliance game with non-transferrable payoffs must have a non-empty core as well. (A game with non-transferrable payoffs corresponds to an alliance in which the allocation for carrier \( i \) is equal to the direct revenue earned by carrier \( i \), or \( x^i = q^i \forall i \in N \).) This is the case because, as is proven below, a solution in which all capacity exchange prices are zero is always feasible for InvLC.

**Theorem 7.** Given a set of carriers \( N \), the set of feasible solutions for InvLC defines a set of allocations that is a subset of the core only if \( v(S) \leq \sum_{i \in S} v_i \forall S \subset N \).

**Proof.** Assume that \( c = 0 \) is a feasible solution to InvLC. The allocation \( x^i \) received by carrier \( i \) is then equal to \( q^i \), the amount of revenue carrier \( i \) receives by delivering loads in accordance with \( f^* \). If there exists a subset \( S \subset N \) such that \( v(S) > \sum_{i \in S} v_i \), then it must also be true that \( v(S) > \sum_{i \in S} x^i \) and \( x \) cannot be a core allocation since it violates (32).

It remains to show that \( c = 0 \) is a feasible solution to InvLC. Consider the Limited Control model for carrier \( i \):

\[
(LC^i) : \quad \max \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)} - \sum_{e \in E^i} c_e \left( \sum_{(o,d,i) \in L^i} f^{(o,d,i)}_e \right)
\]

s.t. \[
\sum_{(u,v) \in E} f^{(u,v)}_{e(v,w)} - \sum_{(v,w) \in E} f^{(v,w)}_{e(v,w)} \leq 0 \quad \forall v \in V, \forall (o,d,i) \in L^i
\]

\[
\sum_{(o,d,i) \in L^i} f^{(o,d,i)}_e \leq k^i_e \quad \forall e \in E
\]

\[
f^{(o,d,i)}_e \leq d^{(o,d,i)} \forall (o,d,i) \in L^i
\]

Let \( f^{*i} \) be the vector of components of \( f^* \) pertaining to the loads of carrier \( i \). That is, \( f^{*i} \) is comprised of \( f^{(o,d,i)}_{(d,o,i)} \) and \( f^{(o,d,i)}_e \), \( \forall e \in E \). We know that \( f^{*i} \) is a feasible solution to \( LC^i \),
since the capacity limits $k_e^i$ were constructed in a manner that ensures $\sum_{(o,d,i) \in L^i} f_c^{(o,d,i)} \leq k_e^i$. Let $\hat{f}$ be an optimal solution to $InvLC^i$ when $c = 0$, and assume $f^*$ is not optimal for carrier $i$ when $c = 0$. $f = \hat{f} \cup \bigcup_{j \in N, j \neq i} f^*$ must be a feasible solution to the centralized problem $C$, since the capacity limits $k_e^i$ were also constructed to ensure that $\sum_{e \in E} k_e^i \leq k_e$ $\forall e \in E$. Furthermore, $\sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^*_d^{(o,d,i)} > \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{(o,d,i)}$ (which must be true due to the optimality of $\hat{f}$) implies that $\sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^*_d^{(o,d,i)} + \sum_{j \in N, j \neq i} \sum_{(o,d,j) \in L^j} r^{(o,d,j)} f^{(o,d,j)} > \sum_{e \in E} \sum_{(o,d,i) \in L^i} u^{(o,d,i)} f^*_{d_e}^{(o,d,i)}$, which contradicts the optimality of $f^*$. We conclude that $f^*$ must be optimal for $LC^i$ when $c = 0$.

Now consider the dual of $LC^i$:

$$(DLC^i): \min \sum_{(u,v) \in E} k_v \alpha^i_{(u,v)} + \sum_{(o,d,i) \in L} d^{(o,d,i)} \beta^{i}_{(o,d,i)}$$

subject to

$$(39) \quad \pi^v_{(o,d,i)} - \pi^u_{(o,d,i)} + \alpha^i_{(u,v)} \geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i$$

$$(40) \quad \pi^u_{(o,d,i)} - \pi^v_{(o,d,i)} + \alpha^i_{(u,v)} \geq -c^i_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i$$

$$(41) \quad \pi^d_{(o,d,i)} - \pi^d_{(o,d,i)} + \beta^i_{(o,d,i)} \geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i$$

$$(42) \quad \pi^v_{(o,d,i)} \geq 0 \quad \forall v \in V, (o,d,i) \in L$$

$$(43) \quad \alpha^i_{(u,v)} \geq 0 \quad \forall (u,v) \in E$$

$$(44) \quad \beta^i_{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L.$$  

Because $LC^i$ has an optimal solution when $c = 0$ (namely, $f^*$), $DLC^i$ must also have an optimal solution when $c = 0$. Let $(\pi^*, \alpha^*, \beta^*)$ be optimal for $DLC^i$ when $c = 0$. $f^*$ and $(\pi^*, \alpha^*, \beta^*)$ must satisfy complementary slackness conditions for $LC^i$ and $DLC^i$; it therefore follows that $(\pi^*, \alpha^*, \beta^*, c = 0)$ must be feasible for $InvLC^i$. (Recall that $InvLC^i$ is constructed by modifying the dual of $LC^i$ to ensure that complementary slackness conditions are satisfied with $f^*$.)

We have shown that $c = 0$ must be feasible for $InvLC^i$. Because $InvLC$ is constructed by combining the constraints of $InvLC^i$ for all $i \in N$, and the exchange prices $c$ are the only components common among the constraints $InvLC^i$ and $InvLC^j$, $\bigcup_{i \in N} (\pi^i, \alpha^i, \beta^i), c = 0$ must be feasible for $InvLC$. \(\square\)

The relationship among the set of allocations that may be obtained using the Strict and Limited Control models and the core of the carrier alliance game is depicted in Figure 3. In summary,

![Figure 3: General Relationship of Allocations](image-url)

we have shown that a core allocation can be obtained using either behavioral model, but that in general the solution space for the Limited Control model includes more core allocations than...
does the solution space for the Strict Control model. Furthermore, even though it is possible to obtain allocations outside the core when using the Limited Control model, the model can easily be adapted to ensure a core allocation is obtained. Thus the Limited Control model, in addition to offering the practical advantage of centralized feasibility, offers desirable theoretical properties as well.

5. Computational Study
We conducted two sets of experiments in order to investigate the benefit to be gained by carrier collaboration. The experiments were not designed to provide a comprehensive look at carrier alliances, but rather to gain general insight into how the benefit associated with collaborating changes with the number and size of the carriers participating in the alliance. Furthermore, by employing our management mechanism using both the Limited and Strict Control models, we were able to analyze the performance of the mechanism under each model. In the first set of experiments, all alliances are comprised of two carriers, while in the second set of experiments each alliance has three carriers. Some instance classes have “carriers” with no legs; these carriers represent freight forwarders and allow us to investigate the impact of freight forwarders on alliances among carriers. This section includes a description of the experimental procedure, as well as a discussion of insights obtained from the results.

5.1. Data Generation
Using data publicly available from the Bureau of Transportation Statistics (Bureau of Transportation Statistics 2006), we identified 5 classes of combination carriers based on network size and capacity of fleet. Network size is approximated by the number of (origin, destination) pairs served by a carrier, while fleet capacity is approximated by the average number of passengers per departure. For each classification listed in Tables 3(a) and 3(b), the actual range represented by the classification is given. Using these classifications, we obtain the general classes of carriers described in Table 3(c).

<table>
<thead>
<tr>
<th>(a) Network Size</th>
<th>(b) Fleet Capacity</th>
<th>(c) Carrier Classifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network Size</td>
<td># of (o,d) Pairs</td>
<td>Fleet Capacity</td>
</tr>
<tr>
<td>large</td>
<td>400 - 775</td>
<td>large 95-150</td>
</tr>
<tr>
<td>medium</td>
<td>131-366</td>
<td>small 15-90</td>
</tr>
<tr>
<td>small</td>
<td>16-100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fleet Capacity per Departure</td>
<td>Network Size</td>
</tr>
<tr>
<td></td>
<td></td>
<td>large</td>
</tr>
<tr>
<td></td>
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<td>95-150</td>
</tr>
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</table>

Each carrier operates a pure hub-and-spoke network, in which the network size and fleet capacity are scaled to reflect, approximately, the relative size relationships among the classes. The number of hubs depends on the size of the carrier; carriers with a large, medium, and small network size operate 3, 2, and 1 hubs, respectively. The number of spoke legs operated by a carrier in class C1 is $12n$, the number of spoke legs operated by a carrier in class C2 or C3 is $5n$, and the number of spoke legs operated by a carrier in class C4 or C5 is $n$, where $n = 5$. The origins for spoke legs operated by carrier $i$ are approximately equally distributed among the hubs operated by carrier $i$, while every spoke leg has a unique destination. Spoke legs operated by carriers with small fleet capacity have 2 units of capacity, while carriers with large fleet capacity operate spoke legs with 5 units of capacity.
The networks of the carriers are completely integrated, meaning that there is a leg from each hub of carrier $i$ to each of hub of carrier $j$ for all pairs of carriers $i$ and $j$. The capacity of the inter-hub legs is large enough to ensure that the benefit of collaborating is not restricted. In this pure hub-and-spoke system, it can easily be seen that the benefit associated with collaborating increases as the capacity on inter-hub legs increases, because any load associated with carrier $i$ that has a destination outside the network of carrier $i$ must travel on an inter-hub leg. It is therefore assumed that carriers participating in an alliance will increase inter-hub capacity to a level that ensures sufficient benefit. Furthermore, in order to simplify analysis, the network is generated such that the decisions about whether to accept a load and how to route that load are dependent solely on network geography and capacity, and not on time. This is accomplished by orienting all spoke legs from hub to spoke, and then setting the origin and destination time of every hub-to-spoke leg as 1 and 2, respectively; every inter-hub leg as 0 and 1, respectively; and every load as 0 and 2, respectively.

The number of loads associated with a carrier is equal to the number of spoke legs operated by that carrier, which approximates a proportional relationship between the size of a carrier’s network and the number of cargo loads booked by that carrier. As any load originating at a spoke must be transported to the hub of that spoke before it can be transported anywhere else, the system is simplified by generating the origin of a load associated with carrier $i$ randomly from the set of carrier $i$’s hubs. For each carrier, the maximum size of a load, $S^i$, is equal to the capacity of that carrier’s legs, and the maximum per-unit revenue of each load is 3. The size and per-unit revenue of each load associated with carrier $i$ are generated according to a uniform distribution over the ranges $[1, S^i]$ and $[1, 3]$, respectively. Two classes of freight forwarders are used in the experiments; a large freight forwarder is represented by class F1, and is associated with $12n$ loads. A small freight forwarder is represented by class F2, and is associated with $5n$ loads. The maximum sizes of loads associated with forwarders of type F1 and F2 are 5 and 2, respectively.

Obtaining accurate demand distributions is very difficult; for this reason we test our mechanism using two different distributions. In the first distribution (D1), a high proportion of a carrier’s loads have destinations within his network, while in the second demand distribution (D2) the proportion of loads a carrier can serve using his own network is low. Let $p^i$ represent the probability that the destination of load associated with carrier $i$ is within the set of destinations reached by spoke legs operated by carrier $i$. All destinations within the network of carrier $i$ are equally likely with probability $\frac{p^i}{\text{spokes}^i}$ where $\text{spokes}^i$ is the number of spoke legs operated by carrier $i$. For a load associated with carrier $i$, all destinations outside the network of carrier $i$ have an equal probability of being selected; this probability is $\frac{1-p^i}{\sum_{j \neq i} \text{spokes}^j}$. Consequently, loads associated with freight forwarders have destinations that are uniformly distributed throughout the network.

5.2. Results and Insights from 2-Carrier Experiments

Information pertaining to the alliance optimal solution is contained in Table 4. For each instance class, the results reported represent the average from 30 instances generated with the same class parameters. The “Carriers” column indicates the class from which each carrier is selected. The change in system revenue is calculated as the total revenue earned by the alliance minus the sum of the revenue each carrier earns by working independently. The percent increase in accepted loads measures the percent difference in the number of loads that can be completely delivered in the local (independent) solution for a carrier and the number of loads associated with that carrier that are completely delivered in the centralized alliance solution.

Analyzing the results in Table 4, we obtain several observations and insights:

- Not surprisingly, the benefit associated with collaborating increases as the probability that a load can be served by its associated carrier decreases. Note that when the benefit under distribution D2 is not higher than the benefit under distribution D1, it is an instance in which the probability that a load can be served by its associated carrier is the same under both distributions.
The benefit associated with collaborating, measured by the increase in system revenue, increases with the size of the network and fleet capacity. Under distribution D1 there are slightly diminishing returns, as the percentage increase in profit declines as network size and fleet capacity increase, while under distribution D2, the percentage increase in profit increases with network and fleet size. Thus we conclude that the marginal benefit associated with increasing network and fleet sizes in collaborating partners increases as the proportion of loads that a carrier can serve using only his network decreases.

Under distribution D1, fleet capacity has more impact than network size on the benefit associated with collaborating. Furthermore, a carrier with large fleet capacity does not experience a significant increase in the number of loads completely accepted for delivery when collaborating with a carrier with small fleet capacity. These observations lead to an interesting insight: consider the relationship between a large national carrier and a smaller subsidiary. If the subsidiary carrier can serve a high proportion of its own demand (as is the case under distribution D1), the parent carrier stands to benefit more by increasing the fleet size of its subsidiary than by increasing the size of the subsidiary network.

The benefit associated with collaborating, measured both by the percentage increase in the number of loads completely accepted for delivery as well as by an improvement over $v(i)$, is strictly positive when a carrier collaborates with another carrier. The number of loads completely accepted

### Table 4 System Revenue and Accepted Loads for Two-Carrier Alliances

<table>
<thead>
<tr>
<th>Instance Class (A,B)</th>
<th>Carriers (A,B)</th>
<th>Chg. in System Revenue</th>
<th>Chg. in Loads Accepted</th>
<th>Demand Distribution D1 (high probability that a carrier can serve own loads)</th>
<th>Demand distribution D2 (low probability that a carrier can serve own loads)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
<td>%</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>C1,C1</td>
<td>40.1</td>
<td>7.2%</td>
<td>7.2%</td>
<td>5.9%</td>
</tr>
<tr>
<td>2</td>
<td>C1,C2</td>
<td>33.0</td>
<td>8.4%</td>
<td>6.7%</td>
<td>16.7%</td>
</tr>
<tr>
<td>3</td>
<td>C1,C3</td>
<td>19.6</td>
<td>5.9%</td>
<td>2.2%</td>
<td>12.8%</td>
</tr>
<tr>
<td>4</td>
<td>C1,C4</td>
<td>30.6</td>
<td>10.2%</td>
<td>6.2%</td>
<td>62.7%</td>
</tr>
<tr>
<td>5</td>
<td>C1,C5</td>
<td>14.6</td>
<td>4.9%</td>
<td>1.9%</td>
<td>44.9%</td>
</tr>
<tr>
<td>6</td>
<td>C1,F1</td>
<td>206.4</td>
<td>68.0%</td>
<td>-24.2%</td>
<td>N/A</td>
</tr>
<tr>
<td>7</td>
<td>C1,F2</td>
<td>55.1</td>
<td>18.2%</td>
<td>-4.5%</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td>C2,C2</td>
<td>33.6</td>
<td>15.3%</td>
<td>14.5%</td>
<td>14.7%</td>
</tr>
<tr>
<td>9</td>
<td>C2,C3</td>
<td>17.8</td>
<td>11.3%</td>
<td>3.6%</td>
<td>18.1%</td>
</tr>
<tr>
<td>10</td>
<td>C2,C4</td>
<td>33.6</td>
<td>28.9%</td>
<td>14.2%</td>
<td>88.5%</td>
</tr>
<tr>
<td>11</td>
<td>C2,C5</td>
<td>15.3</td>
<td>13.2%</td>
<td>-4.4%</td>
<td>86.4%</td>
</tr>
<tr>
<td>12</td>
<td>C2,F1</td>
<td>157.9</td>
<td>124.8%</td>
<td>-41.7%</td>
<td>N/A</td>
</tr>
<tr>
<td>13</td>
<td>C2,F2</td>
<td>54.1</td>
<td>43.9%</td>
<td>-16.2%</td>
<td>N/A</td>
</tr>
<tr>
<td>14</td>
<td>C3,C3</td>
<td>16.3</td>
<td>15.8%</td>
<td>15.8%</td>
<td>14.5%</td>
</tr>
<tr>
<td>15</td>
<td>C3,C4</td>
<td>18.7</td>
<td>29.3%</td>
<td>20.2%</td>
<td>26.8%</td>
</tr>
<tr>
<td>16</td>
<td>C3,C5</td>
<td>13.0</td>
<td>22.3%</td>
<td>15.1%</td>
<td>36.6%</td>
</tr>
<tr>
<td>17</td>
<td>C3,F1</td>
<td>79.6</td>
<td>134.9%</td>
<td>-56.8%</td>
<td>N/A</td>
</tr>
<tr>
<td>18</td>
<td>C3,F2</td>
<td>35.0</td>
<td>60.7%</td>
<td>-28.0%</td>
<td>N/A</td>
</tr>
<tr>
<td>19</td>
<td>C4,C4</td>
<td>23.4</td>
<td>90.1%</td>
<td>58.8%</td>
<td>78.0%</td>
</tr>
<tr>
<td>20</td>
<td>C4,C5</td>
<td>11.1</td>
<td>52.0%</td>
<td>10.4%</td>
<td>55.7%</td>
</tr>
<tr>
<td>21</td>
<td>C4,F1</td>
<td>48.0</td>
<td>185.7%</td>
<td>-95.5%</td>
<td>N/A</td>
</tr>
<tr>
<td>22</td>
<td>C4,F2</td>
<td>33.2</td>
<td>116.9%</td>
<td>-59.8%</td>
<td>N/A</td>
</tr>
<tr>
<td>23</td>
<td>C5,C5</td>
<td>9.8</td>
<td>72.9%</td>
<td>66.7%</td>
<td>65.6%</td>
</tr>
<tr>
<td>24</td>
<td>C5,F1</td>
<td>17.7</td>
<td>147.4%</td>
<td>-98.2%</td>
<td>N/A</td>
</tr>
<tr>
<td>25</td>
<td>C5,F2</td>
<td>16.9</td>
<td>159.4%</td>
<td>-88.2%</td>
<td>N/A</td>
</tr>
</tbody>
</table>
strictly decreases, however, for a carrier collaborating with a freight forwarder. This result suggests that carriers may want to negotiate rules regarding priority of the carrier’s loads relative to the forwarder’s loads in order that the carrier’s customer service level does not decline as a result of entering into collaboration with a forwarder.

Our experimental results also confirm that allocations obtained under the Limited and Strict Control models are as expected with respect to the core. However, the Strict Control model seems to apportion alliance benefit more equally among the carriers, while the Limited Control model can arbitrarily favor one carrier. This behavior occurs because the mechanism has been designed to be indifferent when selecting among all feasible settings for the capacity exchange prices; when implemented using a standard LP solver, therefore, default rules for establishing a starting solution will have a large impact on the final solution obtained, in this case resulting in a tendency to distribute alliance benefit in a disproportionate manner. Clearly, then, it is desirable for an alliance to have more control over the allocations obtained when implementing the mechanism; this can be accomplished by guiding the choice of capacity exchange prices towards those prices that result in desired allocations. Establishing rules for selecting among feasible capacity exchange prices, and the adaptation of the mechanism for incorporating these rules, will be addressed in a subsequent paper.

5.3. Results and Insights from 3-Carrier Experiments

In the second set of experiments, alliances with three carriers were examined. Once again, 30 instances with the same class parameters were generated for each instance class. A total of 80 instance classes were tested; all possible combinations of three carriers were represented, excluding alliances comprised only of freight forwarders. Analyzing data similar to that reported in Table 4, we observe the following:

- Under demand distribution D1, the benefit associated with adding a third carrier to an existing (or potential) two-carrier alliance varies greatly. For example, given an alliance comprised of two C1 carriers, adding a third carrier yields no benefit to the original two C1 carriers. Given an alliance of C1 and C4, adding a third carrier helps C4 (but not C1) if the third carrier has a large fleet capacity. When two carriers of type C5 collaborate, both carriers are helped by the addition of a third carrier with large fleet capacity.

- Under distribution D2, it is in general beneficial to all carriers to grow the alliance.

- As in the two-carrier experiments, we observe that under distribution D1, higher fleet capacity seems to yield higher benefits from collaborating than does size of network.

- While the number of loads each carrier accepts does not always decrease when two carriers collaborate with a freight forwarder, we still observe a dramatic decline in the number of carrier loads accepted as compared to when two carriers collaborate with a third carrier.

We also see even more pronounced in the 3-carrier experiments that collaborating yields much higher benefits as the proportion of loads that can be served entirely by their associated carrier decreases. This effect, in addition to the first two observations discussed above, imply that the properties of demand experienced by carriers can greatly impact how much the carriers can benefit by collaborating. While it is true that under both distributions studied the benefit experienced by any carrier is strictly positive when collaborating with other carriers, it is an important observation that some pairs of carriers are in fact better off (in terms of the number of loads completely accepted) by not adding a third carrier. Before evaluating the benefits of a potential alliance or an additional partner, therefore, it is important to consider the characteristics of the demand associated with each carrier.

6. Summary and Insights

In this paper we have proposed a mechanism that utilizes capacity exchange prices in order to achieve optimal utilization of alliance capacity as well as an implementable distribution of alliance
revenues. The methodology underlying the mechanism employs inverse optimization to determine capacity exchange prices that encourage carriers to behave in a manner that is optimal for the alliance; this tool is dependent on the mathematical model of the behavior of an individual carrier within the alliance. We studied two ways of modeling this behavior, and examined the impact of model selection on mechanism performance. The models studied differ in how the capacity used by other carriers’ loads is acknowledged in an individual carrier’s model.

We found that our proposed model, the Limited Control model, is promising with regard to both practical and theoretical properties of the allocations obtained. The Limited Control model guarantees centralized feasibility whereas the Strict Control model cannot. Furthermore, while both models can guarantee a core allocation, the feasible solution space of the Limited Control model is larger and defines more core allocations than that of the Strict Control model. A key insight obtained from the comparison of mechanism results when two distinct models are used is that the choice of behavioral model can significantly impact not only alliance recommendations (i.e. capacity exchange prices), but also whether the mechanism achieves alliance optimal performance.

Finally, experimental analysis was conducted to determine the benefit to be gained by collaborating, and how this benefit changes as the number and characteristics of the carriers participating in the alliance changes. In general, the benefit a carrier experiences by collaborating increases with the network size and fleet capacity of the partnering carriers. The characteristics of the demand distribution significantly impact the benefit a carrier experiences by participating, however. When a carrier must utilize the networks of partner carriers to deliver a high proportion of his loads, he will always benefit as the alliance grows, while this is not necessarily the case for a carrier who can deliver a high proportion of his loads using only his own network. An interesting insight gained from the experiments is that a carrier may benefit more by increasing the fleet capacity of a subsidiary carrier than by increasing the number of markets served. Finally, the analysis confirmed that the mechanism operates as expected with regard to how the choice of underlying behavioral model impacts the allocations obtained.

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References


