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Leibniz Integral Rule

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The Leibniz integral rule gives a formula for differentiation of a definite integral whose limits are functions of the differential variable,

$$\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)\,dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z}\,dx + f(b(z),z)\frac{\partial b}{\partial z} - f(a(z),z)\frac{\partial a}{\partial z}.$$

It is sometimes known as differentiation under the integral sign.

This rule can be used to evaluate certain unusual definite integrals such as

$$\phi(\alpha) = \int_0^\pi \ln(1 - 2\alpha\cos x + \alpha^2)\,dx = 2\pi\ln|\alpha|$$

for $|\alpha| > 1$ (Woods 1926).

Feynman (1997, pp. 69-72) recalled seeing the method in Woods (1926) and remarked "So because I was self-taught using that book, I had peculiar methods for doing integrals," and "I used that one damn tool again and again."

SEE ALSO: Derivative, Integral, Integration Under the Integral Sign.
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