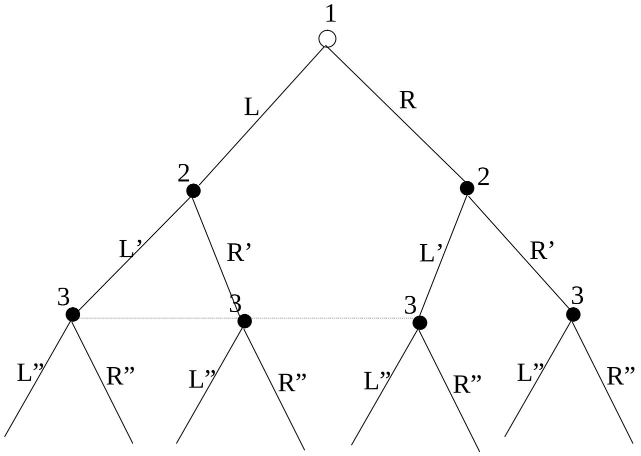
Recap:

- Last class (January 18, 2007)
  - Stackelberg (sequential versus simultaneous)
  - Multi-stage game
  - Backwards induction
- Today (January 23, 2007)
  - Extensive form games
  - Information sets
  - Subgame perfect equilibrium
  - Credible commitment
  - Repeated games

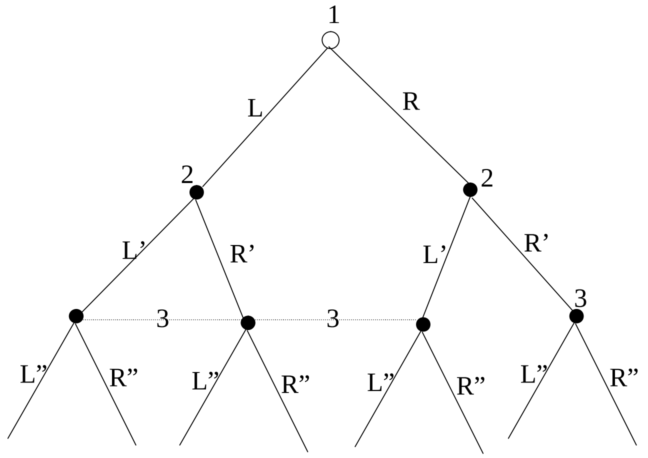
Example (cont.)



Information set

- An information set for a player is a collection of decision nodes satisfying:
  - The player has the move at every node in the information set
  - When the play of the game reaches a node in the information set, the player with the move does not know which node in the information set has (or has not) been reached

Example (cont.)

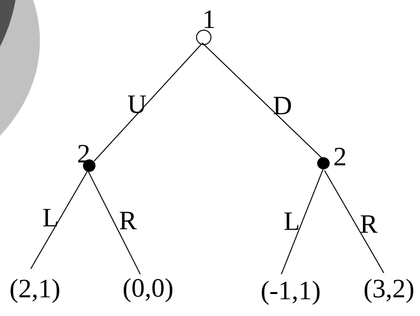


Player 2 has two information sets, both singletons.
Player 3 has two information sets, one of them is singleton.

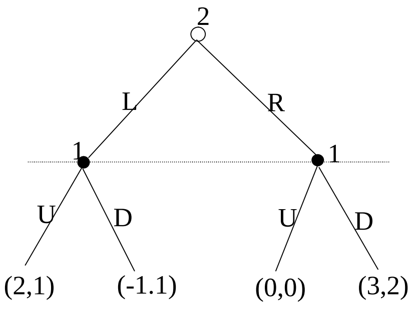
Subgame

- A ***subgame*** in an extensive form game
  - begins at a decision node *n* that is a singleton information set
  - includes all the decision and terminal nodes following *n* in the game tree (but no nodes that do not follow *n*), and
  - does not cut any information sets (i.e., if a decision node *n*' follows *n* in the game tree, then all other nodes in the information set containing *n*' must also follow *n*, and so must be included in the subgame).

Example

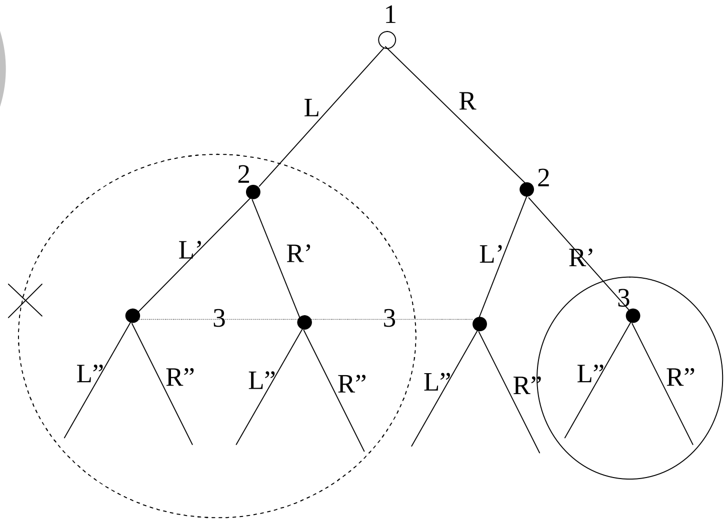


Two subgames, one beginning at each of player 2's decision nodes (+ whole game)



No subgames other than whole game

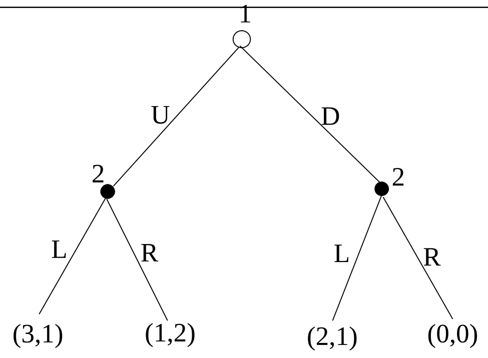
Example (cont.)



Subgame Perfection

- Definition (Selten 1965): A Nash equilibrium is subgame-perfect if the players’ strategies constitute a Nash equilibrium in every subgame.
- (Any finite dynamic game of complete information has a subgame-perfect Nash equilibrium, perhaps in mixed strategies)

Nash/subgame example



Nash/Subgame example

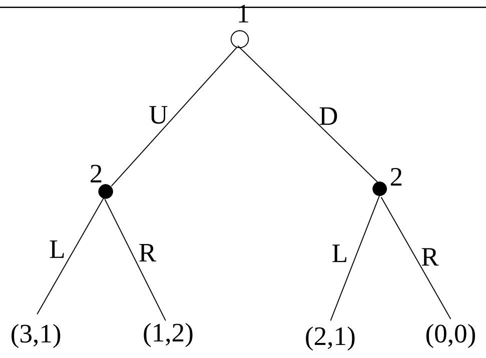
Player 2

Player 1

	(L,L)	(L,R)	(R,L)	(R,R)
U	3,1	3,1	1,2	1,2
D	2,1	0,0	2,1	0,0

- Find the pure strategy Nash Equilibrium(s):

Nash/subgame example



Find the backwards induction solution.

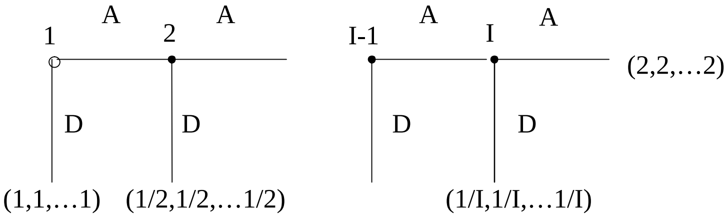
Nash/subgame example

- One Nash Equilibrium is the subgame-perfect Nash equilibrum
- The other one represents a threat or promise that is not credible
  - In which subgame is it not a Nash Equilibrium?

Dr. Strangelove

Critiques

- Backward Induction and Subgame Perfection may be less reasonable if there are multiple players or if each player moves several times
- Example: I players with decision A or D



Multistage Prisoner’s Dilemma

Single stage game:

Prisoner 2

Prisoner 1

	C (cooperate)	D (defect)
C	4, 4	0, $\underline{5}$
D	$\underline{5}$, 0	$\underline{1}$, $\underline{1}$

- The “stage game” will be repeated M times
- The payoffs of the players will be the sum of their payoffs from all stages

Two-stage Prisoner’s Dilemma

Second Stage:

Prisoner 2

Prisoner 1

	C (cooperate)	D (defect)
C	4, 4	0, $\underline{5}$
D	$\underline{5}$, 0	$\underline{1}$, $\underline{1}$

- Stage 2 equilibrium is (D,D) with payoffs (1,1) regardless of stage 1 outcome
- Given the equilibrium outcome of stage 2, update the payoffs of stage 1 and find the equilibrium

Two-stage Prisoner's Dilemma

First stage:

Prisoner 2

Prisoner 1

	C (cooperate)	D (defect)
C	5, 5	1, $\underline{6}$
D	$\underline{6}$, 1	$\underline{2}$, $\underline{2}$

- The unique equilibrium of the modified stage 1 game is also (D,D)
- The unique subgame-perfect equilibrium of the two-stage Prisoner's Dilemma game is (D,D) in the first stage followed by (D,D) in the second stage

Repeated games

- Let $G = \{A^1,...,A^n; p^1,...,p^n\}$ denote a static game of complete information in which players $1,...,n$ simultaneously choose their actions $a^1,...,a^n$ from action spaces $A^1,...,A^n$ and receive payoffs $p^1(a^1,...,a^n),...,p^n(a^1,...,a^n)$. We call G the *stage game* of the repeated game.
- Given a stage game G, let G(T) denote the *finitely repeated game* in which G is played T times, with the outcomes of all preceding plays observed before the next play begins. The payoffs for G(T) are the (discounted) sum of the payoffs from the T stage games.

Repeated games

- **Result**: If the stage game G has a unique Nash equilibrium then for any finite T, the repeated game G(T) has a unique subgame-perfect outcome: the Nash equilibrium of G is played in every stage.

Example

Player 2

Player 1

	L	M	R
L	1, 1	5, 0	0, 0
M	0, 5	4, 4	0, 0
R	0, 0	0, 0	3, 3

- The stage game is played twice
- The first-stage outcome is observed before the second stage begins

Example

Player 2

Player 1

	L	M	R
L	1, 1	5, 0	0, 0
M	0, 5	4, 4	0, 0
R	0, 0	0, 0	3, 3

- Partial strategy for stage 2:
  - Play R in stage 2 if stage 1 outcome is (M,M); otherwise, play L in stage 2.

Example

Player 2

Player 1

	L	M	R
L	2, 2	6, 1	1, 1
M	1, 6	7, 7	1, 1
R	1, 1	1, 1	4, 4

- Modified stage 1 game
- Subgame perfect outcomes:
  [(L,L),(L,L)] [(M,M),(R,R)] [(R,R),(L,L)]

Observation

- Let G be a static game of complete information with multiple Nash equilibria. There may be subgame-perfect outcomes of the repeated game G(T) in which for any $t < T$, the outcome in stage t is not a Nash equilibrium of G.

Definitions

- In the finitely repeated game G(T), a player's *strategy* specifies the player's actions in each stage, for each possible history of play through the previous stages.
- In the finitely repeated game G(T), a *subgame* beginning at stage t+1 is the repeated game in which G is played T-t times, denoted by G(T-t).

Example

Player 2

Player 1	L	M	R
L	$\underline{1}$, $\underline{1}$	$\underline{5}$, 0	0, 0
M	0, $\underline{5}$	4, 4	0, 0
R	0, 0	0, 0	$\underline{3}$, $\underline{3}$

- All possible outcomes (histories) at the end of stage 1:

  (L,L) (L,M) (L,R) (M,L) (M,M) (M,R) (R,L) (R,M) (R,R)
- (M; L, L, L, L, R, L, L, L, L)

  Play M in the first stage; Play L in the second stage unless the first stage outcome is (M,M), in which case play R