Recap

- Principal-Agent problems with incomplete information
  - Information about actions (moral hazard)
  - *Information about type (adverse selection)*
- Solve backwards induction represented by optimization model
- Possible Constraints:
  - Participation constraint
  - Incentive compatibility

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Lagrangian Relaxation

- $\max_x f(x)$
- s.t. $g_i(x)$ · 0 8 $i=1\ldots k$

- → $f(x) - \Sigma_{i=1}^{k} \lambda g_i(x)$

- $\max_x f(x)$
- s.t. $h_i(x)$ ¸ 0 8 $i=1\ldots k$
- (equiv to $-h_i(x)$ · 0

- → $f(x) + \Sigma_{i=1}^{k} \lambda h_i(x)$

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Adverse Selection

- Parties in contracts may not have all of the information about each other
  - Ability, interest, habits, health
  - Equivalent to not knowing “type”
- Examples?
- Information will only be revealed if it is in agent’s best interest

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Examples:

- Hiring carpenter for home renovations
- Driver knows habits better than insurance company
- Firm knows cost of project better than government
- Firm negotiating license agreement may know more than patent holder about market profitability
- Regulated firm knows more than government about market

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Sequence of Events

- Nature determines the *type* of employee
  - Good (G) or Bad (B)
- Firm offers a contract for work
  - Possibly a collection of (wage, effort) pairs, or (w, e)
  - (May need to make a decision based on her belief about the employee's type)
- Employee chooses (wage, effort) pair that maximizes his utility

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Assumptions

- Principal's Profit
  - Pi(e) = $\sum_{i=1}^{n} p_i(e)\, x_i$,
  - $\Pi'(e)>0$ and $\Pi''(e)<0$
- Agent's Utility
  - Reservation utility $\underline{U}$
  - Good agent: $U^G(w,e) = u(w) - v(e)$
  - Bad agent: $U^B(w,e) = u(w) - kv(e)$
  - Assume $v''(e) > 0$ and $k > 1$

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Complete Information

- If principal contracts with G-type:
  - $\max_{e,w} \Pi(e) - w$
  - s.t. $u(w) - v(e) \geq \underline{U}$
- How can we solve this?

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Constrained Optimization

- $L(w,e) = \Pi(e) - w + \mu(u(w) - v(e) - \underline{U})$
- $\partial L/\partial w = -1 + \mu u'(w) = 0$
- $\partial L/\partial e = \partial \Pi'/\partial e - \mu v'(e) = 0$
- Result:
  - $u(w^{G*}) - v(e^{G*}) = \underline{U}$

$$p'(e^{G*}) = \frac{v'(e^{G*})}{u'(e^{G*})}$$

- If agent were “bad” type:
  - $u(w^{B*}) - kv(e^{B*}) = \underline{U}$

$$p'(e^{B*}) = \frac{kv'(e^{B*})}{u'(e^{B*})}$$

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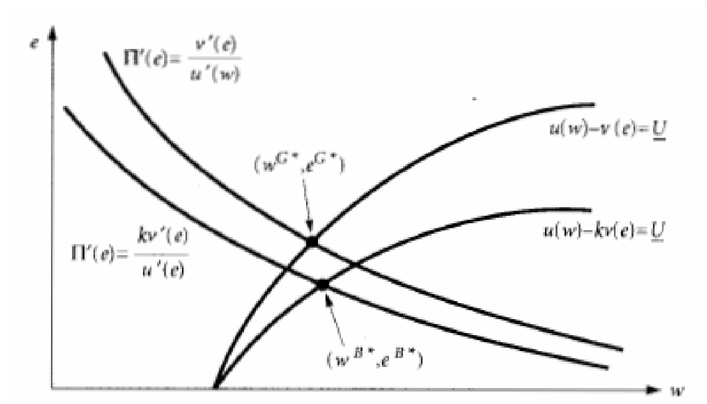
AS-PA with Complete Information

- $e^{G*} > e^{B*}$
- w^{G*} vs w^{B*}?



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AS-PA with Incomplete Information

- Suppose the principal offered the complete information contract and allowed agent to select
  - Type B will choose his contract
  - Type G will also choose the B contract!
    - $U^G(w^{B*},e^{B*}) = u(w^{B*}) - v(e^{B*})$
      $> u(w^{B*}) - kv(e^{B*}) = \underline{U}$
- Conclusion?

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AS-PA with Incomplete Information

- Principal thinks probability of agent being type G is q ($0 < q < 1$)
  - Could design one contract for both agents
  - Could design "menu" of contracts
    - $\{(e^g, w^g), (e^b, w^b)\}$
- Which is better?
- Examples?

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AS-PA with Incomplete Information

- Participation Constraint?
- Incentive compatibility constraint?

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AS-PA with Incomplete Information

$$\max_{[(e^G, w^G),(e^B, w^B)]} q[\Pi(e^G) - w^G] + (1-q)[\Pi(e^B) - w^B]$$

$s.t.$

1) $u(w^G) - v(e^G) \geq U$
2) $u(w^B) - kv(e^B) \geq U$
3) $u(w^G) - v(e^G) \geq u(w^B) - v(e^B)$
4) $u(w^B) - kv(e^B) \geq u(w^G) - kv(e^G)$

AS-PA with Incomplete Information

- What can we say about these constraints? (hint, the first one)

- From 3 and 4, we also have that greater effort is demanded of the most efficient agent ($e^G \geq e^B$)
  - $v(e^G) - v(e^B) \leq u(w^G) - u(w^B) \leq k[v(e^G) - v(e^B)]$, (5)
  - $k > 1$, so $v(e^G) \geq v(e^B)$

Solving the AS-PA Problem with Incomplete Information

$$\max_{[(e^G,w^G),(e^B,w^B)]} q[\Pi(e^G)-w^G]+(1-q)[\Pi(e^B)-w^B]$$

s.t. 2) $u(w^B) - kv(e^B)$ ¸ $\underline{U}$ — λ

3) $u(w^G) - v(e^G)$ ¸ $u(w^B) - v(e^B)$ — μ

4) $u(w^B) - kv(e^B)$ ¸ $u(w^G) - kv(e^G)$ — δ

$L(e^G,w^G,e^B,w^B,\lambda,\mu,\delta) =$

$q[\Pi(e^G) - w^G]+(1-q)[\Pi(e^B) - w^B)]$

$+ \lambda[u(wB) - kv(eB) - \underline{U}]$

$+ \mu[u(wG) - v(eG) - u(wB) + kv(eB)]$

$+ \delta[u(wB) - kv(eB) - u(wG) + kv(eG)]$

s.t. λ, μ, δ ¸ 0

First-Order Conditions

- $-q+\mu u'(w^G) - \delta u'(w^G) = 0$
  - → $\mu - \delta = q/(\mu'(w^G)$ (6)
- $-(1-q) + \lambda u'(w^B) - \mu u'(w^B) + \delta u'(w^B) = 0$
  - → $\lambda - \mu + \delta = (1-q)/u'(w^B)$ (7)
- $q\Pi'(e^G) - \mu v'(e^G) + \delta kv'(e^G) = 0$
  - → $\mu - \delta k = q\Pi'(e^G)/v'(e^G)$ (8)
- $(1-q)\Pi'(e\ )-\lambda kv'(e^B)+\mu v'(e^B)-\delta kv'(e^B) = 0$
  - → $\lambda k - \mu + \delta k = (1-q)\Pi'(e^B)/v'(e^B)$ (9)

First-Order Conditions

- (6) and (7) imply
  - $\lambda = q/u'(w^G) + (1-q)/u'(w^B) > 0$ (10)
- (8) and (9) imply
  - $\lambda k = q\Pi'(e^G)/v'(e^G) + (1-q)\Pi'(e^B)/v'(e^B)$ (11)
- So we know that constraint 2 is binding
- If $\mu=0$, then equation (6) → $\delta<0$, so $\mu > 0$

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First-Order Conditions

- Could we ask for the same effort ($e^G=e^B$)?
  - If so, then $w^G = w^B$ by equation (5)
  - (10) and (11) imply that
    - $\lambda = 1/u'(w) = \Pi'(e)/kv'(e)$
  - (6) and (8) imply that:
    - $\mu = q/u'(w) + \delta = q\lambda + \delta$
    - $\mu = q\Pi'(e)/v'(e) + k\delta = qk\lambda + k\delta = k(q\lambda + \delta)$
- which is impossible since μ ? $k\mu$ with $k > 1$ and $\mu > 0$

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First-Order Conditions

- Since $e^G > e^B$, then (3) and (4) cannot hold simultaneously
- We know that $\mu > 0$, so
  - (3) must be binding
  - (4) is not binding → $\delta = 0$
- We can rewrite (3) to see:
  - $u(w^G) – v(e^G)$
    $= u(w^B) – v(e^B)$
    $= u(w^B) – kv(e^B) + (k-1)v(e^B)$
    $= \underline{U} + (k-1)v(e^B)$

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First-Order Conditions

- Since $\delta = 0$, (6) and (8) →
  - $1/u'(w^G) = \Pi'(e^G)/v'(e^G)$
- Since (7) is equivalent to
  - $-\mu=(1-q)/v'(w^B) - \lambda$
- Then (9) can be written (using (10)) as
  - $\Pi'(e^B) = [q(k-1)/(1-q)]*[v'(e^B)/u'(w^G)] + kv'(e^B)/u'(w^B)$

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Final Solution

- The menu of contracts $\{(e^G, w^G), e^B, w^B)\}$ that solves the AS-PA problem is defined by this set of equations:

$$u(w^G) - v(e^G) = \underline{U} + (k-1)v(e^B)$$

$$u(w^B) - kv(e^B) = \underline{U}$$

$$\Pi'(e^G) = \frac{v'(e^G)}{u'(w^G)}$$

$$\Pi'(e^B) = \frac{kv'(e^B)}{u'(w^B)} + \frac{q(k-1)}{(1-q)} \frac{v'(e^B)}{u'(w^{G)}}$$

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Characteristics

- The participation constraint only binds for the agent with the highest costs (B)
  - The other agent (G) receives an "informational rent" of $(k-1)v(e^B)$
- The incentive condition binds for the high-efficiency agents (G) but the one for low-efficiency agents (B) does not does not

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Characteristics

- The efficiency condition binds for the good agent (i.e., he produces the same output he would if he were the only agent present)
  - Called “non distortion at the top”
  - The high-cost agent (B) produces less output than he would if he were the only agent
- Distortion is introduced into the efficiency condition for the bad agent (to make this contract less attractive to the good agent)

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Comparison to Complete Information

- Agent G
  - Efficiency condition is the same
  - Participation binds under symmetric but not under asymmetric
  - Result: $e^G \cdot e^{G*}$ and $w^G > w^{G*}$
- Agent B
  - $e^B < e^{B*}$ and $w^B < w^{B*}$
- Now we have $e^G > e^B$ and $w^G > w^B$

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Other Comments

- Adverse selection problems turn up independently of the agent's risk-aversion
- We designed a contract for both agents
  - Is this necessarily the most profitable?
  - Could design a contract for only one to choose, which takes place with probability q
  - Principal's profit = $q[\Pi^{G*}-w^{G*}]$

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Supply Chains and Information Asymmetry (Recent Papers)

- Cachon, G. P. and M. A. Lariviere (2001). "Contracting to assure supply: How to share demand forecasts in a supply chain." Management Science **47**(5): 629-646.
- Corbett, C. J. (2001). "Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock." Operations Research **49**(4): 487-500.
- Corbett, C. J. and X. de Groote (2000). "A supplier's optimal quantity discount policy under asymmetric information." Management Science **46**(3): 444-450.
- Gibbons, R. (2005). "Incentives Between Firms (and Within)." Management Science **51**(1).
- Plambeck, E. and S. Zenios (2000). "Performance-Based Incentives in a Dynamic Principal-Agent Model." Manufacturing and Service Operations Management **2**: 240-263.

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Example 1

- Principal (Π(x,w) = x-w) contracts with Agent (U(w,e) = v w - e^2), whose effort determines results
- Prob of each state = 1/3
- Agent reservation utility is $\underline{U}$=114
- What are effort and wage in symmetric information?
- What happens under asymmetric information?
- What kind of problem is this?

			OUTCOMES	
		o_1	o_2	o_3
EFFORT	e=6	60,000	60,000	30,000
	e=4	30,000	60,000	30,000

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Example: Symmetric Information

- Given effort level e,

$$\max_{w} \ (E(x) - w)$$

$$\text{s.t. } \sqrt{w} - e^2 \geq \underline{U}$$

- For e = 6: w=22,500;
  - Π = 1/3 (60K + 60K + 30K) – 22,500 = 27,500
- For e = 4, w=16,900;
  - Π = 1/3 (30K + 60K + 30K) – 16,900 = 23,100

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Example: Asymmetric Information

- For e = 6,

$$\max_{w_6} \left(\frac{1}{3}(60k - w_6) + \frac{1}{3}(60k - w_6) + \frac{1}{3}(30k - w_3) \right)$$

$$\text{s.t. } \frac{1}{3}\sqrt{w_6} + \frac{1}{3}\sqrt{w_6} + \frac{1}{3}\sqrt{w_3} - 6^2 \geq 114$$

$$\frac{1}{3}\sqrt{w_6} + \frac{1}{3}\sqrt{w_6} + \frac{1}{3}\sqrt{w3} - 6^2 \geq \frac{1}{3}\sqrt{w_6} + \frac{1}{3}\sqrt{w_3} + \frac{1}{3}\sqrt{w_3} - 4^2$$

- How to solve?
- For e = 6, w_6 = 28,900 & w_3 = 12,100, Π = 26,700
- For e = 4, w=16,900, Π = 23,100
- Which does principal choose?
- What is loss due to asymmetric information?

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Example 2

- Expert translator is required, but principal doesn't know if translator is slow or fast
  - If fast, then 2 pg / hr; If slow, 1 pg / hr
  - 1 hr translating → disutility of $10 for both
- If asymmetric information, what kind of contract should the principal offer?
- What values should it have?

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Example 2: Symmetric Information

- For each type t, choose w_t and n_t
  - max $\Pi(n_t)$ - w_t
  - s.t. $w_t - n_t$ * (\$10/hr)*(rate$_t$) $\geq \underline{U}$
- For fast type, $w_f = 5n_f$; $\Pi'(n_f^*)=5$
- For slow type, $w_s = 10n_s$; $\Pi'(n_s^*)=10$,
- Assume with complete information, the principal wants $n_s = 50$, $n_f = 80$

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Example 2: Asymmetric Information

- What do we know?
  - Efficient agent:
    - Receives "rent", so $w_f > w_f^*$
    - When agent is risk-neutral, $n_f = n_f^*$ ($n_f \neq n_f^*$ in general)
  - Inefficient agent has distortion:
    - $n_s < n_s^*$
    - $w_s < w_s^*$
  - Both:
    - $n_f > n_s$, $w_f > w_s$

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Example 2: Asymmetric Information

$$\max_{w_f, n_f, w_s, n_s} \quad q[\,?(n_f) - w_f\,] + (1-q)[\,?(n_s) - w_s\,]$$

$$\text{s.t.} \quad w_f\text{-}5n_f \geq 0$$

$$w_s\text{-}10n_s \geq 0$$

$$w_f\text{-}5n_f \geq w_s\text{-}5n_s$$

$$w_s\text{-}10n_s \geq w_f\text{-}10n_f$$

- n_f: $qp'(n_f)=5m-10g$
- n_s: $(1-q)p'(n_s)=10l-5m+10g$
- w_f: $m-g=q$
- w_s: $l-m+g=1-q$
- → $l=1$, $m=q$, $g=0$, $P'(n_f)=5$, $P'(n_s) = 10-5q/(1-q)$
- → $n_f=nf^*$ and $n_s < n_s^*$, incentive constraint for fast will bind, and participation constraint for slow will bind

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