Recap

- Before break
  - Dynamic games with incomplete information
    - Signaling
    - Pooling and Separating equilibria
  - Principal-Agent problems intro
    - Principal has desired outcome but cannot necessarily control the agent's actions directly
- Today
  - PA: P does not know effort level

1

Moral Hazard PA Problem

- In the final stage of the game, the agent decides his action (effort)
  - $\max_a \{\sum_{j=1}^{m} P_{ij}U(w_j) - e(a_i)\}$
- In the next to last stage, the agent decides whether to accept the contract, and will if
  - $\sum_{j=1}^{m} P_{ij}U(w_j) - e(a_i)$ ¸ $\underline{U}$
- In the first stage, the principal designs the contract, anticipating the agent's behavior:

$$\max \sum_{j=1}^{m} P_{ij}(x_j - w_j)$$

$$\text{s.t.} \sum_{j=1}^{m} P_{ij}U(w_j) - e(a_i) \geq \underline{U}$$

$$\sum_{j=1}^{m} P_{ij}U(w_j) - e(a_i) \geq \sum_{j=1}^{m} P_{kj}U(w_j) - e(a_k) \quad \forall k \neq i$$

2

Two actions and two outcomes

- Suppose the agent has two possible actions, a and b, and there are two possible outcomes, x_1 and x_2.
- Suppose that action b is preferred by the principal

For action b:

$$\max \sum_{j=1}^{2} P_{bj}(x_j - w_j)$$

$$P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) \geq \underline{U} \; \ldots\ldots\ldots\ldots\ldots\ldots \; m$$

$$P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) \geq P_{a1}U(w_1) + P_{a2}U(w_2) - e(a) \; \ldots\ldots \; l$$

3

Two actions and two outcomes

Incentive compatibility constraint:

$$P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) \geq P_{a1}U(w_1) + P_{a2}U(w_2) - e(a)$$

When is the agent indifferent between a and b?

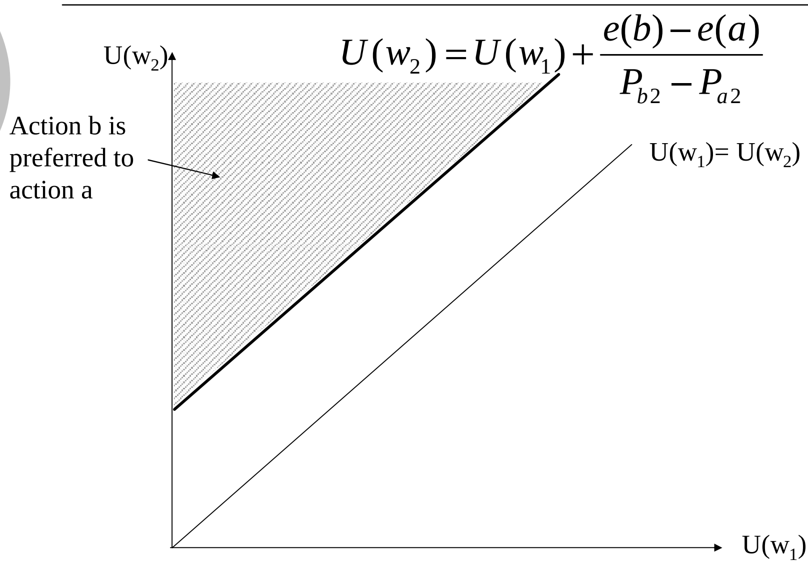
$$P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) = P_{a1}U(w_1) + P_{a2}U(w_2) - e(a)$$

$$U(w_2) = \frac{P_{a1} - P_{b1}}{P_{b2} - P_{a2}} U(w_1) + \frac{e(b) - e(a)}{P_{b2} - P_{a2}} = U(w_1) + \frac{e(b) - e(a)}{P_{b2} - P_{a2}}$$

since $P_{a1} + P_{a2} = P_{b1} + P_{b2} = 1$

4

Two actions and two outcomes



5

Two actions and two outcomes

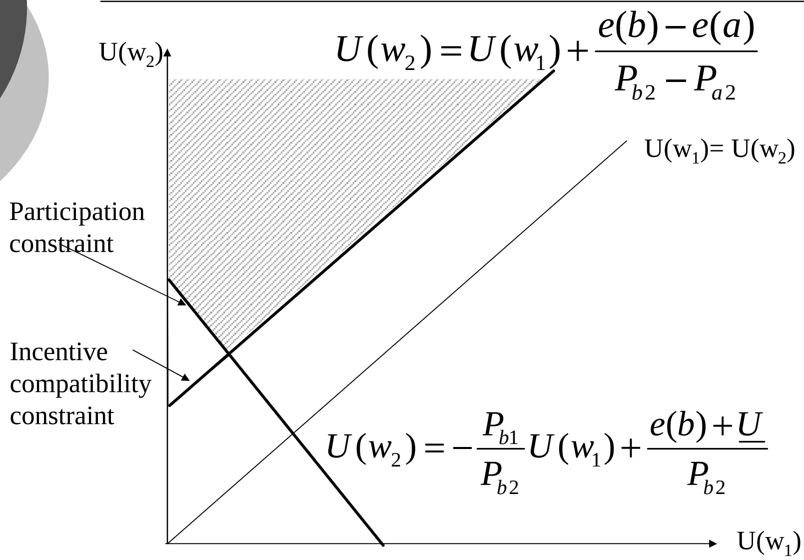
Participation constraints:

Action b: $P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) \geq \underline{U}$

$$\rightarrow\rightarrow\ U(w_2) \geq -\frac{P_{b1}}{P_{b2}}U(w_1) + \frac{e(b)+\underline{U}}{P_{b2}}$$

6

Two actions and two outcomes



7

Two actions and two outcomes

- Principal's objective function

For action b:

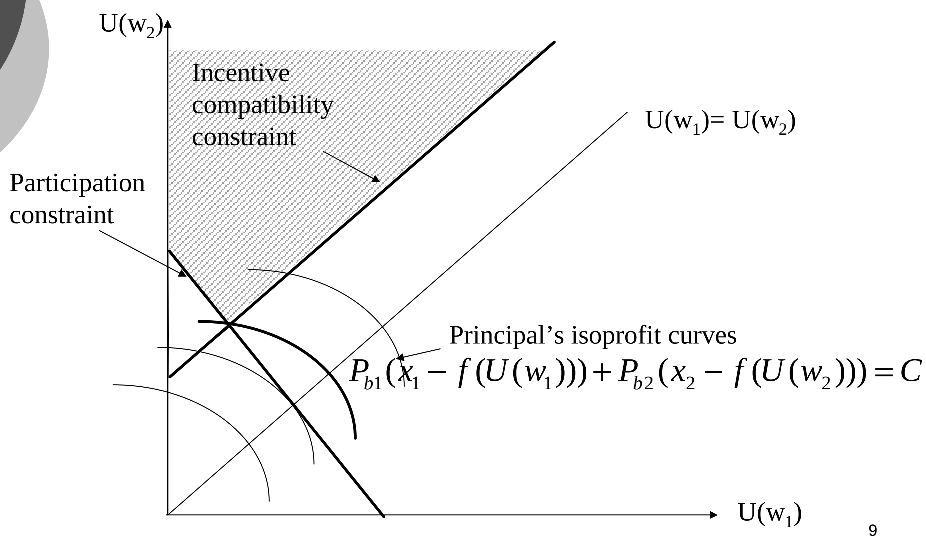
$$\max P_{b1}(x_1 - w_1) + P_{b2}(x_2 - w_2)$$

Let f be the inverse of U: $f(U(w_j)) = w_j$

$$\max P_{b1}(x_1 - f(U(w_1))) + P_{b2}(x_2 - f(U(w_2)))$$

8

Two actions and two outcomes



9

Two actions and two outcomes

$$\max P_{b1}(x_1 - f(U(w_1))) + P_{b2}(x_2 - f(U(w_2)))$$

Slope of the objective function:

$$MRS = -\frac{P_{b1} f'(U(w_1))}{P_{b2} f'(U(w_2))}$$

10

Two actions and two outcomes

If $U(w_1) = U(w_2)$ (45^0 line)

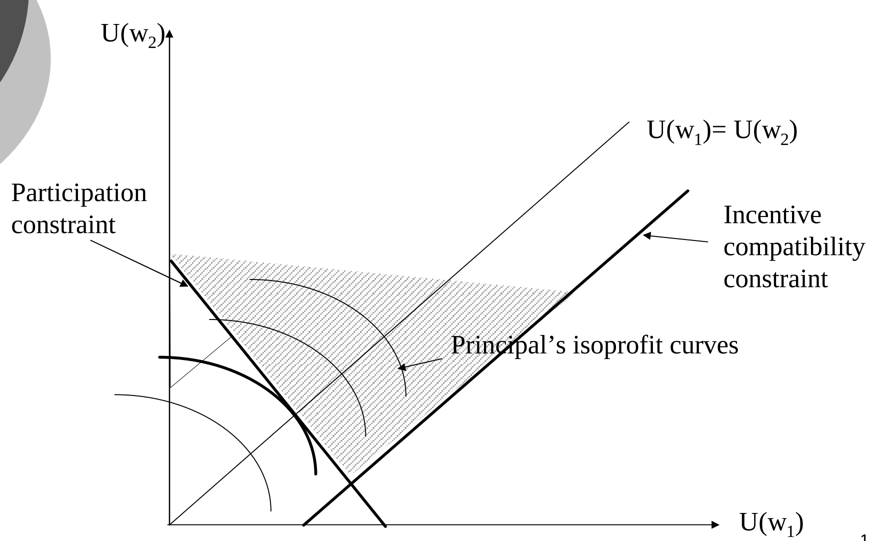
$$MRS = -\frac{P_{b1} f'(U(w_1))}{P_{b2} f'(U(w_2))} = -\frac{P_{b1}}{P_{b2}}$$

Same as the slope of the participation constraint!

The principal's isoprofit curve is tangent to the agent's participation constraint along the 45^0 line.

11

Two actions and two outcomes



12

Two actions and two outcomes

$$\frac{1}{U'(w_j)} = m + l\left(1 - \frac{P_{aj}}{P_{bj}}\right) \qquad \frac{P_{aj}}{P_{bj}}: \text{likelihood ratio}$$

If the incentive constraint is not binding:

$l = 0$, $U'(w_j) = 1/m$ and $w_j = w$.

Substituting into the incentive constraint

(the agent prefers action b over action a):

$$\sum_{j=1}^{n} P_{bj}U(w) - e(b) \geq \sum_{j=1}^{n} P_{aj}U(w) - e(a) \rightarrow e(a) \geq e(b)$$

13

Two actions and two outcomes

$$\frac{1}{U'(w_j)} = m + l\left(1 - \frac{P_{aj}}{P_{bj}}\right) \qquad \frac{P_{aj}}{P_{bj}}: \text{likelihood ratio}$$

If the incentive constraint is binding:

$l > 0$, and $U'(w_j)$ depends on the likelihood ratio (LR).

The optimal incentive scheme is a linear function of LR.

14

Sensitivity analysis: Changes in the agent's costs

- What is the impact of the agent's costs on the outcome?

$$\max L(w, l, m) = P_{b1}(x_1 - w_1) + P_{b2}(x_2 - w_2) + m(P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) - \underline{U}) + l(P_{b1}U(w_1) + P_{b2}U(w_2) - e(b) - P_{a1}U(w_1) - P_{a2}U(w_2) + e(a))$$

$$\frac{\partial L(w, l, m)}{\partial e(b)} = -(m + l) \qquad \frac{\partial L(w, l, m)}{\partial e(a)} = l$$

15

Sensitivity analysis: Changes in the agent's costs

- What is the impact of the agent's costs on the outcome?

$$\frac{\partial L(w, l, m)}{\partial e(b)} = -(m + l) \qquad \frac{\partial L(w, l, m)}{\partial e(a)} = l$$

- "Carrot": Decrease the cost of the desired action, b
- "Stick": Increase the cost of the undesired action, a
- Carrot vs. stick, which one is better?

16

Sensitivity analysis: Changes in probability distribution

- What is the impact on the principal's utility of a change in probability distribution dP_{aj}?

$$\frac{\partial L(w, l, m)}{\partial P_{aj}} = -l \sum_{j=1}^{2} U(w_j) dP_{aj}$$

- When the IC constraint is binding the interests of the principal and agent are opposed

17