A Real Options Approach to Describe Market Entry Investment Decisions: A General Model and A Case Study of the Retail Industry

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Abstract

Many industries such as the retail industry are increasingly competitive in the U.S. since there are many players that offer similar products and services at similar price levels. In a competitive market, the decision to invest in a market is a key strategy which will differentiate a company from its competitors. In this paper, we pursue the market selection problem from an investment perspective and describe a novel option valuation approach used to determine the optimal investment time and the market potential threshold at which a firm should enter the market. We use the retail industry to explain our theoretical model and examine our theoretical results. One important result from our option valuation approach is an explanation of why big retailers invest relatively later and at higher market potential thresholds in markets with small retailers than in markets without small retailers. Findings from an empirical study support this theoretical result.

1. Introduction

The retail industry is an essential component of any industrialized economic system [1]. In 2005, retail was the third largest private industry in the U.S. in terms of number of establishments and employees after ‘educational services, health care, and social assistance’ and ‘professional and business services’. Retail trade accounts for approximately 12.4 percent of all business establishments and for approximately 11.6 percent of employment (see [2] for statistics). The U.S. retail industry generates \$3.8 trillion in retail sales annually (\$4.2 trillion if food service sales are included) [2]. It is also one of the largest industries in the United States in terms of the value added to the Gross Domestic Product (GDP) and accounts for approximately 7% of the U.S. GDP (see [3] for statistics).

The retail industry is increasingly competitive in the U.S, and consequently, retailers constantly seek to improve their operations and business strategies to differentiate themselves from other competitors [4]. In addition to price reductions and running promotions and loyalty programs, one of the most important business strategies is the selection of geographic markets to open new stores [5]. Despite the importance of retail market selection, many retailers have often used qualitative approaches, which are mainly based on expert opinions for evaluating potential markets [6-10]. Many retailers expressed their interests in the development of appropriate methods and techniques that could assist them in systematic evaluation of market potential for new store investment and development [11, 12]. Hence, market selection analysis is a central topic in retail research and many existing marketing studies have introduced and developed qualitative methods for retail market selection (for a comprehensive review of this literature see [5, 13]). These research studies primarily identify the most significant attributes that impact the store’s performance in a particular market or in a particular location. However, they do not address retail market selection from an investment analysis point of view, which is an important aspect of decision-making in the retail industry.

Retailers recognize opening stores in developing markets as investment opportunities whose success enhances the financial position of retail firms. Opening stores is an example of an investment activity since it has three important features of investment decisions as outlined by Dixit and Pindyck [14]. It is *partially irreversible*, it is subject to *uncertainty*, and management may have the *flexibility* to choose investment time. In this paper, we describe *an option valuation approach for investment analysis of a retailer's investment in a dynamic market*. This option valuation approach *explains why big retailers invest relatively later and at higher market potential in markets with small retailers compared to markets without small retailers*. To the best of our knowledge, the research introduced in this paper is novel since it provides a computational framework to *evaluate the financial performance of a retail store in a potential market*. It also provides a means to *determine the retailer's optimal investment exercise time* considering the *competition* effect and the management *flexibility* to defer an investment. In addition, our option valuation approach is the first model that *explains the difference in investment timing of retailers considering their cost structures*. The preliminary results of this work have been appeared in [15, 16].

It is important to recognize the general nature of the problem addressed in this paper. The essential phenomenon of interest is market entry by large and small competitors characterized by differing investment, fixed, and variable costs. These competitors could be selling laundry detergent, airplanes, or consulting services. The general decision of interest is when, and under what conditions, one should enter a new market, as a function of your characteristics and those of existing or potential competitors. This paper addresses retail companies because of the availability of large data sets to support validation of our theoretical conclusions. We expect, however, that these conclusions may apply to entry into other types of markets as well. This paper is structured, as follows.

In section 2, we summarize the similarity between *a retailer's investment opportunity* in a developing market and *a perpetual American call option on a common stock*. Based on this analogy, we develop an option valuation approach to determine the optimal time that a retailer should exercise his investment opportunity and opens a store in a developing market. In section 3, we present several numerical examples of this option valuation approach to assess the impact of competition on retailer's investment decisions. We theoretically demonstrate that big retailers should invest relatively later and at higher market potential in developing markets with small retailers compared to markets without small retailers. In section 4, we develop a practical procedure that can be used by retailers for finding optimal investment times in developing markets. In section 5, we empirically examine the actual investment behavior of a big retailer in retail markets. Our empirical results provide supporting evidence for our theoretical work. Conclusions are provided in section 6.

2. An option valuation approach for the evaluation of a retailer's investment opportunity

Traditional investment analysis is usually performed based on the Net Present Value (NPV) approach. In this approach, a retailer should choose the best market for opening a store that provides the greatest NPV. The NPV approach is based on some implicit assumptions that reflect its basic inadequacy. Limitations of the NPV approach are widely documented in the literature [14, 17, 18]. This approach does not properly address uncertainty in store cash flow analysis and it does not consider the possibility that the retail management may have the flexibility to defer investment in a developing market. Further, the NPV approach does not fully capture the strategic aspect of an investment opportunity in a competitive retail market. For instance, a retailer may decide to open a store earlier in a market to take the first-mover advantage and delay the entry of his competitor.

Limitations of the NPV approach can be overcome by using a different perspective on investment under uncertainty, which is recognized as real options. The real option approach addresses managerial flexibility and strategic behaviors of decision makers under dynamic uncertainty [14, 17, 18]. It also provides an analytical framework to evaluate management flexibility in decision-making regarding whether or how to proceed with business investments while it considers the dynamic uncertainty of the investment underlying factors. A number of existing corporate finance studies have used the real option theory in strategy practice and market performance [19-21].

A retailer's investment opportunity to open a store in a developing market is an example of a real option. It gives the retailer the right – which the retailer needs not exercise – to make an investment outlay (the exercise price of this option) and open a store in a developing market. In this paper, we assume this retailer's investment opportunity is perpetual and never disappears. This is similar to perpetual American call options that can be exercised any time in the future. The financial performance of this store is determined by the store's free cash flow that derives from the size (or potential) of the market where the retail store is located. In this paper, we consider the value of the retail market potential as the underlying factor in the evaluation of a retailer's investment opportunity in a developing market. This is similar to stock price, which is the underlying asset for the valuation of a call option. We use the similarity between a retailer's investment opportunity as a real option and a perpetual American call option on a common stock as a financial option and develop an investment analysis approach for the evaluation of a retailer's investment opportunity. This analogy is summarized in Table 1, as follows.

Table 1. The analogy between a retailer's investment opportunity and a perpetual American call option

Perpetual American Call Option	Retailer's Investment Opportunity
Stock price	**Value of retail market potential**
Expected growth rate of stock price	**Expected growth rate of retail market**
Volatility of stock price	**Volatility of retail market**
Maturity date	**Investment time horizon**
Strike price	**Investment cost to develop a store**
Discounted option payoff	**Discounted expected Net Present Value (NPV) of store free cash flow**
Discount rate	**Discount rate**

The value of retail market potential – the underlying asset of a retailer's investment opportunity – changes randomly over time. Similarly the stock price – the underlying asset of a financial call option – changes randomly over time. In this paper, we describe the dynamic uncertainty of a retail market by two parameters: the expected growth rate and the volatility. These parameters are similar to the expected growth rate and the volatility of stock price, respectively that are used for modeling the dynamic uncertainty of stock price in the financial evaluation of perpetual American call options. We also assume that investment time horizon of a retailer's investment opportunity is infinitive. This is similar to the maturity date of a Perpetual American call option, which is infinity. Further, we assume that a retailer's investment option is free. Exercising an investment option is what costs money. This investment cost to develop a store is similar to the strike price that a call option holder must pay to acquire a share of stock.

The financial problem for a call option holder is to find the optimal time to exercise his call option and acquire a share of common stocks. Similarly, the investment problem for a retailer who holds an investment opportunity in a developing market is to find the optimal time to exercise his investment opportunity and open a store. A retailer exercises his investment opportunity when the net present value of store free cash flow exceeds the value of keeping the investment opportunity alive. Similarly, a call option holder also exercises his perpetual American call option when the discounted option payoff exceeds the value of keeping the option alive. The retail store free cash flow is evaluated according to a discount rate that represents the retailer's time value of money and the riskiness of his investment opportunity. This is similar to the discount rate that is used by the call option holder that represents his time value of money as well as his subjective assessment of the riskiness of the option. In this paper, we assume that the retailer can correctly estimate the discount rate for his investment opportunity.

We use this described analogy to develop an option valuation approach that determines the optimal investment time for a retailer to exercise his investment opportunity. In the next section, we describe our option valuation approach by introducing a demand model for retail markets.

2.1. Demand modeling in retail markets

In this paper, we use a linear demand function [22, 23] to characterize *the inverse relationship between the price of a product and the amount of the product that customers purchase at this price from a retailer in a market.* This demand function is based on the assumption that retailers provide similar products at similar service levels to customers in a market. In modeling the retail demand, we use a typical product (or basket of products) that retailers provide for simplifying our model formulation. Equation (1) summarizes this linear demand function, as follows.

$$P(t) = -\gamma D(t) + X(t) \tag{1}$$

where (P(t)) is the price of the product at time step (t), (D(t)) is the demand for this product at the given price at time step (t), (γ) is the absolute constant slope of this price-demand line, and (X(t)) is the intercept of this line that is modeled to change over time. (X(t)) in Equation (1) represents the intercept of the demand line and determines the maximum price for this product in this retail market at time step (t). (X(t)) can be interpreted as an indicator of the value of retail market potential (or the size of a retail market). The value of (X(t)) increases as the value of retail market potential increases since in large markets there exist customers who pay higher price for a product. Variable (X) in this demand function is modeled to be time-dependent in order to capture the dynamic change of demand in retail markets. Figure 1 shows how the inverse relationship between the price and the demand for this product changes over time as the value of (X(t)) changes over time. In the next section, we describe how a retailer optimizes its profit with respect to the demand in a market.

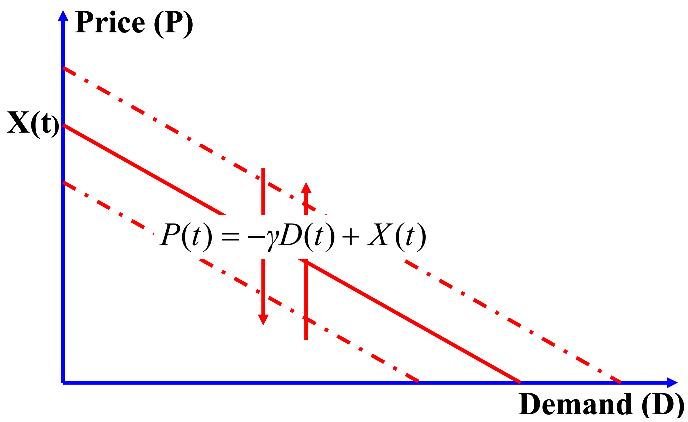

Figure 1. The demand function in retailing

2.2. Retailer's profit function

Retailers seek to maximize their profits according to the market demand, their cost structures, and retail market structure. Retailers are different in many aspects of their operations including store size, supply chain management, service levels, negotiations with suppliers, etc. In this paper, retailers' differences are summarized at higher level of abstraction in terms of their cost structures specified by the following three cost variables.

- The investment cost (IC) to develop and open a store in a market
- The fixed cost (FC) to keep a store open in a market
- The variable cost (VC) to provide one unit of the product in the store

The values of these cost variables are assumed to be known and fixed for retailers. These cost variables are used in the retailer's profit function formulation that is summarized in Equation (2), as follows.

$$\Pi(t) = \left(P(t) - VC\right)Q(t) - FC \quad (2)$$

where (Π(t)) represents the retailer's profit function at time step (t) and (Q(t)) is the amount of the product that customers purchase from a retailer in this market at the given price of (P(t)). In addition, the retailer's investment cost (IC) is a one-time cost that a retailer must pay in order to receive the above profit function in the subsequent time steps. The retailer's financial objective is to maximize its profit considering its cost structure, the demand in the retail market, and the market structure. In this paper, we consider only two types of retail market structures: monopoly and duopoly, but our formulation can be extended to markets with more retailers. The objective is to determine the optimal profit for a retailer in a monopoly or a duopoly retail market. First, we study monopoly retail markets.

2.2.1. Monopoly retail markets

The retailer's profit function in a monopoly market is described in Equation (2). The retailer has the power in this monopoly market and can choose the optimal amount of the product to supply to the market considering the demand in the market. Consequently, we express the price of the product in this monopoly market using Equation (1) by substituting (Q(t)) for (D(t)). Considering this price, the retailer's profit function in the monopoly market is summarized below in Equation (3).

$$\Pi(t) = \left(P(t) - VC\right)Q(t) - FC = \left(\left(-\gamma Q(t) + X(t)\right) - VC\right)Q(t) - FC$$

$$= -\gamma\left(Q(t)\right)^2 + \left(X(t) - VC\right)Q(t) - FC \quad (3)$$

The retailer maximizes its profit by providing the optimal amount of the product to this monopoly market, i.e., the optimal amount of the product maximizes the above profit function. The retailer's optimal profit in the monopoly market – denoted by (ΠM(t)) – is specified below in Equation (4).

$$\Pi^M(t) = \begin{cases} \dfrac{\left(X(t)\right)^2}{4\gamma} - \dfrac{\left(X(t)\right)\left(VC\right)}{2\gamma} + \dfrac{\left(VC\right)^2}{4\gamma} - FC & \text{When } (X(t) \geq VC) \\ -FC & \text{Otherwise} \end{cases} \quad (4)$$

Next, we study duopoly retail markets.

2.2.2. Duopoly retail markets

In a duopoly retail market, we assume that there are only two retailers in the market: retailer 1 and retailer 2. Retailer 1's and retailer 2's fixed costs are denoted by (FC_1) and (FC_2) while their variable costs are denoted by (VC_1) and (VC_2), respectively. Their amounts of the product in this duopoly market are denoted by (Q_1(t)) and (Q_2(t)), respectively. These two retailers have the power in this duopoly market and can choose the optimal amounts of the product to supply to the market considering the demand in the market. For a duopoly retail market, we express the price of the product using Equation (1) where we substitute (Q_1(t)) + (Q_2(t)) for (D(t)). Considering this price, retailer 1's and retailer 2's profit functions in this duopoly market – which are denoted by (Π1(t)) and (Π2(t)), respectively – are summarized in Equation (5).

$$\begin{cases} \Pi_1(t) = -\gamma\left(Q_1(t)\right)^2 + \left(X(t) - VC_1 - \gamma Q_2(t)\right)\left(Q_1(t)\right) - FC_1 \\ \Pi_2(t) = -\gamma\left(Q_2(t)\right)^2 + \left(X(t) - VC_2 - \gamma Q_1(t)\right)\left(Q_2(t)\right) - FC_2 \end{cases} \quad (5)$$

These two retailers maximize their profits by providing optimal amounts of the product to this duopoly market, i.e., optimal amounts of the product maximize above profit functions simultaneously. Retailer 1's and retailer 2's optimal profits in the duopoly market – (Π_1^D(t)) and (Π2D(t)), respectively – are specified below in Equation (6).

$$\begin{cases} \text{when } (\text{VC}_2 - 2\text{VC}_1 + X(t)) \geq 0 \,\&\, (\text{VC}_1 - 2\text{VC}_2 + X(t)) \geq 0 \\ \begin{cases} \Pi_1^D(t) = \dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1 \\ \Pi_2^D(t) = \dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2 \end{cases} \\ \text{when } (\text{VC}_2 - 2\text{VC}_1 + X(t)) \geq 0 \,\&\, (\text{VC}_1 - 2\text{VC}_2 + X(t)) < 0 \\ \begin{cases} \Pi_1^D(t) = \dfrac{(X(t))^2}{4\gamma} - \dfrac{2(VC_1)(X(t))}{2\gamma} + \dfrac{(VC_1)^2}{4\gamma} - FC_1 \\ \Pi_2^D(t) = -FC_2 \end{cases} \\ \text{when } (\text{VC}_2 - 2\text{VC}_1 + X(t)) < 0 \,\&\, (\text{VC}_1 - 2\text{VC}_2 + X(t)) \geq 0 \\ \begin{cases} \Pi_1^D(t) = -FC_1 \\ \Pi_2^D(t) = \dfrac{(X(t))^2}{4\gamma} - \dfrac{2(VC_2)(X(t))}{2\gamma} + \dfrac{(VC_2)^2}{4\gamma} - FC_2 \end{cases} \\ \text{when } (\text{VC}_2 - 2\text{VC}_1 + X(t)) < 0 \,\&\, (\text{VC}_1 - 2\text{VC}_2 + X(t)) < 0 \\ \begin{cases} \Pi_1^D(t) = -FC_1 \\ \Pi_2^D(t) = -FC_2 \end{cases} \end{cases} \tag{6}$$

From Equations (4) and (6) the retailer's optimal profit in the monopoly or duopoly retail market is a function of (X(t)), which is the state variable in our decision space. Since the value of (X(t)) changes randomly over time *it is critical to define a mathematical model for (X(t))to captures its dynamic uncertainty*. This mathematical formulation also helps us determine the optimal time at which a retailer decides to open a store in a market. This optimal time is when the value of (X(t)) reaches a critical threshold value. In the next section, this formulation is presented.

2.3. Modeling the dynamic uncertainty

The value of retail market potential (X(t)) is the underlying factor in the evaluation of a retailer's investment opportunity. We assume that the value of (X(t)) grows exponentially at some positive rate plus some random variation due to economic noise. We use a *Geometric Brownian Motion (GBM)* [24] *to model the dynamic uncertainty of* $\{X(t);\ t\in[0,\infty]\}$ according to Equation (7).

$$dX = \alpha X dt + \sigma X dz \tag{7}$$

where (dz) is an increment of a Wiener process, ($\alpha>0$) is the drift parameter, and ($\sigma>0$) is the volatility parameter of this stochastic process. The GBM process is a useful model to characterize the dynamic uncertainty, which is widely used in finance to model the dynamic variation of the stock price [24]. The GBM process is a good model to reflect incremental movements of our stochastic variable $\{X(t);\ t\in[0,\infty]\}$. It is not a good model to represent large swings. In this paper, we assume that the value of retail market potential $\{X(t);\ t\in[0,\infty]\}$ changes incrementally and therefore, a GBM process is an appropriate model to capture its dynamic uncertainty.

Although GBM models are popular in finance the calculation of the expected option payoff is difficult for many financial options such as perpetual American call options on dividend paying stocks. Also the optimal exercise time for these options cannot be computed analytically. Therefore, numerical methods such as finite difference [25], lattice methods [26], and Monte Carlo simulation [27] are used to estimate the exercise time. In this paper, we chose the lattice method for simplicity as our approximation approach. In the next section, we present a numerical lattice model for the GBM process to approximate the dynamic uncertainty of $\{X(t);\ t\in[0,\infty]\}$ in a discrete time fashion. This discrete approximation helps us evaluate a retailers' investment opportunity as a decision tree analysis problem.

2.4. The numerical lattice model

We use a particular form of lattice methods called the trinomial lattice developed by Kamrad and Ritchken [28] for approximating the stochastic variation of $\{X(t);\ t\in[0,\infty]\}$ in a discrete random walk fashion as shown in Figure 2. Suppose the value of $\{X(t);\ t\in[0,\infty]\}$ at the beginning of the first time step is (X_0). For the next time step, this value may increase by the ratio of (u>0), stay constant, or decrease by the ratio of (d = 1/u) with probabilities of (p_1, p_2, and p_3), respectively. Suppose the length of each time increment is (Δt). Hence, the value of (X) at the next time step ($X(t+\Delta t)$) are summarized below in Equation (8).

$$X(t+\Delta t)=\begin{cases} uX(t) & \text{with probability } p_1 \\ X(t) & \text{with probability } p_2 \\ dX(t) & \text{with probability } p_3 \end{cases} \tag{8}$$

where probabilities (p1, p2, and p3) and jump ratios (u and d) are identified in Equation (9).

$$\begin{cases} u=e^{\lambda\sigma\sqrt{\Delta t}} \\ d=e^{-\lambda\sigma\sqrt{\Delta t}} \end{cases} \quad \text{and} \quad \begin{cases} p_1=\dfrac{1}{2\lambda^2}+\dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \\ p_2=1-\dfrac{1}{\lambda^2} \\ p_1=\dfrac{1}{2\lambda^2}-\dfrac{\mu\sqrt{\Delta t}}{2\lambda\sigma} \end{cases} \tag{9}$$

where ($\mu = \alpha - (\sigma 2/2)$) and ($\lambda \geq 1$). The values of these parameters are approximated based on the method of moments. The mean and the variance of the above approximating discrete distribution are chosen to be equal to the mean and the variance of the normal distribution that characterizes the stochastic nature of the GBM process. A range of values for the above probabilities is generated according to different values of (λ) through which option valuation can be carried out [28]. Any value for ($\lambda \geq 1$) results in a feasible set of probabilities where ($\lambda = 1$) reduces the trinomial lattice formulation into the binomial lattice formulation, which is another standard approximating approach for the GBM process.

The above pattern continues for the subsequent time steps until it reaches the last time step. The total number of time steps – denoted by N – is selected to be large enough to cover several possible values of our state variable $\{X(t);\ t\in[0,\infty]\}$ since the trinomial lattice is an appropriate representation of the GBM process when the time increment (Δt) is small enough and the process occurs over a long time [28]. In this paper, we use the following numerical values for the time increment (Δt = 0.125 Year) and the total number of time steps (N = 10,000).

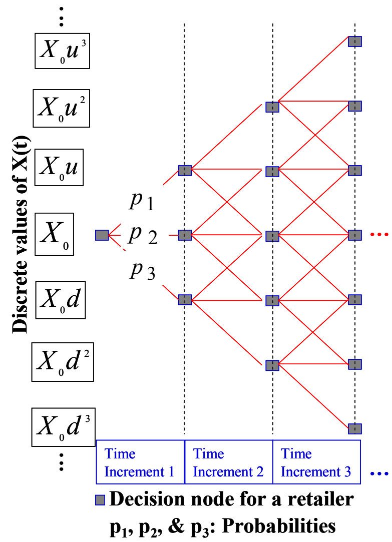

Figure 2. A trinomial lattice model to approximate the dynamic uncertainty of (X(t))

Figure 2 shows how this lattice model is used to evaluate a retailer's investment opportunity as a decision tree problem. Every node in this lattice is a decision node for a retailer who must choose between two alternatives: to exercise its investment opportunity and open a store or wait and defer its investment opportunity to the next time step. In the next section, we describe an evaluation procedure based on this decision tree to determine when it is optimal for a retailer to open a store in a developing market.

2.5. A procedure to evaluate a retailer's investment opportunity in a developing market

To describe our computational evaluation procedure, we consider two retailers (retailer 1 the market whose potential changes according to the GBM model of Equation (7). This is similar to holding two perpetual American call options on a single underlying asset, which is the retail market potential. We assume that these two retailers use the same discount rate for the evaluation of their investment opportunities. We denote this common discount rate by parameter (ρ) in this paper. The problem is to determine the optimal investment time for these retailers to exercise their investment opportunities and open stores in this developing market. We describe the evaluation algorithm, based on the decision tree analysis approach and the concept of the Nash equilibrium in game theory [23], to solve this investment problem, as follows.

1. Input values for the model parameters: ((VC_1), (FC_1), (IC_1), (VC_2), (FC_2), (IC_2), (ρ), (X_0), (γ), (α), (σ), (Δt = 0.125 Year), and (N = 10,000))
2. Compute (p_1, p_2, and p_3) and (u and d) based on Equation (9)
3. Construct a trinomial lattice with (N) time steps of length (Δt) to represent the stochastic variation of (X) according to Equation (8)
4. Set the current time step as the last time step: time step (N)
5. For every node in the current time step
    5.1. Calculate retailer 1's and retailer 2's expected NPVs in the following four market structures

5.1.1. The market, in which both retailers exercise their investment opportunities and open stores (see Appendix A for details of the calculation procedure)

5.1.2. The market, in which retailer 1 exercises his investment opportunity and opens a store while retailer 2 defers its investment opportunity to the next time step (see Appendix B for details of the calculation procedure)

5.1.3. The market, in which retailer 2 exercises his investment opportunity and opens a store while retailer 1 defers its investment opportunity to the next time step (see Appendix B for details of the calculation procedure)

5.1.4. The market, in which both retailers defer their investment opportunities to the next time step or drop their investment opportunities if the current node under consideration is in time step (N) (see Appendix C for details of the calculation procedure)

5.2. Find the stable state of the market (the Nash equilibrium of the market) from the aforementioned four market structures at the current node

5.3. Mark this node for retailer 1 when either market structure 5.1.2 or market structure 5.1.4 is the Nash equilibrium state of this competitive market, i.e., retailer 1 should exercise his investment opportunity at the current node when either market structure 5.1.2 or market structure 5.1.4 is the Nash equilibrium state of this competitive market.

5.4. Mark this node for retailer 2 when either market structure 5.1.3 or market structure 5.1.4 is the Nash equilibrium state of this competitive market, i.e., retailer 2 should exercise his investment opportunity at the current node when either market structure 5.1.3 or market structure 5.1.4 is the Nash equilibrium state of this competitive market.

6. Move back in the lattice to the time step before the current time step. Set the current time step to this new time step and repeat step (5) until the process reaches the first time step

7. For any time step (i) where (i = 1, 2, …, N):

7.1. Find the node with the lowest value of (X) that retailer 1 exercises its investment opportunity. Save this minimum value in ($X_1^*(i)$)

7.1.1. If retailer 1 does not exercise his investment opportunity in none of the nodes in the current time step set ($X_1^*(i) = \infty$)

7.2. Repeat step (7.1) for retailer 2 and save the minimum value in ($X_2^*(i)$)

7.2.1. If retailer 2 does not exercise his investment opportunity in none of the nodes in the current time step set ($X_2^*(i) = \infty$)

8. Find $T_1 = \min \{k \mid k \in \{1, 2, \ldots, N\} \;\&\; (X_1^*(k) \neq \infty)\}$. Return ($T_1$) as the optimal time step that retailer 1 should stop holding his investment opportunity and exercises his option. Also return ($X_1^*(T_1)$) as the optimal investment threshold for retailer 1. Retailer 1 should exercise his investment opportunity when the value of $\{X(t);\, t \in [0,\infty]\}$ reaches or exceeds this threshold value.

9. Repeat step (8) and return (T_2) as the optimal time step that retailer 2 should stop holding his investment opportunity and exercises his option. Also return ($X_2^*(T_2)$) as the optimal investment threshold for retailer 2.

Our evaluation procedure can be readily modified for finding the optimal investment threshold for a retailer (only retailer 1 or retailer 2) in a monopoly market. Only step five in the aforementioned procedure needs to be revised for the calculation of a retailer's expected NPV in a monopoly retail market, as follows.

5. For every node in the current time step

    5.1. Calculate the retailer's expected NPV in the market, in which this retailer exercises its investment opportunity (see Appendix B for details of the calculation procedure)

    5.2. Calculate the retailer's expected NPV if a retailer decides to defer its investment opportunity for the next time step or drop its investment opportunity if the node under consideration is in time step (N) (see Appendix B for details of the calculation procedure)

    5.3. Mark this node for the retailer if the expected NPV of the exercise is greater than the expected NPV of deferring investment opportunity

In the next section, we use the above algorithm and conduct several numerical examples to assess the impact of the competition and the change in the values of model parameters on the retailer's optimal investment threshold. We also compare our approach with the traditional NPV approach.

3. Numerical examples

The first numerical example is conducted to assess how the competition impacts the retailer's optimal investment threshold.

3.1. The competition effect

Consider two retailers with different cost parameters. The first retailer (retailer 1) is a big discount retailer (e.g. Wal-Mart), which has a low variable cost but high fixed and investment costs to open a store. The second retailer (retailer 2) is a small discount retailer (e.g. Dollar General) that has a high variable cost but low fixed and investment costs. To illustrate our option valuation approach, we choose some notional values for our model parameters. We use the following numerical values for retailers' cost parameters (IC_1 = \$400,000 > IC_2 = \$200,000, FC_1 = 200,000 \$/Year > FC_2 = 100,000 \$/Year, and VC_1 = 80 \$/Item Sold < VC_2 = 100 \$/Item Sold). The following numerical values are assigned to our model parameters: (ρ = 15 %/Year, σ = 10 %/Year, X_0 = 700, γ = 1, Δt = 0.125 Year, and N = 10,000 Time Steps). We carry out our analysis at four levels of parameter (α = 2, 4, 6, and 8 %/Year). We determine these two retailers' optimal investment thresholds in the *duopoly* vs. the *monopoly* market in order to explore *the significance of the competition effect* in the retailer's entry decision.

The vertical axis in Figure 3 (X*) indicates the big and the small retailer's optimal investment threshold in monopoly and duopoly markets. These optimal thresholds are critical values of (X^*) that are computed according to the algorithm in section 2.5. The horizontal axis in Figure 3 specifies different values of parameter (α) for which these critical values are calculated. Figure 3 shows that the small retailer's optimal investment threshold in the monopoly and the duopoly market is lower than the big retailer's optimal investment threshold in the monopoly and the duopoly market, respectively. This result is expected since the small retailer has the overall cost advantage, which puts him at the better position than the big retailer to enter this market at the relatively lower value of (X^*).

The competition effect can be also observed in Figure 3. The big retailer's optimal investment threshold in the duopoly market is greater than this retailer's optimal investment threshold in the monopoly market because the small retailer has already opened a store in the duopoly market and hence, the big retailer should wait longer to exercise its investment opportunity. However, the small retailer's optimal investment threshold in the monopoly market is the same as this retailer's optimal investment threshold in the duopoly market since the small retailer is the first retailer who can open a store in this market. Thus, *our option valuation approach indicates the strategic aspect of early investment by small retailers in competitive markets*.

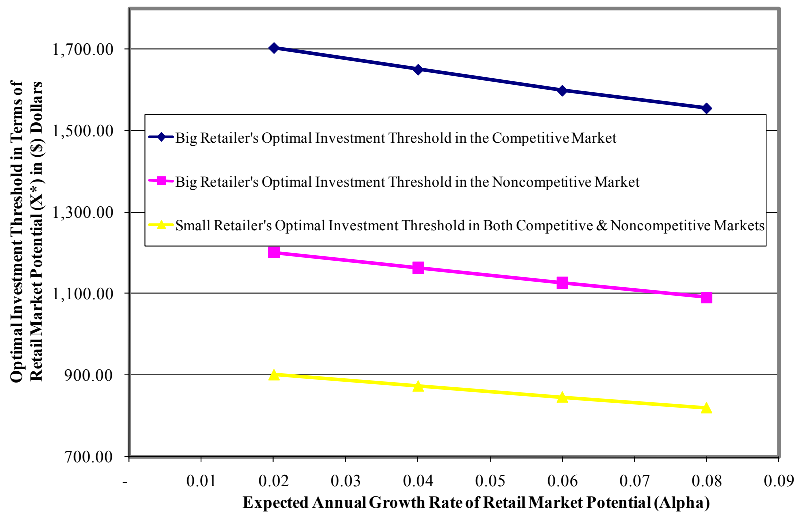

Figure 3. The competition effect on the retailer's optimal investment threshold

In the next section, we conduct several sensitivity analyses to determine how the change in the values of some important model parameters impacts the retailer's optimal investment threshold.

3.2. Sensitivity analysis

The first sensitivity analysis is conducted to explore how the change in the value of the expected growth rate of retail market potential impacts the retailer's optimal investment threshold. Figure 3 shows that the value of the retailer's optimal investment threshold in the duopoly market decreases when the value of parameter (α) increases from 2 %/Year to 8 %/Year. The retail market tends to grow at a higher rate when the value of parameter (α) increases, and hence, the value of a retailer's investment in this market increases. *This motivates a retailer to invest earlier and at lower level of retail market potential.*

The second sensitivity analysis is conducted to explore how the change in the value of the volatility of retail market potential impacts the retailer's optimal investment threshold. Figure 4 shows that the retailer's optimal investment threshold in the duopoly market increases as the value of parameter (σ) increases from 10 %/Year to 30 %/Year. The dynamic uncertainty of the retail market increases when the value of parameter (σ)

increases, and hence, the riskiness of an investment in this market increases. This further raises the critical investment threshold for the retailer in this risky market.

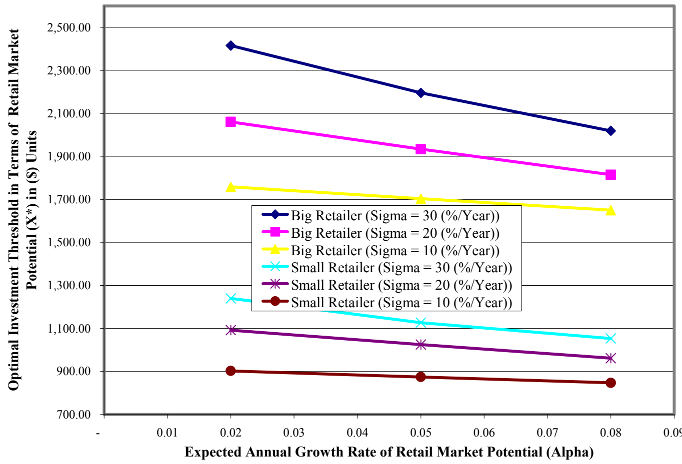

Figure 4. Sensitivity analysis for parameter (σ)

The third sensitivity analysis is conducted to explore how the change in the value of the discount rate impacts the retailer's optimal investment threshold. Figure 5 shows that the retailer's optimal investment threshold in the duopoly market increases as parameter (ρ) increases from 8 %/Year to 12 %/Year. The discount rate shows the retailer's subjective assessment of the riskiness of an investment opportunity. When the riskiness of an investment in a retail market increases, the retailer requires relatively higher rate of return (i.e., discount rate) for investment in this market and invests when the value of retail market potential market reaches satisfactorily high level that compensates for the riskiness of the investment. Next, we compare our options investment analysis approach with the traditional NPV analysis approach to explore the significance of considering management flexibility in investment valuation.

3.3. Comparison between our option valuation approach and the traditional NPV approach

The traditional NPV approach is based on the assumption that retailers have to exercise their investment opportunities immediately or their investment opportunities disappear. The decision rule in this approach is simple; a retailer opens a store in a market when the expected NPV of its investment exceeds zero. This approach does not consider the possibility that a retailer may have the flexibility to defer its investment in a developing market. We use our numerical example in section 3.1 to determine the retailer's investment threshold under the NPV approach and compare this threshold with the threshold under our options analysis approach.

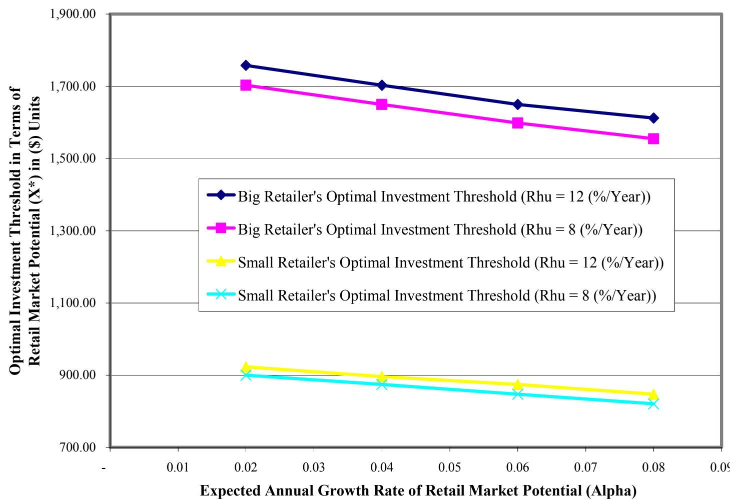

Figure 5. Sensitivity analysis for parameter (ρ)

Figure 6 shows that the big retailer's and the small retailer's optimal investment thresholds under the NPV approach are lower than the optimal investment thresholds under our option valuation approach, respectively. The (X^*) value that triggers an investment decision in our option valuation approach takes into account the balance between waiting for higher (X), where the upside will inevitably happen, and foregoing revenue while the retailer is waiting. On the other hand, the (X^*) value that triggers an investment decision in the NPV approach is one for which the NPV is greater than zero since this approach does not allow for management flexibility to defer an investment opportunity in the market. This makes the NPV approach a more aggressive investment decisions. The NPV approach does not appreciate the value of waiting with only two possible decisions: now or never. Our option valuation approach addresses the value of waiting as an important value component of a retailer's investment opportunity. This result is consistent with the findings of Dixit and Pindyck [14] and Trigeorgis [18] who indicate that in the real world an investor requires a higher return than what is required by the NPV approach for investments in risky projects. However, the choice of the investment analysis approach is not important in the following situations:

- *When a retailer does not have any flexibility to defer its investment opportunity*
- *When a retailer's investment opportunity expires in a short time*
- *When the value of retail market potential is substantially larger than the retailer's investment optimal thresholds based on the NPV and the real option valuation approach*

In other market situations, the optimal retailer's investment threshold is different under these two investment analysis approaches and a retailer should use our option valuation approach for the appropriate investment evaluation. In the next section, we describe a practical procedure that shows how a retailer can use our options investment analysis approach for the real world decision-making.

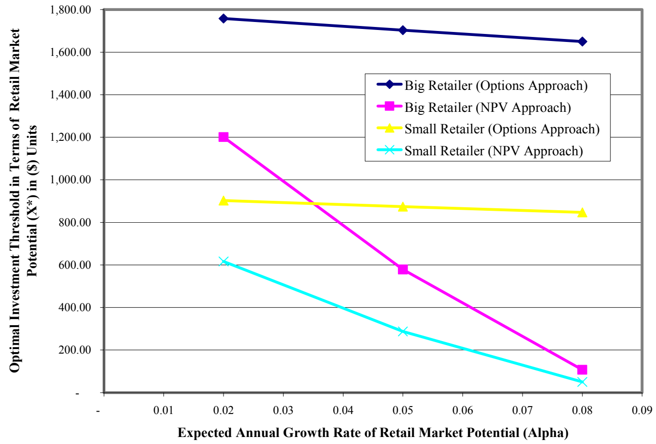

Figure 6. Comparison of the NPV approach and our option valuation approach

4. A practical procedure for investment decision-making

Retailers can use our option valuation approach to determine their optimal investment thresholds in developing markets. The first step is to determine the values of parameters that are required as inputs in our option valuation approach.

4.1. Inputs

A retailer first needs to determine the values of parameters (α) and (σ) as essential inputs for our option valuation approach. In this paper, we assume that a retailer can use historical data for retail market population – denoted by $\{R(t);\, t\in[0,\infty]\}$ – to estimate the values of the above parameters since. The value of retail market population represents the size of a retail market (or the value of retail market potential), which is characterized by variable $\{X(t);\, t\in[0,\infty]\}$ in our demand model in Equation (1). The value of $\{X(t);\, t\in[0,\infty]\}$ increases as the value of retail market population $\{R(t);\, t\in[0,\infty]\}$ increases since in large markets there exist customers who pay higher price for a product. Suppose that populations of this retail market are (R_0), (R_1), …, and (R_N) for time step (0), (1), …, and (N), respectively. Luenberger [29] describes an estimation procedure to use these (N+1) time points of data to estimate the value of parameters (α) and (σ). The sample average and the sample standard deviation of the log-ratio of these (N+1) time points of demand data (denoted by ($\hat{\alpha}$) and ($\hat{\sigma}$) in Equation (10), respectively) are used as unbiased estimators of the expected growth rate and the volatility of $\{R(t);\, t\in[0,\infty]\}$, and consequently, variable $\{X(t);\, t\in[0,\infty]\}$ in our options valuation approach. Equation (10) shows the formula for this estimation process.

$$\begin{cases} \hat{\alpha} = \dfrac{1}{N}\left(\displaystyle\sum_{k=0}^{N-1} \ln\left(\dfrac{R_{k+1}}{R_k}\right)\right) = \dfrac{1}{N}\ln\left(\dfrac{R_N}{R_0}\right) \\ \hat{\sigma} = \sqrt{\dfrac{1}{N-1}\left(\displaystyle\sum_{k=0}^{N-1}\left(\ln\left(\dfrac{R_{k+1}}{R_k}\right) - \hat{\alpha}\right)^2\right)} \end{cases} \tag{10}$$

In addition, the retailer must determine the values of its own cost parameters as well as the values of its competitor's cost parameters if the retail market is competitive. Since a retailer may not be confident about his estimate for the values of its competitor's cost parameters, he should conduct sensitivity analysis to explore the impact of the incorrect estimation on his optimal investment threshold in a developing market.

Finally, the retailer needs to specify the discount rate that to be used for the evaluation of its investment opportunity. The retail firm's cost of capital is a good estimate for this discount rate. However, sensitivity analysis must be conducted for different possible estimates for this factor based on the retailer's subjective assessment of the riskiness of an investment opportunity. Next, we describe what outputs are our option valuation approach yields.

4.2. Outputs

The most important output of our option valuation approach is the retailer's optimal investment threshold in terms of the value of retail market potential that guides him to exercise his investment opportunity and open a store. The retailer should open a store in this market when the value of retail market potential $\{X(t); t\in[0,\infty]\}$ reaches this optimal threshold. In addition, our option valuation approach determines the optimal time that a retailer should exercise its investment opportunity. Further, our model provides the optimal investment threshold of the other retailer, if a competitor is present in this market.

In addition, a retailer can use our option valuation approach to determine how likely it is that the other retailer opens a store in a developing market where this retailer has already opened a store. Suppose that the value of (X) in this market changes according to the GBM process of Equation (7) with parameters (α) and (σ). Also suppose that the first and the second retailer's optimal investment thresholds in this market are (X_1^*) and (X_2^*), respectively. Variable (W = ln(X)) follows a Brownian motion process with the expected growth rate of (ν = α – (σ2/2)) and the volatility of (σ), as summarized in Equation (11).

$$dW = \upsilon dt + \sigma dz = (\alpha - (\sigma^2 / 2))dt + \sigma dz \tag{11}$$

According to Ross's hitting time distribution for a Brownian motion process [30], random variable (Mt), which denotes the maximum value of (W) on the interval [0, t], has the following cumulative density function that is summarized in Equation (12).

$$P\left(\max_{0\le\tau\le t} W_\tau < y\right) = P(M_t < y) = \Phi\left(\frac{y-\upsilon t}{\sigma\sqrt{t}}\right) - e^{\left(2\upsilon y/\sigma^2\right)}\Phi\left(\frac{-y-\upsilon t}{\sigma\sqrt{t}}\right) \tag{12}$$

where (Φ) represents the cumulative distribution function of standard normal random variables and (y) is equal to ($\ln(X_2^*/ X_1^*)$). This is the probability of the event that within time window (t) the second retailer does not open a store in this developing market. We elaborate this calculation through conducting a numerical example. Suppose that the small retailer enters the market at the critical value of ($X^* = 900$) and the big retailer enters the market at the critical value of ($X^* = 1700$). We want to know how likely it is for the big retailer to open a store within five-year time window after the small retailer opens his store in a developing market that is identified by parameters (α = 0.04 and σ = 0.1). Values of other model parameters are ($\nu = \alpha - (\sigma^2/2) = 0.04 - ((0.1)^2/2) = 0.035$, t = 5, and y = ln((1700)/(900)) = 0.63). According to the probability calculation below, it is very likely that the big retailer does not open a store in this market within the five years after the small retailer.

$$P\left(\max_{0\le\tau\le t} W_\tau < y\right) = P(M_t < y) = \Phi\left(\frac{y-\upsilon t}{\sigma\sqrt{t}}\right) - e^{\left(2\upsilon y/\sigma^2\right)}\Phi\left(\frac{-y-\upsilon t}{\sigma\sqrt{t}}\right)$$

$$= \Phi\left(\frac{0.63-(0.035)(5)}{(0.1)\sqrt{5}}\right) - e^{\left(2(0.035)(0.63)/(0.1)^2\right)}\Phi\left(\frac{-0.63-(0.035)(5)}{(0.1)\sqrt{5}}\right) = 0.96$$

This probability calculation is an important output of our option valuation approach since it helps a retailer assess the entry threat of the competing retailer. In the next section, we provide empirical evidence for our theoretical result regarding the impact of competition on big retailers' investment decisions.

5. Empirical evidence

Our theoretical result in section 3.1 indicates that the preferred strategy for big retailers is to invest relatively later and at higher market potential in markets with small retailers compared to markets without small retailers. We empirically examine this theoretical result by looking at the actual investment behavior of a big discount retailer (Wal-Mart in our case study) in two types of retail markets. The first type is markets in which a small discount retailer (Dollar General in our case study) has a store opened. The second type is markets in which Dollar General does not have a store opened. In this paper, we call the first retail markets competitive and the second markets noncompetitive.

We chose Wal-Mart and Dollar General for our case study since these two retail firms have much in common in geography and customer base [31-33]. The average Wal-Mart customer's annual income is $35,000, which is below the national average [34, 35]. Dollar General also serves low income communities whose average household annual income is below $35,000 [31]. Approximately 80 percent of Dollar General customers earn below $25,000 a year [32]. David A. Perdue, Dollar General former chief executive describes Dollar General competitive relationship to Wal-Mart "We compete on price and convenience while Wal-Mart competes on price and assortment [33]."

5.1. Data collection

We use Wal-Mart and Dollar General stores in the state of Georgia to conduct our empirical study. As of December 2006, there are 118 Wal-Mart stores opened in Georgia. The number of Wal-Mart stores in the state of Georgia is large enough to provide a meaningful sample for our case study. In addition, Wal-Mart opened its first store in Georgia in 1981 and then gradually expanded its operation across the state. Currently, Wal-Mart operates in many geographical locations across the state and therefore, our sample represents a variety of markets in this state. Data for the Wal-Mart store location (its physical address as well as its latitude/longitude information) and the year store opened were acquired from the Wal-Mart corporation and the Wal-Mart store locator website [36]. In addition, the same data for Dollar General stores were acquired from the Dollar General corporation and the Dollar General store locator website [37]. As of December 2006, there are 347 Dollar General stores serving several markets across the state of Georgia, which provide a meaningful sample for our case study.

We use a standard approach based on the concept of market mile [38, 39] to define the retail market around a Wal-Mart store. Market mile determines the maximum distance that a customer must travel to reach a Wal-Mart store. We define a store's market be a notional circle centered at the store's location with radius of market mile (here 15 miles). We define a Wal-Mart store's market competitive when there is at least one Dollar General store located in this notional circle and it is opened before the Wal-Mart store. On the other hand, we define a Wal-Mart store's market noncompetitive when either there is no Dollar General located in this notional circle or there is a Dollar General located in this notional circle but it is opened later in time than the Wal-Mart store. Figure 7 shows competitive and noncompetitive Wal-Mart markets on the map of the state of Georgia. Out of 118 Wal-Mart stores in Georgia there are 60 stores located in competitive markets and 58 stores located in noncompetitive markets. Competitive and noncompetitive markets are shown by red and blue circles on the map, respectively. In our empirical study presented in the next section, we use Wal-Mart opening year data and market population at the opening time where population data were acquired from The ESRI Community Sourcebook America [40].

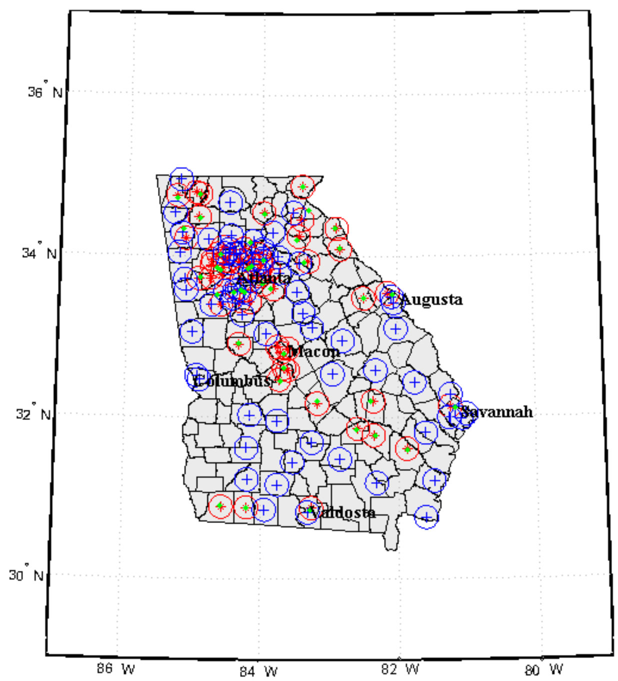

Figure 7. Competitive and noncompetitive Wal-Mart retail markets in Georgia

5.2. Data analysis

We test two hypotheses in this paper to explore the competition effect on Wal-Mart entry decisions in Georgia. The first hypothesis is that Wal-Mart opens stores relatively later in competitive markets than noncompetitive markets. The second hypothesis is that Wal-Mart opens stores in relatively higher market population area in competitive markets than noncompetitive markets. These hypotheses are based on our theoretical result regarding the competition impact on big retailers' entry decisions in competitive markets that was discussed in section 3.1. We use market population as a proxy for the value of retail market potential (variable (X(t))) in our hypothesis testing.

Since our sample data do not support the assumption of the population normality we cannot use standard t-tests for comparing Wal-Mart's behavior in competitive vs. noncompetitive markets. Instead, we use Wilcoxon rank-sum test [41, 42] for comparing our two variables of interest, opening years of Wal-Mart stores and opening-year market populations of Wal-Mart stores in competitive versus noncompetitive markets. The Wilcoxon rank-sum test is a nonparametric test to examine whether two sample data come from the same distribution. The null hypothesis in this test is that two sets of observations are independently sampled from an identical distribution. The alternative hypothesis in this one-sided test is that the first set of observations has the higher median value than the second set of observations. The results of our hypothesis testing are summarized, as follows.

- The null hypothesis that '*the years Wal-Mart opened stores in competitive markets and the years Wal-Mart opened stores in noncompetitive markets have the same medina*' is strongly rejected at 5% significance level with the very low p-value of (2.055e-12). The greater median of opening years of Wal-Mart stores in competitive markets compared to opening years of Wal-Mart stores in noncompetitive markets can also be observed in Figure 8 that shows the box-plot of opening years of Wal-Mart stores in these two market types.

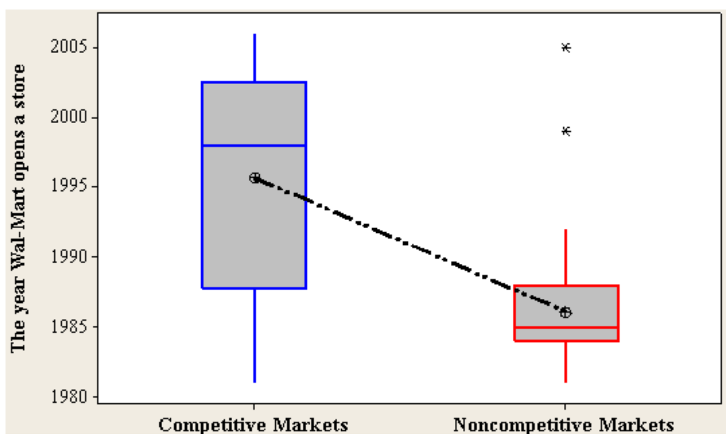

Figure 8. The box-plot of the opening years of Wal-Mart stores in competitive versus noncompetitive markets

- The null hypothesis that '*the market populations when Wal-Mart opened stores in competitive markets and the market populations when Wal-Mart opened stores in noncompetitive markets have the same median*' is rejected at 5% significance level with the p-value of (0.01997). The greater median of the opening-year market populations of Wal-Mart stores in competitive markets compared to the opening-year market populations of Wal-Mart stores in noncompetitive markets can also be observed in Figure 9 that shows the box-plot of the opening-year market populations of Wal-Mart stores in these two market types.

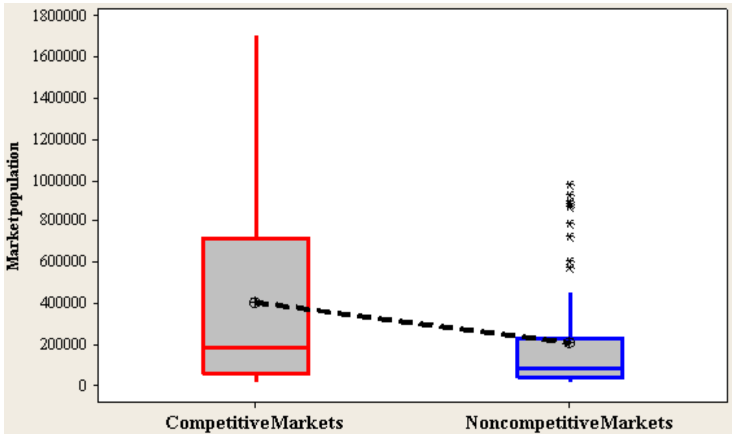

Figure 9. The box-plot of the opening-year market populations of Wal-Mart stores in competitive versus noncompetitive markets

Therefore, Wal-Mart's investment behavior is different in markets with Dollar General compared to markets without Dollar General. In practice, Wal-Mart follows the preferred strategy for big retailers where this strategy is to open stores relatively later and at higher market population in markets in which small retailers have already opened stores compared to markets in which small retailers have not yet opened stores.

6. Conclusions

In this paper, we describe a real option valuation approach that provides a means to determine the investment exercise time and market potential threshold. This options framework provides a strategic perspective in which retail stores are evaluated as flexible assets of a retail firm. Our option valuation approach considers retail management flexibility, the dynamic uncertainty of retail markets, and the competition effect within a comprehensive computational framework. Also, our theoretical model indicates that big retailers ought to invest relatively later and at higher market potential in markets with small retailers compared to markets without small retailers.

We conducted an empirical study in the state of Georgia, which shows how in practice, Wal-Mart follows this preferred strategy of big retailers and opens stores relatively later and at higher population in markets with Dollar General compared to markets without Dollar General. However, to support this strategy for general practice, our empirical study needs to be extended to other retail markets located in other states and similar case studies need to be conducted for other discount retailers to examine their investment behaviors under the competition effect. For instance, one may study the competition between retailers with similar cost structures such as Target and Wal-Mart or Dollar General and Family Dollar.

Our option valuation approach is based on many assumptions that are used for simplifying the investment evaluation procedure. Relaxing any of these assumptions may provide opportunities for further research. In this paper, we use a simple demand function, which is exogenous to our investment analysis model to characterize the inverse relationship between price and quantity in retail markets. This model is based on an important assumption that retailers provide a product at the same price to retail markets. We also assume that customers only consider the price of a product at their decision of choosing a retail store for shopping. Future research is needed to develop more appropriate demand models that describe the relationship between price, quantity, and service for the retail industry. Perhaps, works such as [43, 44] could provide a good start to understand the economics of retail firms.

Last but not least, this paper is the first step to understand the dynamics of investments in retail markets. The conceptual framework and the evaluation procedure are general and can be adjusted and applied to investments opportunities in other services. The choice of the retail industry in this paper has been driven by the availability of data for a large number of decisions. Importantly, similar studies may be conducted in other service industries and compared with our analysis in the retail industry.

Appendix A: The procedure to compute retailers' expected NPVs in the duopoly market

We describe a procedure to compute the expected NPVs of two retailers who simultaneously open stores in a dynamic, duopoly market whose potential $\{X(t); t\in[0,\infty]\}$ changes according to the GBM process of Equation (7). The initial value of this process is the value of (X) at the current node, which is denoted by (X_c) in this section. We use the trinomial lattice model to approximate the dynamic uncertainty of this retail market in a discrete fashion. In this section, we build a trinomial lattice with 10,000 time steps (N' = 10,000 months) to approximate the infinite time horizon of retailers' investments. This lattice is used as a basis for discounting future uncertain cash flows to compute retailers' expected NPVs.

Retailer 1's and retailer 2's free cash flows are equal to retailer 1's and retailer 2's optimal profits – that will be calculated according to Equation (6) – and then, multiplied by the time increment (Δt). This calculation must be done for any intermediate node (nodes in time steps 1 to (N'-1) on the lattice) corresponding to the value of (X) at that node. For the nodes in the last time step (time step (N')) retailer 1's and retailer 2's free cash flows are calculated based on the assumption that the value of (X) remains constant for ever after the last time step. Hence, retailer 1's and retailer 2's terminating value (denoted by (TV1D) and (FV2D), respectively) are computed according to the following integral in Equation (A1).

$$TV_i^D = \int_0^\infty \Pi_i^D(\tau) e^{-\rho\tau} d\tau \qquad i = 1, 2 \qquad \text{(A1)}$$

We substitute retailers' optimal profits from Equation (6) in this integral and complete the calculation. Retailer 1's and retailer 2's terminating values are summarized in Equation (A2), as follows.

$$
\begin{cases}
\text{when } (\mathrm{VC}_2 - 2\mathrm{VC}_1 + X(t)) \geq 0 \;\&\; (\mathrm{VC}_1 - 2\mathrm{VC}_2 + X(t)) \geq 0 \\
\begin{cases}
TV_1^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_2 - 2VC_1)(X(t))}{9\gamma} + \dfrac{(VC_2 - 2VC_1)^2}{9\gamma} - FC_1\right) \\
TV_2^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X(t))^2}{9\gamma} + \dfrac{2(VC_1 - 2VC_2)(X(t))}{9\gamma} + \dfrac{(VC_1 - 2VC_2)^2}{9\gamma} - FC_2\right)
\end{cases} \\
\text{when } (\mathrm{VC}_2 - 2\mathrm{VC}_1 + X(t)) \geq 0 \;\&\; (\mathrm{VC}_1 - 2\mathrm{VC}_2 + X(t)) < 0 \\
\begin{cases}
TV_1^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X(t))^2}{4\gamma} - \dfrac{2(VC_1)(X(t))}{2\gamma} + \dfrac{(VC_1)^2}{4\gamma} - FC_1\right) \\
TV_2^D(t) = \dfrac{1}{\rho}(-FC_2)
\end{cases} \\
\text{when } (\mathrm{VC}_2 - 2\mathrm{VC}_1 + X(t)) < 0 \;\&\; (\mathrm{VC}_1 - 2\mathrm{VC}_2 + X(t)) \geq 0 \\
\begin{cases}
TV_1^D(t) = \dfrac{1}{\rho}(-FC_1) \\
TV_2^D(t) = \dfrac{1}{\rho}\left(\dfrac{(X(t))^2}{4\gamma} - \dfrac{2(VC_2)(X(t))}{2\gamma} + \dfrac{(VC_2)^2}{4\gamma} - FC_2\right)
\end{cases} \\
\text{when } (\mathrm{VC}_2 - 2\mathrm{VC}_1 + X(t)) < 0 \;\&\; (\mathrm{VC}_1 - 2\mathrm{VC}_2 + X(t)) < 0 \\
\begin{cases}
TV_1^D(t) = \dfrac{1}{\rho}(-FC_1) \\
TV_2^D(t) = \dfrac{1}{\rho}(-FC_2)
\end{cases}
\end{cases}
\tag{A2}
$$

Retailers' free cash flows and terminating values are discounted back to the initial time step in order to calculate retailers' expected NPV at the current node. This procedure is summarized in the following algorithm.

1. Take the input values for the model parameters: ((VC_1), (FC_1), (IC_1), (VC_2), (FC_2), (IC_2), (ρ), (X_c), (γ), (α), (σ), ($\Delta t = 0.125$ Year), and ($N' = 10{,}000$))
2. Compute (p_1, p_2, and p_3) and (u and d) based on Equations (12) and (13), respectively
3. Construct a trinomial lattice with (N') time steps of the length (Δt) to represent the stochastic variation of (X) considering the values of (p_1, p_2, and p_3) and (u and d).
4. Set the current time step as the last time step: time step (N')
5. For every node in the current time step:
    5.1. Calculate retailer 1's and retailer 2's free cash flows
    5.2. If the current time step is the last time step (time step (N')) use retailer 1's and retailer 2's cash flows as retailer 1's and retailer 2's total cash flows
    5.3. If the current time step is not the last time step compute retailer 1's and retailer 2's total cash flows by adding retailer 1's and retailer 2's cash flows at the current node to the expected retailer 1's and retailer 2's total cash flows in the next time step that are discounted back for one time step
6. Move back in the lattice to the time step before the current time step. Set the current time step to this new time step and repeat step (5) until the process reaches the first time step
7. Return retailer 1's and retailer 2's total cash flows in the only node at the first time step as retailer 1's and retailer 2's expected NPVs

Appendix B: The procedure to compute a retailer's expected NPV in the monopoly market

We describe a procedure to compute the expected NPV of a retailer who opens a store in a dynamic, monopoly market whose potential $\{X(t);\, t \in [0,\infty]\}$ changes according to the

GBM process of Equation (7). The initial value of this process is the value of (X) at the current node, which is denoted by (X_c) in this section. The procedure will be very similar to the evaluation process described in Appendix A. The only difference is that we should use the retailer's optimal profit in the monopoly market according to Equation (5) in the calculation of the retailer's free cash flow. Further, the retailer's terminating value (denoted by (TVM)) is computed according to the following integral in Equation (B1).

$$TV^M = \int_0^\infty \Pi^M(\tau)e^{-\rho\tau}d\tau$$

$$= \begin{cases} \frac{1}{\rho}\left(\frac{(X(t))^2}{4\gamma} - \frac{(X(t))(VC)}{2\gamma} + \frac{(VC)^2}{4\gamma} - FC\right) & \text{When } (X(t) \geq VC) \\ \frac{1}{\rho}(-FC) & \text{Otherwise} \end{cases} \quad \text{(B1)}$$

Appendix C: The procedure to compute retailers' expected NPVs when retailers decide to defer their investment opportunities

If the current node is in any intermediate time steps (any time step between 1 and (N-1)) retailer 1's and retailer 2's expected NPVs are equal to retailer 1's and retailer 2's expected investment values at the stable state of the market (the Nash equilibrium of the market) in the next time step that are discounted back for one time increment. If the current node is in the last time step retailer 1's and retailer 2's expected NPVs are equal to zero.

Reference:

[1] C. A. Ingene and R. F. Lusch, "Estimation of a department store production function," *International Journal of Physical Distribution & Logistics Management,* vol. 29, pp. 453 - 465 Sept. 1999.

[2] M. Vargas, "Retail Industry Profile," in *Your Guide to Retail Industry*, Retrieved on April 2008.

[3] "Bureau of Economic Analysis, U.S. Department of Commerce " 2007.

[4] M. Vargas, "Top 100 Retailers Rankings," in *Your Guide to Retail Industry* Retrieved on April 16, 2008.

[5] T. A. Arentze, "A spatial decision support system for the planning of retail and service facilities." vol. PhD Eindhoven, NL: Eindhoven University of Technology., 1999.

[6] I. Clarke, M. Horita, and W. MacKaness, "The spatial knowledge of retail decision makers: capturing and interpreting group insight using a composite cognitive map," *International Review of Retail, Distribution & Consumer Research,* vol. 10, pp. 265-285, July 2000.

[7] T. Hernandez and D. Bennison, "The art and science of retail location decisions," in *The 10th International Conference on Research in the Distributive Trades*, University of Stirling, 1999, pp. 26–28.

[8] D. Rogers, "Shop location analysis," in *The Changing Face of British Retailing*, E. McFayden, Ed. London: Newman Books, 1987.

[9] S. Bowlby, M. J. Breheny, and D. Foot, "Store location: problems and methods 1: is locating a viable store becoming more difficult?," *Retail & Distribution Management,* vol. 12, pp. 31–33, 1984.

[10] M. J. Breheny, "Practical methods of retail location analysis," in *Store Choice, Store Location and Market Analysis*, N. Wrigley, Ed. London: Wiley, 1988, pp. 39–86.

[11] J. R. Beaumont, "Spatial decision support: some comments with regard to their use in market analysis," *Environment and planning A,* vol. 23, pp. 311-317, 1991.

[12] P. Longley and G. Clarke, "Applied geographical information systems developments and prospects," in *GIS for business and service planning*, P. L. Longley and G. Clarke, Eds. Cambridge: Geoinformation International, 1995, pp. 3-9.

[13] C. S. Craig, A. Ghosh, and S. McLafferty, "Models of the retail location process: a review," *Journal of Retailing,* vol. 60, pp. 5-36, 1984.

[14] A. Dixit and R. Pindyck, *Investment under uncertainty*. Princeton, NJ: Princeton University Press, 1994

[15] B. Ashuri, "A Real Options Approach to Modeling Investments in Competitive, Dynamic Retail Markets," in *The H. Milton Stewart School of Industrial and Systems Engineering*. vol. Ph.D. Atlanta, GA: Georgia Institute of Technology, 2008.

[16] B. Ashuri, W. B. Rouse, and D. A. Bodner, "A real options approach to investment decisions in competitive, dynamic retail markets," in *2008 Hawaii International Conference on System Sciences (HICSS-41)*, Waikoloa, Big Island, HI, 2008.

[17] H. Smit and L. Trigeorgis, *Strategic Investment: Real Options and Games*: Princeton University Press, 2004.

[18] L. Trigeorgis, *Real Options: managerial flexibility and strategy in resource allocation*. Cambridge, Massachusetts: The MIT Press, 1996.

[19] "Keeping all options open," in *The Economist Newspaper limited* London, Aug 12th 1999.

[20] "Optional investment and market performance," in *The Economist Newspaper limited* London, 1994.

[21] N. A. Nichols, "Scientific Management at Merck: An Interview with CFO Judy Lewent," *Harvard Business Review,* vol. 72, pp. 88-99, Jan/Feb94 1994.

[22] H. R. Varian, *Microeconomic analysis*, 3 ed. London: WW Norton and Company, Inc., 1992.

[23] H. R. Varian, "Intermediate Microeconomics A Modern Approach," New York: Norton, 2003.

[24] S. E. Shreve, *Stochastic Calculus for Finance II Continuous-Time Models*. New York: Springer-Verlag, 2004.

[25] M. Brennan and E. Schwartz, "Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis," *Journal of Financial and Quantitative Analysis,* vol. 13, pp. 461-474, 1978.

[26] J. C. Cox, S. A. Ross, and M. Rubinstein, "Option pricing: a simplified approach," *Journal of Financial Economics,* vol. 7, pp. 229-263, 1979.

[27] F. A. Longstaff and E. S. Schwartz, "Valuing American Options by Simulation: A Simple Least-Squares Approach," *Review of Financial Studies,* vol. 14, pp. 113-147, 2001.
[28] B. Kamrad and P. Ritchken, "Multinominal Approximating Models for Options with k State Variables," *Management Science,* vol. 37, pp. 1640-1652, 1991.
[29] D. Luenberger, *Investment science*. New York: Oxford University Press, 1998.
[30] S. Ross, *Stochastic Processes*, 2 ed.: John Wiley & Sons Inc, 1996.
[31] E. SHAPIRO, " Market Place; At Dollar General, What Recession?," in *New York Times* New York, January 6, 1992.
[32] J. Stilgoe, "Wal-Mart giant can be tamed," in *The Boston Globe*, November 23, 2003.
[33] R. Berner and B. Grow, "Out-Discounting The Discounter Why dollar stores that sell at cut-rate prices are giving Wal-Mart pause," in *BusinessWeek*, MAY 10, 2004.
[34] L. P. Weston, "The Basics: National Bank of Wal-Mart?," in *MSN Money*, Retrieved on April 16, 2008.
[35] S. Mallaby, "Progressive Wal-Mart. Really.," in *The Washington Post*, November 28, 2005.
[36] I. Wal-Mart Stores, "Wal-Mart store locator," Retrieved on April 16, 2008.
[37] D. General, "Dollar General store locator," Retrieved on April 16, 2008.
[38] P. Davis, "Spatial Competition in Retail Markets: Movie Theaters," *RAND JOURNAL OF ECONOMICS,* vol. 37, pp. 964-982, 2006.
[39] P. Ellickson, S. Houghton, and C. Timmins, "Estimating Network Economies in Retail Chains: A Revealed Preference Approach," Duke University, 2008.
[40] ESRI, "2005 Community Sourcebook·America CD-ROM with ArcReader," 2005.
[41] H. B. Mann and D. R. Whitney, "On a test of whether one of two random variables is stochastically larger than the other," *Annals of Mathematical Statistics,* vol. 18, pp. 50-60, 1947.
[42] F. Wilcoxon, "Individual comparisons by ranking methods," *Biometrics Bulletin,* vol. 1, pp. 80-83, 1945.
[43] R. Betancourt and D. Gautschi, "The Economics of Retail Firms," *Managerial and Decision Economics,* vol. 9, pp. 133-144, 1988.
[44] R. R. Betancourt, M. CortiÁas, M. Elorz, and J. M. Mugica, "The Role of the Consumer in Retailing, Distribution Services and the Determinants of Customer Satisfaction," College Park, MD, USA: Department of Economics, University of Maryland, 2004.