Recap

- Last class (January 11, 2007)
  - Normal form game
  - Dominant and dominated actions
  - Best response
  - Nash equilibrium
  - Mixed strategies
- Today (January 16, 2007)
  - Pareto dominance
  - Examples of games with continuous action sets
  - Duopoly models: Cournot and Bertrand

Example

- A scarce manufacturing resource is required by two departments, A and B
- $Y_j \geq 0$: quantity of the resource used by department j
- Payoff to department j from using one unit of the resource

  $200-(Y_A+Y_B)^2$
- Department j's maximization problem

  $\text{Max } Y_j[200-(Y_j+Y_{-j})^2]$
- From FOC:

  $200-(Y_j+Y_{-j})^2-2Y_j(Y_j+Y_{-j}) =$

  $200-3Y_j^2-4Y_jY_{-j}-Y_{-j}^2=0$

Example (cont.)

$200 - 3Y_A^2 - 4Y_AY_B - Y_B^2 = 0$ (1)
$200 - 3Y_B^2 - 4Y_AY_B - Y_A^2 = 0$ (2)

[(2)-(1)]/2: $2Y_A^2 - 2Y_B^2 = 0 \rightarrow Y_A=Y_B$

From (1):

$200 - 3Y_A^2 - 4Y_A^2 - Y_A^2 = 0 \rightarrow 8Y_A^2 = 200 \rightarrow$

$Y_A = Y_B = 5 \rightarrow$ Payoff for department j: 500

A solution with higher payoff?

$Y_A = Y_B = 4 \rightarrow$ Payoff for department j: 544

The Tragedy of the Commons

- Immigrant villages in New England in the 17th century
  - privately owned homesteads and gardens
  - community-owned pastures, called commons, where all of the villagers' livestock could graze
- Incentive to avoid overuse of their private lands, so they would remain productive in the future
- Result? The commons were overgrazed and degenerated to the point that they were no longer able to support the villagers' cattle
- The failure of private incentives to provide adequate maintenance of public resources is known to economists as "the tragedy of the commons."

Examples of the Tragedy of the Commons

- Congestion on urban highways
- Overpopulation
- Pollution
- The depletion of fish stocks in international waters

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Duopoly models

- Two competing firms, selling a homogeneous good
- The *marginal cost* of producing each unit of the good: c_1 and c_2
- The market price, P is determined by (inverse) market demand:
  - P=a-bQ if a>bQ, P=0 otherwise.
- Both firms seek to maximize profits
- Cournot: Firms set quantities simultaneously
- Bertrand: Firm set prices simultaneously
- Stackelberg: Firms set quantities, firm 1 followed by firm 2

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Cournot Competition

- The market price, P is determined by (inverse) market demand:
  - $P=a-bQ$ if $a>bQ$, $P=0$ otherwise.
- Each firm decides on the quantity to sell (market share): q_1 and q_2
- $Q= q_1+q_2$ total market demand
- Both firms seek to maximize profits

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Cournot Competition: Best response of Firm 1

- Suppose firm 2 produces q_2
- Firm 1's profits, if it produces q_1 are:

$$\pi_1 = (P-c_1)q_1 = [a-b(q_1+ q_2)]q_1 \quad - c_1q_1$$

= Own revenue – Cost

- How to choose q_1 to maximize π_1?
- First note that π_1 is concave: $d^2p_1/dq_1^2 = -2b <0$
- First order conditions (FOC):

$$d\, p_1/dq_1 = a - 2bq_1 - bq_2 \quad - \quad c_1$$

= Own marginal revenue – Marginal cost

$= 0 \rightarrow \quad q_1=(a-c_1)/2b – q_2/2= R_1(q_2)$

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Cournot Competition: Best response of Firm 2

- Suppose firm 1 produces q_1
- Firm 2's profits, if it produces q_2 are:

  $\pi_2 = (P-c_2)q_2 = [a-b(q_1+q_2)]q_2 \quad - c_2q_2$

  $= \text{(own) revenue} \; - \text{Cost}$
- First order conditions:

  $d\,p_2/dq_2 = a - 2bq_2 - bq_1 - c_2 =$

  $= \quad MR \quad - \quad MC = 0 \rightarrow$

  $q_2 = (a-c_2)/2b - q_1/2 = R_2(q_1)$

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Cournot Equilibrium

- $q_1 = (a-c_1)/2b - q_2/2$
- $q_2 = (a-c_2)/2b - q_1/2$
- Solving together for q_1 and q_2 :

  $q^C_1 = (a-2c_1+c_2)/3b \quad q^C_2 = (a-2c_2+c_1)/3b$
- Market demand and price:

  $Q^C = q^C_1 + q^C_2 = (2a- c_1 - c_2)/3b$

  $P = a - bQ^C = (a+c_1+c_2)/3$

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Example: Cournot Competition

- $P = 130-(q_1+q_2)$, so $a=130$, $b=1$
- $c_1 = c_2 = c = 10$
- The firms' best response functions:
  $q_1=(a-bq_2-c)/2b = (130-q_2-10)/2=60-q_2/2$
  $q_2=(a-bq_1-c)/2b = (130-q_1-10)/2=60-q_1/2$
- Solving for q_1 and q_2 :
  $q_1 = q_2 = 40$ $Q=80$ $P = 50$
- Firms' profits:
  $\pi_1 = \pi_2 = (50-10)40 = 1600$

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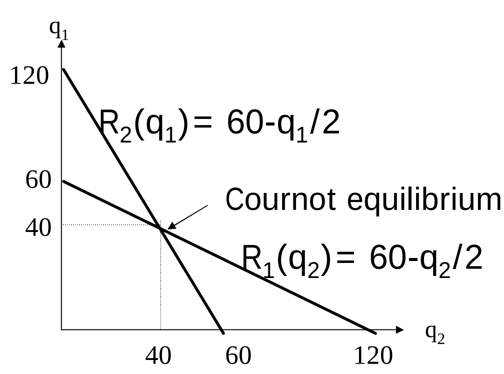
Cournot Competition: Graphical solution

- $q_1= R_1(q_2)= 60-q_2/2$ $q_2 = R_2(q_1)= 60-q_1/2$



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Cournot Equilibrium with N firms

$$\max_{q_i} \pi_i(q_i, q_{-i}) = [a - bq_i - b\sum_{j \neq i} q_j]q_i - c_i q_i$$

First order conditions:

$$a - 2bq_i - b\sum_{j \neq i} q_j - c_i = 0 \quad \forall i = 1, \ldots, N$$

Substitute $Q=\sum q_i$:

$$a - bq_i - bQ - c_i = 0 \quad \forall i = 1, \ldots, N$$

Sum over N:

$$Na - bQ - bNQ - \sum_i c_i = 0$$

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Cournot Equilibrium with N firms

$$Q^C = \frac{Na}{(N+1)b} - \frac{\sum_i c_i}{(N+1)b}$$

$$P^C = \frac{a}{N+1} + \frac{\sum_i c_i}{N+1}$$

If each firm has the same cost $c_i = c$:

$$q_i^C = \frac{Q^C}{N} = \frac{a-c}{(N+1)b} \qquad P^C = \frac{a+Nc}{N+1}$$

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Bertrand Equilibrium Model

- Firms set prices rather than quantities
  - $P=a-bQ$
- Customers buy from the firm with the cheapest price
- The market is split evenly if firms offer the same price

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Best response

- Firm 1's profit function:

  $\pi(P_1)=(P_1-c_1)\,q_1$
- To ensure $q_1>0$ (recall: $P=a-bQ$ and $Q=(a-P)/b$)

  $P_1 \leq a$
- To ensure nonnegative profits

  $P_1 \geq c_1$
- Firm 1 should choose

  $c_1 \leq P_1 \leq a$
- Similarly, firm 2 should choose

  $c_2 \leq P_2 \leq a$

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Best response (cont.)

- Firm i's demand depends on the relationship between P_1 and P_2

$$q_i = \begin{cases} 0, & \text{if } P_i > P_j \\ \dfrac{a - P_i}{b}, & \text{if } P_i < P_j \\ \dfrac{a - P}{2b}, & \text{if } P_i = P_j = P \end{cases} \qquad i = 1,2$$

- Firm 1 should choose $c_1 \leq P_1 \leq P_2$ (if possible)
- Firm 2 should choose $c_2 \leq P_2 \leq P_1$ (if possible)

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Bertrand equilibrium

- For both firms to sell positive quantities profitably

$$c_1 \leq P_1 \leq P_2 \text{ and } c_2 \leq P_2 \leq P_1$$

- Suppose $c = c_1 = c_2$

$$P = c \qquad q_1 = q_2 = (a-c)/2b$$

- Suppose $c_1 < c_2$

$$P_1 = c_2 - \varepsilon \qquad P_2 \geq c_2$$

$$q_1 = (a - c_2 + \varepsilon)/b \qquad q_2 = 0$$

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Example

- $P = 130-(q_1+q_2)$ (a=130, b=1)
- $c_1 = c_2 = c = 10$
- P=10
- $q_1 = q_2 = (a-P)/2b = 60$ Q=120
- Firms' profits:

  $\pi_1 = \pi_2 = 0$

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Quantity-setting monopolist

- Single firm (monopolist), selling a single good
- The *marginal cost* of producing each unit of the good: c
- The firm decides on the quantity to sell: Q (market demand)
- The market price, P is determined by (inverse) market demand:
  - P=a-bQ if a>bQ, P=0 otherwise.
- The firm seeks to maximize profits

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Quantity-setting monopolist

- The firm's profits, if it produces Q are:

  π = (P-c)Q = [a-bQ]Q – cQ

  = Revenue – Cost

- How to choose Q to maximize π?
- First note that π is concave: $d^2p/dQ^2 = -2b < 0$
- First order conditions (FOC):

  d p/dQ = a - 2bQ – c

  = Marginal revenue – Marginal cost

  = 0 $\rightarrow$ Q = (a-c)/2b

  P = (a+c)/2

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Example

- P = 130-Q (a=130, b=1)
- c = 10
- Q = (a-c)/2b = 60

  P = (a+c)/2 = 70

- Monopolist's profits:

  π = (70-10)60 = 3600

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Monopoly vs. Cournot vs. Bertrand

	Bertrand	Cournot	Monopoly
Price	10	50	70
Quantity	120	80	60
Total Firm Profits	0	3200	3600

- Here, firm profits and prices: Bertrand ≤ Cournot ≤ Monopoly
- Quantities: Monopoly ≤ Cournot ≤ Bertrand

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Monopoly vs. Cournot vs. Bertrand

	Bertrand	Cournot	Monopoly
Price	c	(a+2c)/3	(a+c)/2
Quantity	(a-c)/b	2(a-c)/3b	(a-c)/2b
Total Firm Profits	0	$2(a-c)^2/9b$	$(a-c)^2/4b$

- Firm profits and prices: Bertrand ≤ Cournot ≤ Monopoly

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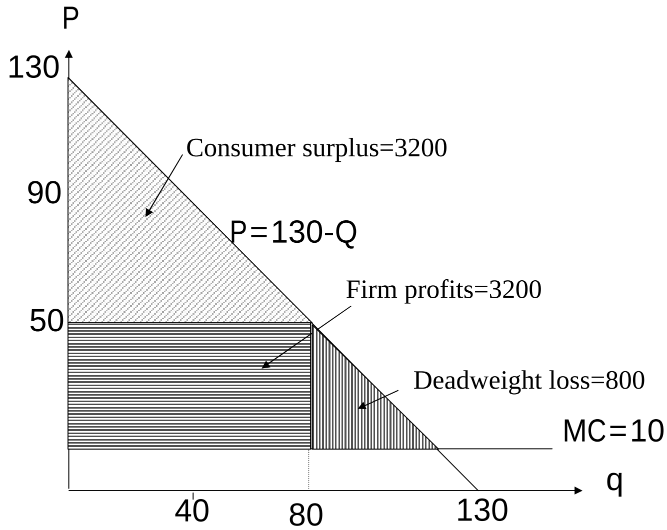
Cournot competition



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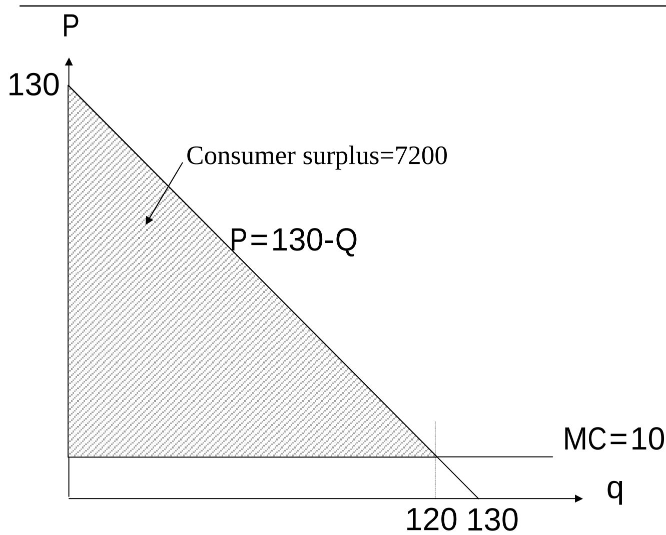
Bertrand competition



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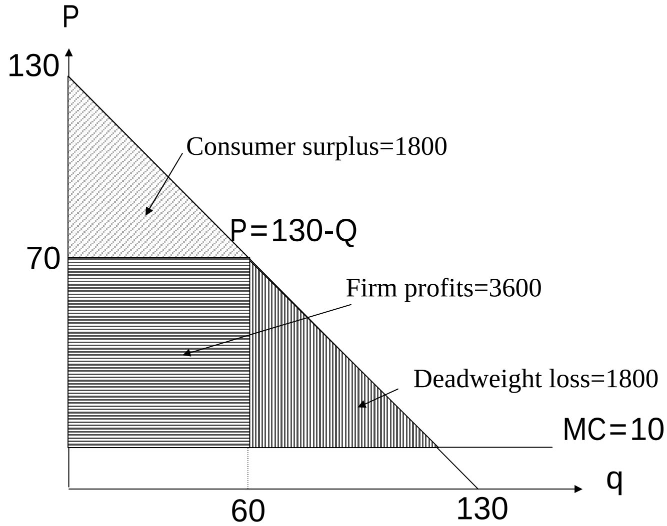
Monopoly



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Monopoly vs. Cournot vs. Bertrand

	Bertrand	Cournot	Monopoly
Consumer surplus	7200	3200	1800
Deadweight loss	0	800	1800
Total Firm Profits	0	3200	3600

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